

Unit-1: (Units & Dimensions)

Units:-

Units are standards for measurement of physical quantities that need clear definitions to be useful.

Characteristics of a unit:-

1. It should be Invariable.
2. It should be easily available for comparison with various measurements.
3. It should be convenient in size.

Measurement:-

A measurement is said to be complete, iff.

- (i) Name of the unit (U)
- (ii) How many times the unit is contained in that measurement (n)

Thus, Measurement is represented as

$$M = nU = \text{constant.}$$

Greater the size of the unit (U), smaller is its numerical value & vice versa.

Physical Quantity:-

A physical quantity is a property of a material or system that can be quantified by measurement.

A physical quantity can be expressed as the combination of a numerical value & a unit.

Ex:- Mass can be quantified as $n\text{kg}$,

where 'n' is the numerical value & 'kg' is the unit

Fundamental Quantity :-

Fundamental quantity is independent physical quantity that is not possible to express in other physical quantity.

There are seven (07) fundamental quantities.

- i) Mass
- ii) Length
- iii) Time
- (iv) Temperature
- v) Electric Current
- vi) Luminous Intensity
- vii) Amount of substance

Derived quantities :-

Derived quantities are quantities that are derived from base or fundamental quantities.

Ex:- Area, volume, speed, velocity, acceleration, force.

System of Units :-

$$UTM = M$$

a) C.G.S. system (French system/Gaussian system)

C - cm - Distance

G - Gram - Measurement of mass

S - sec - Measurement of Time.

M.K.S system (Metric system)

M - metre - Measurement of Distance

K - kilogram - Measurement of mass

S - second - Measurement of Time

Fundamental Units:-

Following seven units were defined as fundamental units for SI

(a) Fundamental Quantity

SI Unit

Mass

Kg

Length

m

sec

Time

Kelvin (K)

Temperature

Ampere (A)

Electric current

Candela (Cd)

Luminous Intensity

mole (mol)

Amount of substance

F. P. S. system:-

F - Foot - length measurement

P - Pound - measurement of mass

S - Second - measurement of Time

Derived units:-

Units of derived quantities can be expressed in terms of fundamental units.

Ex:- Area = $l \times b$

MKS

$$m \times m = m^2$$

Derived unit

CGS

$$cm \times cm = cm^2$$

Derived unit

Derived Quantities	Formula	Units		
		MKS	CGS	SI
Area	$l \times b$	m^2	cm^2	m^2
Volume	$l \times b \times h$	m^3	cm^3	m^3
Speed	$\frac{\text{Distance}}{\text{Time}}$	m/sec	cm/sec	m/sec
Velocity	$\frac{\text{Displacement}}{\text{Time}}$	m/sec	cm/sec	m/sec
Acceleration	$\frac{\text{velocity}}{\text{Time}}$	m/sec^2	cm/sec^2	m/sec^2
Force	$m \times a$	Newton (N)	Dyne	Newton
Pressure	Force/area	N/m^2	$Dyne/cm^2$	N/m^2
Work	Force $\times$ distance	Joule	Erg	Joule
Power	$\frac{\text{Work}}{\text{Time}}$	J/sec	Erg/sec	Watt
Stress	Force/area	N/m^2	$Dyne/cm^2$	N/m^2
Strain	$\frac{\text{Change in length}}{\text{Original length}}$	Unit less	Unit less	Unit less

Derived Quantities	Formula	Units		
		MKS	CGS	SI
Gravitational Force const. (G)	$\frac{\text{Force} \times (\text{distance})^2}{(\text{mass})^2}$	$\frac{\text{N} \times \text{m}^2}{\text{kg}^2}$	$\frac{\text{Dyne} \times \text{cm}^2}{\text{gm}^2}$	$\text{Nm}^2 \text{kg}^{-2}$
Impulse	Force $\times$ time	N $\times$ s	Dyne $\times$ sec	N $\times$ s
Frequency	$\frac{1}{\text{Time Period}}$	sec^{-1}	sec^{-1}	Hertz
velocity gradient	$\frac{\text{velocity}}{\text{distance}}$	sec^{-1}	sec^{-1}	sec^{-1}
Moment of force	Force $\times$ distance	N $\times$ m	Dyne $\times$ cm	N $\times$ m
Torque	Force $\times$ distance	N $\times$ m	Dyne $\times$ cm	N $\times$ m
Momentum	mass $\times$ velocity	kg $\times$ m/sec	gm $\times$ cm/sec	kg $\times$ m/sec
Wavelength (λ)	$\frac{\text{velocity}}{\text{Frequency}}$	m	cm	m
g	Acceleration	m/sec^2	cm/sec^2	m/sec^2
Density (ρ)	$\frac{\text{mass}}{\text{Volume}}$	kg/m^3	gm/cm^3	kg/m^3

Dimensions:-

Dimensions of a physical quantity are the powers to which the fundamental units be raised in order to represent that quantity.

Dimensional equation:-

Dimensional formula of a physical quantity is the formula which tells us how & which of the fundamental units have been used for the measurement of that quantity.

Dimensional formula of any physical quantity is written as :- $[M^a L^b T^c]$, where a, b & c are the dimensions of mass, length & time

Momentum = mass $\times$ velocity

$$= [M] \times [L \bar{T}^{-1}]$$

$$= [M L \bar{T}^{-1}]$$

Strain = $\frac{\text{change in length}}{\text{original length}}$

$$= \frac{[L]}{[L]} \text{ (dimensionless quantity)}$$

Frequency = $\frac{1}{\text{Time Period}}$

$$= \frac{1}{[T]} = [\bar{T}^{-1}]$$

Angle = $\frac{\text{arc}}{\text{radius}} = \frac{[L]}{[L]} \text{ (dimless quantity)}$

Torque = Force $\times$ distance

$$= [M L \bar{T}^{-2}] \cdot [L]$$

$$= [M L^2 \bar{T}^{-2}]$$

$g \rightarrow$ Acceleration due to gravity

= Acceleration

$$[g] = [L \bar{T}^{-2}]$$

$G \rightarrow$ gravitational force constant

$$= \frac{\text{Force} \times (\text{distance})^2}{(\text{mass})^2} = \frac{[M L \bar{T}^{-2}] \times [L]^2}{[M]^2}$$

$$[G] = [M^{-1} L^3 \bar{T}^{-2}]$$

Application of Dimensional Analysis:-

1. To check the correctness of a given relation.

To check the correctness of a given relation, we find the dimensional formula of every term on either side of the relation. If the dimensions are identical, then the relation is said to be correct.

Principle of Homogeneity:-

It states that "the dimensional formulae of every term on the two sides of a correct relation must be same."

Ex:- $V = u + at$ is correct iff

$$[V] = [u] = [at]$$

Q. Check the correctness of

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \text{by using dimensional analysis}$$

Ans:-

Dim. Formula of $T = [T']$

" " $l = [L']$

" " $g = [L' T'^{-2}]$

LHS

Dim formula of

$$T = [T']$$

RHS

Dim. Formula of

$$2\pi \sqrt{\frac{l}{g}} = \sqrt{\frac{[L']}{[L' T'^{-2}]}}$$

$$= \sqrt{\frac{L}{[T^2]}} = [T']$$

So LHS = RHS.

According to principle of homogeneity, the above relation is correct dimensionally.

MCQ

- (1) Write the SI unit of Frequency.
- (2) Name the physical quantities which are dimensionally similar.
- (3) Name the physical quantities which neither possess unit nor the dimensions.
- (4) Name the physical quantities which possess unit but no dimension.
- (5) Find the odd one
 - a) Current
 - b) Temperature
 - c) Mass
 - d) Volume

Unit-2: (Scalars and Vectors)

Scalar quantities:-

Scalar quantities are those quantities which possess only magnitude for their complete specification.

Ex:- Mass, length, time, temperature, electric current, Volume, density, energy, work, electric potential etc..

Vector quantities:-

Vector quantities are the quantities which possess both magnitude & direction for their complete specification.

Ex:- Displacement, weight, velocity, acceleration, force, momentum, torque, g , electric intensity etc...

Representation of a vector:-

A vector can be represented by an "arrow" like $\vec{x}$.

Types of Vectors:-

① Equal Vectors:-

Two vectors are said to be equal if they possess the same magnitude & direction.

$$\vec{A} = \vec{B} \text{ iff}$$

$$|\vec{A}| = |\vec{B}| \text{ \& with same}$$

direction i.e., $(\theta = 0^\circ)$



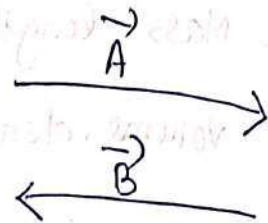
② Negative Vectors:-

A vector is said to be the negative vector of other, if it has same magnitude & is directed in opposite direction.

$$\vec{A} = -\vec{B}$$

iff $|\vec{A}| = |\vec{B}|$ & is in

opposite direction.



③ Null vector ($\vec{0}$):-

It is a vector having zero magnitude & arbitrary direction.

Properties of a null vector:-

(i) It has zero magnitude.

(ii) It has arbitrary direction.

(iii) It is represented by a point/dot.

(iv) When a null vector is added or subtracted from a given vector, the resultant vector is same as the given vector.

④ Collinear Vectors:-

There are two types of collinear vectors

a) Parallel vectors ($\theta = 0^\circ$)

Two vectors (magnitude may or may not be equal) acting along same direction are called parallel vectors. ($\theta = 0^\circ$).

(b) Anti-parallel vectors ($\theta = 180^\circ$)

Two vectors which are directed in opposite directions are called anti-parallel vectors.

$$(\theta = 180^\circ)$$

Addition of Vectors :

Vectors can be added geometrically ~~but~~ not algebraically.

Triangle's law of vector addition

Statement :-

If two vectors are represented by the two sides of a triangle, taken in the same order, then their resultant is represented by the third side of the triangle taken in the opposite order.

Mathematically,

$$\vec{A} + \vec{B} = \vec{R}$$

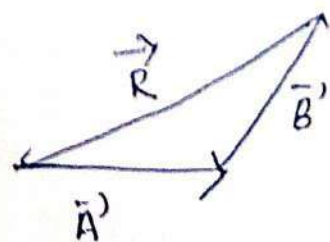
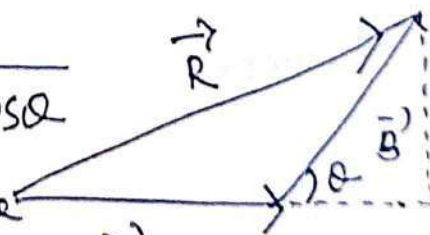
$$|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$$

If the resultant is in the same order with $\vec{A}$ & $\vec{B}$, then

$$\vec{A} + \vec{B} = -\vec{R}$$

$$\Rightarrow \boxed{\vec{A} + \vec{B} + \vec{R} = 0}$$

Condition of equilibrium



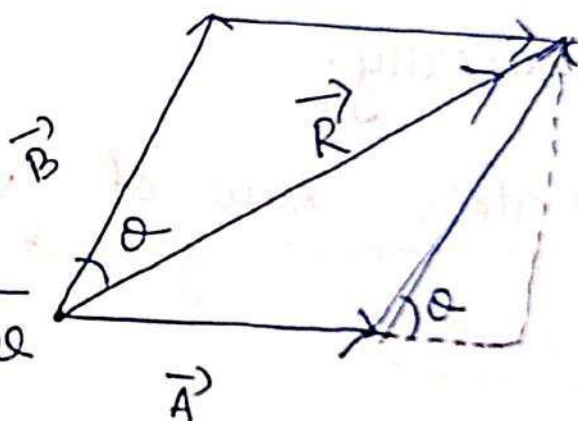
Parallelogram's law of vector addition:-

Statement:-

If two vectors acting simultaneously, at a point are represented by the two sides of a parallelogram, their resultant is given by the diagonal of the parallelogram passing through that point.

$$\text{ie, } \vec{A} + \vec{B} = \vec{R}$$

$$|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$$



Equilibrant:-

- * A vector having same magnitude & opposite direction to that of the resultant.
- * Negative vector of the resultant of a number of vectors.

Resolution of Vectors in a plane:-

Splitting of a vector into its no. of components.

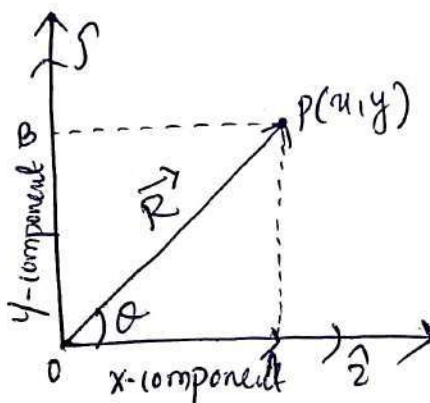
Resolution of vectors is the process of obtaining the component vectors which when combined, according to laws of vector addition, produce the given vector.

$$\vec{OP} = \vec{R}$$

OA = x-component

OB = y-component

$$\text{So } \boxed{\vec{R} = \vec{OA} + \vec{OB}}$$



OA & OB are the Rectangular components of $\vec{R}$.

where $\hat{i}$ & $\hat{j}$ are called unit vectors along x-axis & y-axis.

$$\& |\hat{i}| = |\hat{j}| = 1$$

Let $\vec{R}$ makes an angle ' θ ' with the x-axis,

$$\text{then, x-component} = R \cos \theta$$

$$\text{y-component} = R \sin \theta$$

$$\text{So } \boxed{\vec{R} = (R \cos \theta) \hat{i} + (R \sin \theta) \hat{j}}$$

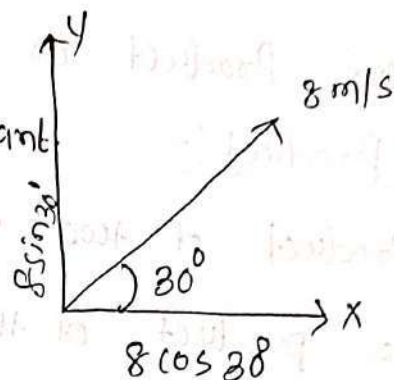
So $\vec{R}$ is resolved into two components as $R \cos \theta$

& $R \sin \theta$ in xy plane.

Numerical

1. Find the rectangular components of a velocity of 8 m/s when one of the components makes an angle of 30° with the resultant.

Solⁿ Let x -component makes an angle 30° with the resultant.



$$\begin{aligned}\text{So } x\text{-component} &= 8 \cos 30^\circ \\ &= 8 \times \frac{\sqrt{3}}{2} \\ &= 4\sqrt{3} \text{ m/s}\end{aligned}$$

$$y\text{-component} = 8 \sin 30^\circ = 8 \times \frac{1}{2} = 4 \text{ m/s}$$

So, the rectangular components of 8 m/s are $4\sqrt{3} \text{ m/s}$ & 4 m/s .

2. One of the rectangular components of a force of 65 N is 25 N . Find the other component.

Solⁿ Given $|\vec{R}| = 65 \text{ N}$, $|\vec{A}| = 25 \text{ N}$.

$$|\vec{B}| = ?$$

$$|\vec{R}|^2 = |\vec{A}|^2 + |\vec{B}|^2$$

$$\Rightarrow |\vec{B}|^2 = |\vec{R}|^2 - |\vec{A}|^2$$

$$\begin{aligned}\Rightarrow |\vec{B}| &= \sqrt{(65)^2 - (25)^2} \\ &= 60 \text{ N}.\end{aligned}$$

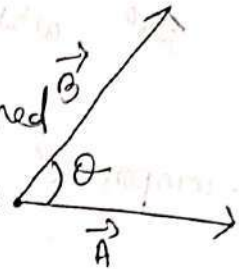
Product of Two Vectors:-

There are two ways in which two vectors can be multiplied together.

1. Dot Product or Scalar Product
2. Cross Product or Vector Product.

Dot Product:-

Dot Product of two vectors is defined as the product of their magnitudes & the cosine of the smaller angle between the two.



$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

*

The dot product between two vectors gives a scalar quantity.

Properties of dot Product:-

1. Commutative

Dot Product between two vectors is commutative in nature.

$$\text{ie, } \boxed{\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}}$$

2. Distributive

$$\vec{A} \cdot (\vec{B} + \vec{C} + \vec{D} + \dots) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} + \vec{A} \cdot \vec{D} + \dots$$

3) ^{imp} Perpendicular Vectors

For two perpendicular vectors $|\vec{A}|$ & $|\vec{B}|$, $\theta = 90^\circ$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 90^\circ$$

$$\boxed{\vec{A} \cdot \vec{B} = 0} \text{ condition of Perpendicularity.}$$

NB
If two vectors are perpendicular, then dot product between them is zero.

Since $\hat{i}, \hat{j}$ & $\hat{k}$ are mutually perpendicular,

$$\text{So } \hat{i} \cdot \hat{j} = 0 = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i}$$

4. Equal Vectors

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

Two vectors are equal if they possess same magnitude & direction.

$$\text{ie, } \vec{A} = \vec{B} \text{ if } |\vec{A}| = |\vec{B}| \text{ \& } \theta = 0^\circ$$

$$\text{So } \vec{A} \cdot \vec{A} = |\vec{A}| |\vec{A}| \cos 0^\circ$$

$$\boxed{\vec{A} \cdot \vec{A} = |\vec{A}|^2}$$

Dot Product of two equal vectors is equal to the square of the magnitude of the either.

In case of unit vectors

$$\hat{i} \cdot \hat{i} = |\hat{i}|^2 = 1$$

$$\hat{j} \cdot \hat{j} = |\hat{j}|^2 = 1$$

$$\hat{k} \cdot \hat{k} = |\hat{k}|^2 = 1$$

5. Collinear Vectors

(a) Parallel vectors ($\theta = 0^\circ$) (b) Anti-parallel vector ($\theta = 180^\circ$)

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$= |\vec{A}| |\vec{B}| \cos 0$$

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}|}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 180$$

$$\boxed{\vec{A} \cdot \vec{B} = -|\vec{A}| |\vec{B}|}$$

6. Dot Product in terms of rectangular components:-

$$\text{Let } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

where A_x, A_y, A_z & B_x, B_y, B_z are the rectangular components of $\vec{A}$ & $\vec{B}$.

$$\begin{aligned}\vec{A} \cdot \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= A_x B_x (\hat{i} \cdot \hat{i}) + A_x B_y (\hat{i} \cdot \hat{j}) + A_x B_z (\hat{i} \cdot \hat{k}) \\ &\quad + A_y B_x (\hat{j} \cdot \hat{i}) + A_y B_y (\hat{j} \cdot \hat{j}) + A_y B_z (\hat{j} \cdot \hat{k}) \\ &\quad + A_z B_x (\hat{k} \cdot \hat{i}) + A_z B_y (\hat{k} \cdot \hat{j}) + A_z B_z (\hat{k} \cdot \hat{k})\end{aligned}$$

AT condition of perpendicularity,

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0, \hat{i} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0, \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = 0$$

AT condition of collinearity

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\text{So } \boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

Formula

1. To find the magnitude of any vector

$$\text{If } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\text{then } |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

2. To determine the angle between two vectors.

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \quad \text{So } \boxed{\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)}$$

Cross Product / Vector Product :-

Cross Product of two vectors $\vec{A}$ & $\vec{B}$ is defined as

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \hat{n}$$

where $\hat{n}$ is the unit vector in a direction $\perp$ to the plane containing $\vec{A}$ & $\vec{B}$.

NB

Cross Product between two vectors gives a Vector quantity.

Properties of Cross Product :-

1. Not commutative

Cross Product of two vectors is not commutative in nature.

$$\text{ie, } \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\Rightarrow \vec{A} \times \vec{B} = - \vec{B} \times \vec{A}$$

2. Distributive

$$\vec{A} \times (\vec{B} + \vec{C} + \vec{D} + \dots) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} + \vec{A} \times \vec{D} + \dots$$

3. Perpendicular Vectors

For two perpendicular vectors, $\theta = 90^\circ$.

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin 90^\circ \hat{n}$$

$$= |\vec{A}| |\vec{B}| \hat{n}$$

$$\text{so } |\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}|$$

In case of orthogonal unit vectors,

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

4. Equal Vectors

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \hat{n}$$

For equal vectors, $\vec{A} = \vec{B}$, $|\vec{A}| = |\vec{B}|$ & $\theta = 0^\circ$

$$\begin{aligned}\text{So } \vec{A} \times \vec{A} &= |\vec{A}| |\vec{A}| \sin 0 \hat{n} \\ &= 0 \hat{n} \\ &= \vec{0} \text{ (Null Vector)}\end{aligned}$$

Cross Product of two equal vectors is always a null vector.

$$\text{So } |\hat{i} \times \hat{i}| = 0, |\hat{j} \times \hat{j}| = 0 \text{ \& } |\hat{k} \times \hat{k}| = 0$$

5) Collinear Vectors

a) Parallel Vectors:-

For two parallel vectors, $\theta = 0^\circ$

$$\begin{aligned}\vec{A} \times \vec{B} &= |\vec{A}| |\vec{B}| \sin 0 \hat{n} \\ &= 0 \hat{n} \\ &= \vec{0} \text{ (Null Vector)}\end{aligned}$$

b) Anti-parallel Vectors:-

$$\theta = 180^\circ$$

$$\begin{aligned}\vec{A} \times \vec{B} &= |\vec{A}| |\vec{B}| \sin 180 \hat{n} \\ &= 0 \hat{n} \\ &= \vec{0} \text{ (Null Vector)}\end{aligned}$$

$\therefore$ Cross Product of two collinear vectors is always a null vector.

6) Cross Product in terms of Rectangular components :-

Let A_x, A_y, A_z & B_x, B_y, B_z be the rectangular components of $\vec{A}$ & $\vec{B}$

Then, $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ &

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= A_x B_x (\hat{i} \times \hat{i}) + A_x B_y (\hat{i} \times \hat{j}) + A_x B_z (\hat{i} \times \hat{k}) \\ &\quad + A_y B_x (\hat{j} \times \hat{i}) + A_y B_y (\hat{j} \times \hat{j}) + A_y B_z (\hat{j} \times \hat{k}) \\ &\quad + A_z B_x (\hat{k} \times \hat{i}) + A_z B_y (\hat{k} \times \hat{j}) + A_z B_z (\hat{k} \times \hat{k}) \end{aligned}$$

$$= A_x B_y \hat{k} - A_x B_y \hat{j} - A_y B_x \hat{k} + A_y B_z \hat{i} + A_z B_x \hat{j} - A_z B_y \hat{i}$$

$$= \hat{i} (A_y B_z - A_z B_y) - \hat{j} (A_x B_y - A_z B_x) + \hat{k} (A_x B_y - A_y B_x)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Differentiate between dot Product & Cross Product

Dot Product

1. Definition

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

2. Dot Product between two vectors gives a scalar quantity.

3. Commutative

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

4. For two perpendicular vectors

$$\boxed{\vec{A} \cdot \vec{B} = 0}$$

5. In case of orthogonal unit vectors, $\hat{i} \cdot \hat{j} = 0 = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i}$

6. For equal vectors

$$\vec{A} \cdot \vec{A} = |\vec{A}|^2$$

In case of orthogonal unit vectors

$$\hat{i} \cdot \hat{i} = 1 = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k}$$

7. In terms of rectangular components,

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Cross Product

1. Definition

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \hat{n}$$

2. Cross Product between two vectors gives a vector quantity.

3. Not commutative

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\text{ie } \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

4. For two perpendicular vectors

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}|$$

5. In case of orthogonal unit

$$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

6. For equal vectors,

$$\vec{A} \times \vec{A} = 0$$

In case of orthogonal unit vectors

$$\hat{i} \times \hat{i} = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$$

7. In terms of rectangular components,

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Numericals

1. Given $\vec{A} = \hat{i} - 2\hat{j} - 5\hat{k}$ & $\vec{B} = 2\hat{i} + \hat{j} - 4\hat{k}$

a) Find $\vec{A} \cdot \vec{B}$ b) Find $|\vec{A}|$ & $|\vec{B}|$

c) Find the angle between (d) Find $\vec{A} \times \vec{B}$

them

Solⁿ
1a) Given $\vec{A} = \hat{i} - 2\hat{j} - 5\hat{k}$, $\vec{B} = 2\hat{i} + \hat{j} - 4\hat{k}$
So $A_x = 1, A_y = -2, A_z = -5, B_x = 2, B_y = 1, B_z = -4$

So $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
 $= 1 \times 2 + (-2)(1) + (-5)(-4)$
 $= 2 - 2 + 20$

$$\boxed{\vec{A} \cdot \vec{B} = 20}$$

(b) $|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2} = \sqrt{1^2 + (-2)^2 + (-5)^2}$
 $= \sqrt{1 + 4 + 25}$

$\therefore |\vec{A}| = \sqrt{30}$ $|\vec{A}| = \sqrt{30}$

$$|\vec{B}| = \sqrt{(B_x)^2 + (B_y)^2 + (B_z)^2}$$

$$= \sqrt{2^2 + 1^2 + (-4)^2}$$

$$= \sqrt{4 + 1 + 16}$$

$$|\vec{B}| = \sqrt{21}$$

(c) To find the angle (θ) between $\vec{A}$ & $\vec{B}$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{20}{\sqrt{30} \sqrt{2}}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{20}{\sqrt{30} \sqrt{2}} \right) =$$

$$\boxed{\theta = 37.17^\circ}$$

(d)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -5 \\ 2 & 1 & -4 \end{vmatrix}$$

$$= \hat{i}(8 - (-5)) - \hat{j}(-4 - (-10)) + \hat{k}(1 - (-4))$$

$$\vec{A} \times \vec{B} = 13\hat{i} - 6\hat{j} + 5\hat{k}$$

(2) For what value of ' λ ' two vectors

$2\hat{i} + \lambda\hat{j} + \hat{k}$ & $\hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular?

Sol

$$\text{Let } \vec{A} = 2\hat{i} + \lambda\hat{j} + \hat{k} \text{ & } \vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$$

Since $\vec{A}$ is $\perp$ to $\vec{B}$, then

$$\Rightarrow 5 - 2\lambda = 0$$

$$\vec{A} \cdot \vec{B} = 0$$

$$\Rightarrow 2\lambda = 5$$

$$\Rightarrow A_x B_x + A_y B_y + A_z B_z = 0$$

$$\Rightarrow 2 \times 1 + \lambda(-2) + 1 \times 3 = 0$$

$$\Rightarrow 2 - 2\lambda + 3 = 0$$

$$\Rightarrow \boxed{\lambda = \frac{5}{2}}$$

Ans

(3) Are the two vectors represented by $2\hat{i} + 4\hat{j} + 2\hat{k}$ & $\hat{i} + 2\hat{j} - 5\hat{k}$ perpendicular to each other.

Ans $\vec{A} = 2\hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - 5\hat{k}$

$$A_x = 2, A_y = 4, A_z = 2, B_x = 1, B_y = 2, B_z = -5$$

$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z \\ &= 2 \times 1 + 4 \times 2 + 2(-5) \\ &= 2 + 8 - 10\end{aligned}$$

$$\boxed{\vec{A} \cdot \vec{B} = 0}$$

Since $\vec{A} \cdot \vec{B} = 0$, $\vec{B} \cdot \vec{A} = 0$ (By commutative law)
So two vectors are mutually $\perp$ to each other.

Assignment

- ① State Triangle's law of vector addition.
- ② Write the condition of perpendicularity.
- ③ It is found that for two vectors $\vec{A}$ & $\vec{B}$
 $\vec{A} \cdot \vec{B} = -AB$. What is the angle between $\vec{A}$ & $\vec{B}$?
- (4) $\vec{A} = a\hat{i} + 2\hat{j} + 3\hat{k}$ & $\vec{B} = 2\hat{i} - 3\hat{j} + 4\hat{k}$.
Find the value of 'a' such that $\vec{A}$ is $\perp$ to $\vec{B}$.
- (5) If $\vec{A} \cdot \vec{B} = \vec{A} \times \vec{B}$, then what is the angle between $\vec{A}$ & $\vec{B}$?
- (6) The minimum number of unequal forces whose vector sum can be zero is —
a) 2 b) 5
c) 3 d) 4
- (7) The magnitude of resultant of two equal forces equal to the magnitude of either force. Find the angle between them.
- (8) Which of the following quantities is a scalar?
a) Velocity b) Momentum
c) Kinetic energy d) Acceleration
- (9) What is the component of a force of 10N in a direction perpendicular to it?
- (10) Two forces having magnitudes F_1 & F_2 act simultaneously on a particle. If the forces are at right angle to each other, then their resultant is —
a) $F_1 + F_2$ b) $F_1 - F_2$ c) $F_1^2 + F_2^2$ d) $\sqrt{F_1^2 + F_2^2}$

UNIT-3 : (KINEMATICS)

Rest Body:-

A body is said to be at rest if it does not change its position w.r.t. displacement.

Motion:-

A body is said to be in motion if it changes its position w.r.t. time.

Displacement:-

- * It is a vector quantity.
- * The displacement can have positive, negative or zero value.
- * The displacement is never greater than the actual distance travelled.
- * The displacement has unit of length.
- * The dimensional formula of displacement is $[L]$

Speed:-

It is the distance covered by the body in one sec.

$$\therefore \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Dim. Formula} = [L T^{-1}]$$

Units	
<u>MKS</u>	<u>CGS</u>
m/sec	cm/sec.

Velocity:-

It is the rate of change of displacement.

$$\therefore \text{Velocity} = \frac{\text{change in displacement}}{\text{Time}}$$

* It is a vector quantity.

* Dimensional formula = $[L^1 T^{-1}]$

* Units

MKS

m/sec

CGS

cm/sec

Uniform Velocity:-

Velocity of a body is said to be uniform if it covers equal displacement in equal intervals of time.

Acceleration (a):-

It is defined as the rate of change of velocity.

$$\therefore \text{Acceleration (a)} = \frac{\text{change in Velocity}}{\text{Time}}$$

* It is a vector quantity.

* Dim. Formula = $[L^1 T^{-2}]$

* Unit

MKS

(m/sec²)

CGS

(cm/sec²)

Force ! -

Force is the basic cause of motion.

If we want to produce motion in a body already at rest or if we want to destroy the motion of a moving body, we have to apply 'effort' known as Force.

Formula

$$\text{Force (F)} = \text{mass (m)} \times \text{acceleration (a)}$$

Units

(1) MKS : Newton

(2) CGS : Dyne

(3) SI : Newton

$$1 \text{ N} = 10^5 \text{ Dyne}$$

Newton's laws of motion:-

1. Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to change the state.
2. The rate of change of momentum of a body is directly proportional to the force & takes place in the direction of force.
3. To every action, there is an equal & opposite reaction.

Equation of motion under gravity:-

1. Velocity - time relation

$$V = u + gt, \text{ where } u \rightarrow \text{Initial velocity}$$
$$V \rightarrow \text{Final velocity}$$

2. Displacement - time relation

$$S = ut + \frac{1}{2}gt^2$$

3. Velocity displacement relation.

$$V^2 = u^2 + 2as$$

Numericals

Q.1 A body starts from rest & acquires a velocity of 12 m/s in 5 sec. Calculate the acceleration & the distance travelled.

Ans:- Given data, $u = 0$

$$V = 12 \text{ m/sec}$$

$$t = 5 \text{ sec.}$$

$$a = ?$$

$$s = ?$$

$$V = u + at \Rightarrow a = \frac{V - u}{t} = \frac{12 - 0}{5}$$

$$a = 2.4 \text{ m/s}^2$$

$$S = ut + \frac{1}{2}at^2$$

$$= 0 \times 5 + \frac{1}{2} \times 2.4 \times (5)^2$$

$$S = 30 \text{ m}$$

Home Work

- ① The velocity of a car increases from 26 km/hr to 54 km/hr in 1 min . Find the acceleration & distance travelled by the car.
- ② The velocity of a body increases, at a constant rate from 10 m/s to 25 m/s in 6 min . Find the distance & the acceleration travelled.

CIRCULAR MOTION:

A body is said to move in circular motion if its distance from a fixed point always remain constant.

The path of a particle undergoing rotational motion is a Circle.

The fixed point is called the Centre & the distance is called radius of the circle.

Angular displacement ($\vec{\theta}$):-

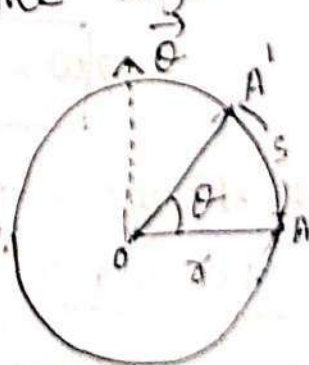
Angular displacement is the angle turned by its radius vector.

As the particle displaces from A to A' with linear displacement s .

the radius vector shifts from OA to OA' by making an angle θ .

* It is a vector quantity.

* It is directed along the axis of rotation.



Relation between linear displacement ($\vec{s}$) & Angular displacement

We know that, angle (θ) = $\frac{\text{Arc}}{\text{radius}}$

From $\triangle OAA'$, angle (θ) = $\frac{AA'}{OA}$

$$\Rightarrow \theta = \frac{s}{r}$$

$$\text{or } s = r\theta \quad \text{scalar form}$$

$\therefore$ Linear displacement = radius $\times$ angular displacement

Vector form :-

$$\vec{s} = \vec{\omega} \times \vec{r}$$

Angular Velocity ($\vec{\omega}$) :-

It is defined as the rate of change of angular displacement.

$$\therefore \text{Angular velocity } (\omega) = \frac{\text{change in ang. displacement}}{\text{Time}}$$

$$\Rightarrow \omega = \frac{\theta_2 - \theta_1}{T}$$

Unit of ' ω ' :- rad/sec.

Dim. Formula :- $[T^{-1}]$

Relation between linear velocity ($\vec{v}$) & angular velocity ($\vec{\omega}$)

We know that, linear velocity (v) = $\frac{\text{Linear displacement}}{\text{Time}}$

$$\Rightarrow v = \frac{s}{T}$$

$$= \frac{r\theta}{T}$$

($\because s = r\theta$)

using the

$$V = r \left(\frac{\theta}{t} \right)$$

$$\boxed{V = r \omega} \quad \text{Scalar form}$$

$$\boxed{\text{Linear Velocity} = \text{radius} \times \text{angular velocity}}$$

Vector form

Since $\vec{r}$, $\vec{v}$ & $\vec{\omega}$ are all vector quantities

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

Angular acceleration (α) :- $\vec{\omega} + \vec{r} \times \vec{\alpha} = \frac{d\vec{v}}{dt}$

It is defined as the rate of change of its angular velocity.

$\therefore$ Angular acceleration (α) = $\frac{\text{change in angular velocity}}{\text{Time}}$

$$\Rightarrow \boxed{\alpha = \frac{\omega}{t} = \frac{\theta}{t^2}}$$

Unit of ' α ' :- rad/sec^2

Dim. Formula of $[\alpha] = [T^{-2}]$

It is a vector quantity.

Relation between linear accelⁿ (a) & Ang. accelⁿ (α)

We know that, linear accelⁿ (a) = $\frac{\text{Linear velocity}}{\text{Time}}$

$$\Rightarrow a = \frac{v}{t}$$

$$= \frac{r\omega}{t} \quad (\because v = r\omega \text{ where } \omega = \text{ang. velocity})$$

$$= r \left(\frac{\omega}{t} \right) \quad (\because \frac{\omega}{t} = \alpha \text{ by definition})$$

$$\boxed{a = r\alpha}$$

Vector form:-

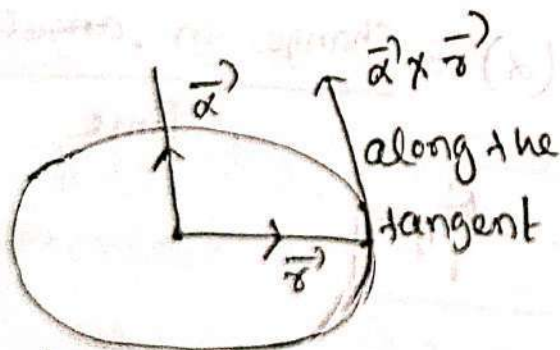
We know that, $\vec{v} = \vec{\omega} \times \vec{r}$

$$\therefore \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} (\vec{\omega} \times \vec{r})$$
$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

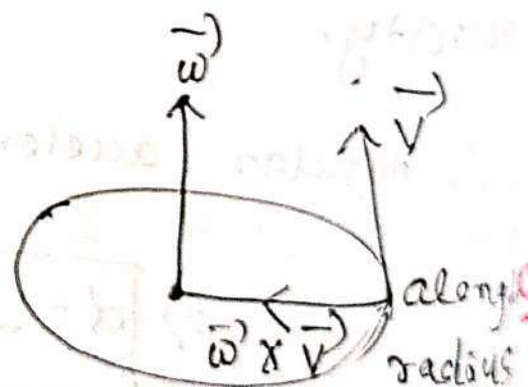
but $\frac{d\vec{\omega}}{dt} = \vec{\alpha}$ & $\frac{d\vec{r}}{dt} = \vec{v}$

So $\boxed{\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}}$

Tangential component Radial component



(Tangential component)



(Radial component)
(towards the centre)

Time Period (T) :-

It is the time taken by the particle to complete one rotation.

$$\boxed{T = \frac{2\pi}{\omega}}$$

Relation between time period & angular velocity.

Solved Numericals

Q.1 A car moves in a circular path of radius 2m with a velocity 4 m/sec. Find the angular velocity.

Solⁿ Given data,

$$r = 2\text{m}$$

$$V = 4\text{m/sec}$$

$$\omega = ?$$

Using the relation, $V = r\omega$

$$\omega = \frac{V}{r}$$

$$= \frac{4\text{m/sec}}{2\text{m}}$$

$$\boxed{\omega = 2\text{sec}^{-1}} \quad \underline{\text{Ans.}}$$

Q.2 The angular velocity of a body changes from 4π rad/sec to 5π rad/sec in 10sec. What is its angular acceleration.

Solⁿ Given $\omega_1 = 4\pi$ rad/sec

$$\omega_2 = 5\pi \text{ rad/sec}$$

$$t = 10 \text{ sec.}$$

$$\alpha = ?$$

$$\alpha = \frac{\omega_2 - \omega_1}{T} = \frac{5\pi - 4\pi}{10} = \left(\frac{\pi}{10}\right) \text{ rad/sec}^2$$

$$\therefore \boxed{\alpha = \frac{\pi}{10} \text{ rad/sec}^2} \quad \underline{\text{Ans.}}$$

PROJECTILE MOTION:-

Projectile:-

A projectile is any object that once projected continues in motion by its own inertia & is influenced only by the downward force of gravity.

Examples:-

1. A cricket ball thrown into space
2. A small bag dropped from an aeroplane
3. A bullet fired from a rifle.

Facts about Projectiles

1. Every Projectile experiences only one signal force that is due to gravity only.
2. Horizontal velocity of a projectile remains the same throughout its flight.
3. Horizontal acceleration $(a) = 0$ & vertical acceleration $a = -g \text{ m/s}^2$
4. The path of projectile is Parabolic when the projectile is fired horizontally
5. The path is a straight line when the projectile is fired vertically.
6. The horizontal motion & vertical motion of Projectile are independent of each other.

Projectile fired at an angle ' θ ' with the horizontal

ong
ype

Find the a) Equation of Trajectory

b) Maximum height

c) Time of ascent (t)

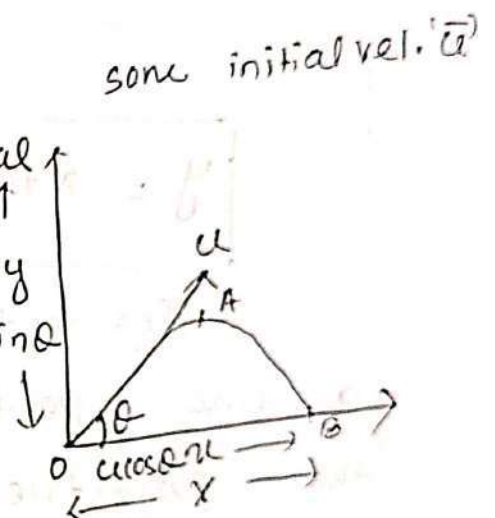
d) Total time of flight (T)

e) Horizontal range

when the projectile is fired at an angle ' θ ' with the horizontal.

Ans:-

Consider a Projectile fired with some initial vel. ' u ' at an angle ' θ ' with the horizontal rises to the highest Point 'A' & falls back at B lying on the level of Projection.



$\vec{u}$ is resolved into two components

Horizontal initial velocity = $u \cos \theta$ (uniform)

Vertical initial velocity = $u \sin \theta$ (non-uniform)

a) Equation of Trajectory

Horizontal eqn of motion:-

horizontal accelⁿ $a = 0$

horizontal initial velocity $u = u \cos \theta$

distance travelled = x

$$\therefore \boxed{x = u \cos \theta \cdot t}$$

Vertical eqn of motion:-

Vertical acceleration $a = -g$

Vertical initial vel. $u = u \sin \theta$

Vertical distance = y

$$\therefore y = u \sin \theta t - \frac{1}{2} g t^2$$

$$= u \sin \theta \left(\frac{x}{u \cos \theta} \right) - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

$$\left(\because x = u \cos \theta t \Rightarrow t = \frac{x}{u \cos \theta} \right)$$

$$\boxed{y = x \tan \theta - \frac{g}{2 u^2 \cos^2 \theta} x^2}$$

This is the eqn of parabola symmetry abt a line parallel to y-axis. Hence the motion of the projectile is parabolic.

b) Maximum height:-

It is the maximum distance travelled by the projectile in vertical direction.

Considering the motion of the projectile in vertical direction only.

At O, initial vertical velocity $u = u \sin \theta$

At A, Final " " " $v = 0$

Vertical accelⁿ $a = -g$

Using $v^2 = u^2 + 2as$ eqn of motion

$$0 = u^2 \sin^2 \theta - 2gs$$

$$\Rightarrow \boxed{s = \frac{u^2 \sin^2 \theta}{2g}}$$

c) Time of ascent (t) :-

It is the time taken by the projectile to reach the highest point.

Considering the motion in vertical direction only.

At O, initial vertical velocity $u = u \sin \theta$

At A, Final " " " " $v = 0$

vertical accelⁿ $a = -g$

using $v = u + at$ eqⁿ of motion

$$0 = u \sin \theta - gt$$

$$\Rightarrow t = \frac{u \sin \theta}{g}$$

(d) Total time of flight (T) :-

It is the total time taken by the projectile to come back to the same level from which it was projected.

$\therefore T = 2 \times \text{time of ascent}$

$$T = \frac{2u \sin \theta}{g}$$

(e) Horizontal range (x) :-

It is the maximum distance travelled by the projectile in horizontal direction.

$\therefore x = \text{horizontal velocity} \times \text{Total time of flight}$

$$= u \cos \theta \times T = u \cos \theta \times \frac{2u \sin \theta}{g}$$

$$\therefore x = \frac{u^2 \sin 2\theta}{g} = u^2 \left(\frac{\sin 2\theta}{g} \right)$$

$$\therefore \text{Horizontal range } (X) = \frac{u^2 \sin 2\theta}{g}$$

Condition for maximum horizontal range

Horizontal range (X) is given by

$$X = \frac{u^2 \sin 2\theta}{g}$$

$X = X_{\max}$ when $\sin 2\theta$ is max^m

$$\sin 2\theta = \max^m$$

$$= +1$$

$$\sin 2\theta = 1 = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ$$

$\therefore$ When the projectile is fired at an angle of 45° with the horizontal, it will cover maximum distance in horizontal direction.

$$\sin 2\theta = 1$$

Very Short -type Question

- (1) Can the average velocity of a particle, moving in one dimensional motion be equal to its instantaneous velocity.
- (2) Can a particle have the distance travelled to be (i) Greater than (ii) Equal to (iii) less than the displacement.
- (3) What do you mean by circular motion.
- (4) Two projectiles are projected with same speed at 30° & 60° respectively with the horizontal. What is the ratio of their horizontal.
- (5) How is angular velocity connected with
 - a) Time Period
 - b) Frequency
- (6) As a body rotates in circular motion which of two, its linear velocity or its angular velocity, is same for all of its points.
- (7) Two bombers running parallel to ground with same velocities at different heights from the ground want to destroy a room on the ground. Which of them has to release the bomb earlier.
- (8) Two bombers running parallel to the ground with different velocities, at same height from ground want to destroy a room on the ground. Which of them has to release the bomb earlier.

Unit-4: (Work & Friction)

Work

Work is said to be done if a force, acting on a body, displaces the body through a certain distance & the force has some component along the displacement.

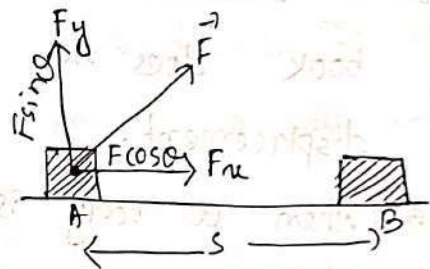
Work done by a constant force:-

Let the body be displaced from A to B through a distance 'u' experiencing a force $\vec{F}$ in a direction inclined at an angle ' θ ' with x-axis.

Work done is defined as the dot product of force & displacement.

$$\text{So } W = \vec{F} \cdot \vec{S}$$

$$W = FS \cos \theta$$



Types of Work

1. Positive work

If $\theta = 0^\circ$, then $\cos \theta = 1$.

So work done $W = FS \cos \theta$

$$W_{\max} = FS$$

ie, When the force & displacement are in same direction, work done is positive.

Ex:-

1. Engine of a car exerts a force on the car in the direction of propagation.
2. A body falling freely under the action of gravity has positive work done by the gravitational force.

2) Zero Work

When $\theta = 90^\circ$, $\cos\theta = 0$

So work done $W = FS \cos 90^\circ$

$$\boxed{W = 0}$$

When the force acts in a direction at right angle to the direction of displacement, no work is said to be done.

Ex:-

a) A person carrying a box over his head & walks along a horizontal road.

b) A student sitting on a chair & studying a book does no work since there is no displacement.

c) When a body is displaced along a horizontal surface, no work is done by its weight, since it is always perpendicular to the displacement.

3) Negative Work

If $\theta = 180^\circ$, $\cos\theta = -1$

Then work done $W = FS \cos\theta$

$$\boxed{W = -FS}$$

When force & displacement are in opposite direction, work done is negative.

Ex:-

a) When brakes are applied to a moving car, the frictional force supplied by the brakes retard the motion of the wheel.

b) When two similar charges approach each other, they repel. So force & displacement are opposite to each other.

Units of Work

$$W_{\max} = FS$$

MKS

$$Nm = \text{Joule}$$

CGS

$$\text{Dyne} \times \text{cm} = \text{Erg}$$

SI

Joule

$$1 \text{ Joule} = 10^7 \text{ erg}$$

Dim. Formula

$$[W] = [F][S] = [M^1 L^1 T^{-2}][L^1] = [M^1 L^2 T^{-2}]$$

Numerical

1.

What work is performed in dragging a block 50m horizontally when a 60N force is applied by a rope making an angle of 30° with the ground.

Ans:- Given data,

$$S = 50\text{m}$$

$$F = 60\text{N}$$

$$\theta = 30^\circ$$

$$\text{So work done } (W) = FS \cos \theta$$
$$= 60 \times 50 \times \cos 30^\circ$$

$$= 60 \times 50 \times \frac{\sqrt{3}}{2}$$

$$W = 1500\sqrt{3} \text{ Joule}$$

Q.2 A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle cause a displacement $\vec{s} = -4\hat{i} + 2\hat{j} + 3\hat{k}$ in its own direction. If the work done is 6 J, then the value of 'c' is —.

Solⁿ

$$\text{Given } \vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$$

$$\vec{s} = -4\hat{i} + 2\hat{j} + 3\hat{k}$$

$$W = 6 \text{ J}$$

$$W = \vec{F} \cdot \vec{s}$$

$$\Rightarrow 6 = (3\hat{i} + c\hat{j} + 2\hat{k}) \cdot (-4\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\Rightarrow 6 = 3 \times (-4) + c \times 2 + 2 \times 3$$

$$\Rightarrow 6 = -12 + 2c + 6$$

$$\Rightarrow 2c - 6 = 6$$

$$\Rightarrow 2c = 12$$

$$\Rightarrow \boxed{c = 6} \text{ (Ans)}$$

Q.3 A body gets displaced through a distance of 5m under the action of a force 10N, acting at a right angle to the displacement. Calculate the work done.

Since $\theta = 90^\circ$ (force is at right angle to the displacement)

$$\therefore \text{Work done } \boxed{W = 0}$$

FRICITION:-

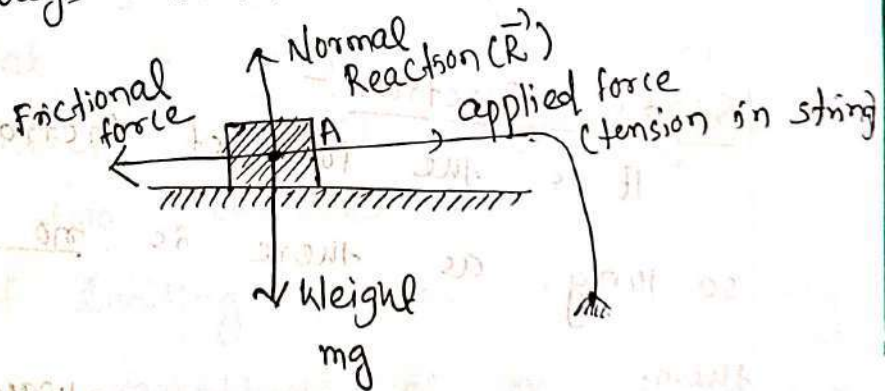
It is an opposition which comes into play between two surfaces when one tends to slide/roll over the other.

Types of Friction:-

1. Sliding Friction
2. Rolling Friction
3. Fluid Friction

Sliding Friction:-

It is the force of friction which comes into play when one surface tends to slide over another surface in which the point of contact between two surface always remains the same.



(Sliding Friction)

A body 'A' of mass 'm' placed over a surface.

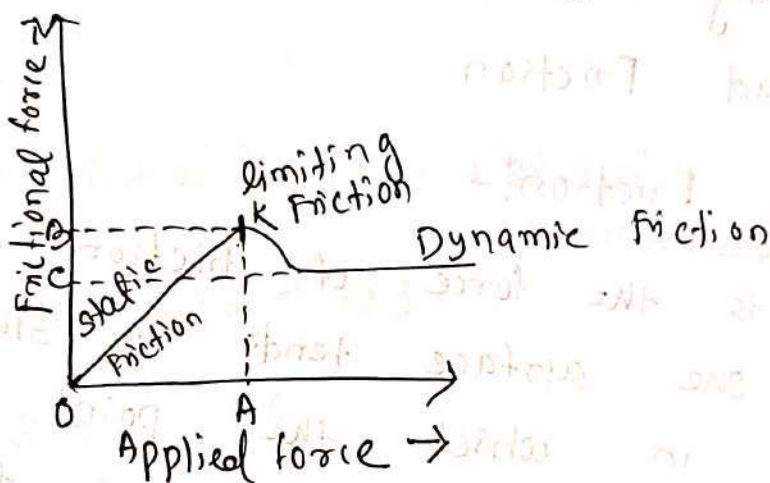
A string tied to the body has a scale pan suspended from its other end.

Various forces acting on the body are as follows

1. Weight mg acting vertically downward.
2. Normal Reaction R vertically upward.
3. Force P , due to tension in the string in forward
4. Force of friction, in the backward direction.

Since sliding occurs in horizontal direction, weight mg & Normal Reaction (R) being vertical do not contribute to the sliding tendency.

* As applied force increases, frictional force also increases automatically keeps itself equal to the applied force & the body does not slide. so Friction is a self adjusting force.



Static Friction:-

It is the force of friction between two surfaces so long as there is no relative motion between them.

- * static friction is always equal to applied force
- * The body doesn't slide & is at rest.
- * The nature is a straight line.

Limiting Friction:-

It is the maximum value of force of friction between two surfaces so long as there is no relative motion between them.

On increasing applied force beyond OA , the body tends to slide & the frictional force decreases & then remains constant throughout.

Dynamic Friction:-

It is the force of friction which comes into play when there is some relative motion between them.

* Dynamic Friction is always less than the limiting Friction.

LAWS OF LIMITING FRICTION:

1. The direction of force of friction is always opposite to the direction of motion.
2. The force of limiting friction depends upon nature of material & state of polish of the surfaces in contact & act tangentially to the interface between two surfaces.
3. The magnitude of limiting Friction ' F ' is directly proportional to the magnitude of normal reaction ' R ' between the two surfaces in contact.

$$\text{ie, } |\vec{F}| \propto |\vec{R}|$$

4. The magnitude of limiting friction is independent of area & shape of the surfaces in contact.

Coefficient of Friction:-

Since $|\vec{F}| \propto |\vec{R}|$

$$\Rightarrow |\vec{F}| = \mu |\vec{R}|$$

where $\mu \rightarrow$ coefficient of friction

So
$$\mu = \frac{|\vec{F}|}{|\vec{R}|}$$

* It is a unitless quantity.

Coefficient of friction is defined as the ratio between the force of limiting friction to the normal reaction.

Methods of Reducing Friction:-

1. By Polishing & rubbing
2. By using lubricants:- A lubricant is an oil or grease which when spread over the surface fills the irregularities & forms a thin layer between them.
3. By converting sliding into Rolling friction:-
Rolling friction is much smaller than the sliding friction.
4. By streamlining:-
Fluid friction depends upon the shape of the body & it is minimum for a streamlined shape. All high speed bodies, aeroplanes, rockets etc have pin-pointed shape.

Numericals:-

- ① Find the horizontal force required to move a body weighing 200kg on a rough horizontal surface having co-efficient of friction 0.35 .

Given data

$$m = 200\text{kg}$$

$$\mu = 0.35$$

$$F = ?$$

We know that, $F = \mu R$

$$\text{So } R = mg$$

$$\therefore F = \mu mg$$

$$= 0.35 \times 200 \times 9.8$$

$$\boxed{F = 686\text{N}}$$

UNIT - 5 GRAVITATION

Gravitation:-

It is a natural phenomenon by which all things with mass or energy including planets, stars, galaxies & even light are brought towards one another.

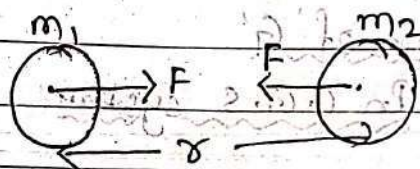
It is the force of attraction between any two bodies in the universe.

Newton's law of Gravitation:-

Statement:-

Every particle of matter in this universe attracts every other particle with a force which varies directly as the product of the masses of the two particles & inversely as the square of the distance between them.

Derivation:-



Consider two bodies of masses m_1 & m_2 separated by a distance ' r ' from each other. Then the force of attraction ' F ' between two bodies is given by

$$F \propto m_1 \quad \dots (1)$$

$$F \propto m_2 \quad \dots (2)$$

$$F \propto \frac{1}{r^2} \quad \dots (3)$$

Combining the above factors,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\Rightarrow F = G \frac{m_1 m_2}{r^2} \quad \text{Newton's Force / Newton's laws of gravitation.}$$

where $G \rightarrow$ Universal gravitational force constant & has the same value everywhere.
 Newton's force $F = G \frac{m_1 m_2}{r^2}$ is mutual & acts

along the line joining the centres of two bodies.
Define G?

According to Newton's laws of gravitation

$$F = G \frac{m_1 m_2}{r^2}$$

Let $m_1 = m_2 = 1 \text{ kg}$, $r = 1 \text{ m}$

Then Newton's force is given by

$$F = G$$

So, the gravitational force constant is defined as the force of attraction between two bodies of unit mass & separated by a unit distance each other.

Unit of 'G'

In C.G.S system

In S.I

$$G = \frac{F \times r^2}{m^2}$$

$$\frac{\text{Dyne} \times \text{cm}^2}{\text{kg}^2}$$

$$\frac{\text{N} \times \text{m}^2}{\text{kg}^2}$$

Dimensional formula of 'G'

$$[G] = \frac{[F] \times [r]^2}{[m]^2}$$

$$= \frac{[M L T^{-2}] [L]^2}{[M]^2} = [M^{-1} L^3 T^{-2}]$$

$$[G] = [M^{-1} L^3 T^{-2}]$$

Relation between 'g' & 'G'

Force between earth & a body near it is called gravity.

Force with which a body is attracted towards earth is called weight.

$$\text{Gravity, } F = mg \quad (1)$$

But according to Newton's law,

$$F = G \frac{Mm}{R^2} \quad (2)$$

From eqn (1) & (2),

$$mg = G \frac{Mm}{R^2}$$

$$\Rightarrow \boxed{g = \frac{GM}{R^2}} \quad \text{Relation between } g \text{ \& } G.$$

Numerical:-

A mass of 2kg experience a weight of 18N on a planet. What is the value of 'g' on the planet?

Sol

$$\text{Weight, } mg = 18\text{N}$$

$$\Rightarrow g = \frac{18}{2} = 9 \text{ m/s}^2$$

Q.

Find the gravitational force of attraction between two bodies of masses 2kg & 5kg separated by a distance of 0.40m. Assuming that the forces acting on them are due to gravitational interaction, find their initial acceleration.

Ans:-

$$m_1 = 2\text{kg}, m_2 = 5\text{kg}$$

$$r = 0.40\text{m}$$

$$F = G \frac{m_1 m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 2 \times 5}{(0.4)^2}$$

$$F = 41.7 \times 10^{-10} \text{ N}$$

$$\text{Acceleration, } a_1 = \frac{F}{m_1} = \frac{41.7 \times 10^{-10}}{2}$$

$$= 20.85 \times 10^{-10} \text{ m/s}^2$$

$$a_2 = \frac{F}{m_2} = \frac{41.7 \times 10^{-10}}{5}$$

$$= 8.34 \times 10^{-10} \text{ m/s}^2$$

$$\frac{F}{m_1} = \frac{F}{m_2} = a$$

Differentiate between Mass & Weight.

Mass

It is the amount of matter contained in a body.

Weight

(1) It is the force with which a body is attracted towards the centre of earth.

It is a scalar quantity.

(2) It is a vector quantity.

It is a const. quantity.

(3) It varies from place to place, it depends upon the value of 'g'.

It is never zero for a body.

(4) It is zero at the centre of earth.

In SI, it is measured in kg.

(5) In SI, it is measured in Newton.

Variation of 'g' with Altitude :-

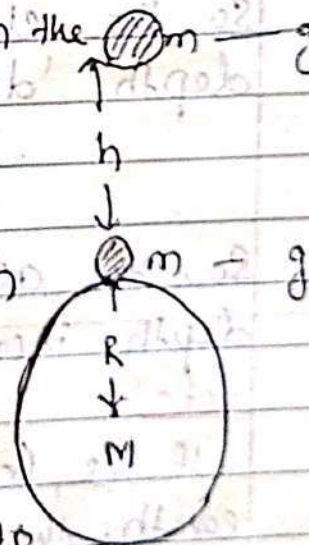
A body of mass 'm' placed on the surface of the earth. $g' = ?$

M - Mass of the earth

R $\rightarrow$ Radius of the earth

g $\rightarrow$ Acceleration due to gravity on the surface of earth.

Then $g = \frac{GM}{R^2}$ ----- (1)



The body of mass 'm' is taken to a height 'h' above the surface of earth.

The acceleration due to gravity at height 'h' is

$$g' = \frac{GM}{(R+h)^2} \text{ ----- (2)}$$

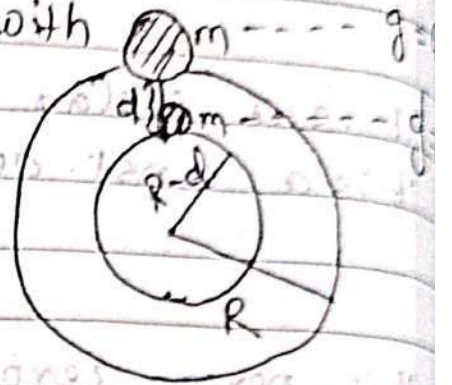
Change in 'g' due to increase of height from above the surface of earth is

$$g - g' \propto h$$

Thus, the value of acceleration due to gravity decreases with increase in height above the surface of earth.

Variation of 'g' with depth (d):

Consider the body of mass 'm' lying on the surface of earth with acceleration due to gravity 'g'.



$$\text{So } g = \frac{GM}{R^2}$$

If the body is taken to a depth 'd' below the surface of earth, then acceleration due to gravity at the depth 'd' is

$$g' = \frac{GM}{(R-d)^2}$$

So the change in acceleration due to gravity at depth 'd' is

$$g - g' \propto d$$

So, the acceleration due to gravity decreases as depth increases.

Note:-

If the body of mass 'm' is placed at the centre of earth, then

$$\text{Weight (W)} = mg = 0$$

The weight of the body at the centre of earth is equal to zero.

Numerical Related to 'g'

Q.1) A body weighs 90 kgwt on the surface of earth. How much will it weigh on the surface of mars, radius is $\frac{1}{2}$ & mass $\frac{1}{9}$ of that of earth.

Ans:-

Given that, $W_e = 90 \text{ kgwt}$
 $W_{\text{mars}} = ?$

We know that $g_e = \frac{GM}{R^2}$

So $g_m = \frac{G(M/9)}{(R/2)^2} = \frac{4}{9} \frac{GM}{R^2}$

$$\Rightarrow g_m = \frac{4}{9} g_e$$

So $\frac{W_e}{W_m} = \frac{mg_e}{mg_m} = \frac{g_e}{g_m}$

$$\Rightarrow \frac{W_e}{W_m} = \frac{g_e}{g_m} = \frac{9}{4}$$

$$\Rightarrow W_m = \frac{4}{9} W_e = \frac{4}{9} \times 90$$

$$W_m = 40 \text{ kgwt}$$

A body has a weight of 81N on the surface of earth. How much will it weigh when taken to height, equal to half the radius of earth.

$$F_1 = \text{Weight of the body on the surface of earth} \\ = \frac{GMm}{R^2} = 81 \text{ N}$$

When taken to a height $R/2$ from the surface, then the distance of the body from the centre of earth is $R + R/2 = \frac{3}{2} R$

So, weight $F_2 = \frac{G M m}{\left(\frac{3}{2} R\right)^2} = \frac{4}{9} \frac{G M m}{R^2}$

$\Rightarrow F_2 = \frac{4}{9} F_1$

$= \frac{4}{9} \times 81$

$F_2 = 36 \text{ N}$

Home work :-

- ① The earth's mass is 80 times that of moon. Their diameters are in the ratio 4:1 respectively. What is the value of 'g' moon.
- ② At what height from the surface of earth, will the value of 'g' be reduced by 40% of the value at the surface? Radius of earth is 6400 km.
- ③ Acceleration due to gravity decreases with an increase in depth. What type of graph will be obtained between change in acceleration due to gravity against depth.
- ④ State & Explain Newton's laws of gravitation.
- ⑤ Define G . Write its SI unit & dimensional formula. Derive the relation between g & G .

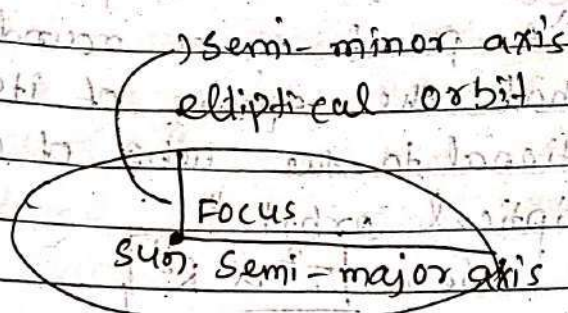
Kepler's laws of Planetary Motion

Kepler gave 3 laws of planetary motion

1. About the shape of the orbit.
2. Velocities of the planet in the orbit.
3. About the Time period of the planet

1st law:- Law of Elliptical orbit.

Statement:- "A planet moves around the sun in an elliptical orbit with sun situated at one of its foci."



Semi-major axis $>$ Semi-minor axis.

2nd law:- The focus of an ellipse is not equidistant from the point on the orbit.

Rotation is the reason for a change of seasons from winter to summer & repetition of the seasons after one year.

3rd law (Law of areal velocities):-

Statement:- "A planet moves round the sun in such a way that its areal velocity is constant. (i.e., the line joining the planet with the sun sweeps equal areas in equal interval of time.)"

Let 't' be the time taken

by the planet to move

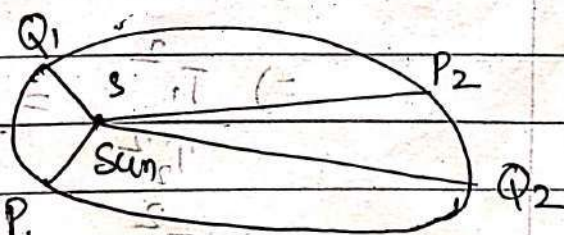
from P_1 to Q_1 so that

the line SP_1 covers an

area SP_1Q_1 .

While going from P_2 to Q_2 , the planet covers an

area SP_2Q_2 in time 't'.



$$\text{Area } P_1 S Q_1 = \text{Area } P_2 S Q_2$$

$$\text{Since } SP_2 > SP_1$$

$$\text{So } P_2 Q_2 < P_1 Q_1$$

The orbital velocity of a planet is not uniform. It is largest when the planet is nearest to sun & is least when the planet is at maximum distance away from sun.

3rd law:- Law of Time period (Harmonic Law)

"A planet moves round the sun in such a way that the square of its Time period is proportional to the cube of semi-major axis of elliptical orbit."

$$T^2 \propto R^3$$

where $T \rightarrow$ Time period of the planet

$R \rightarrow$ Radius of semi-major axis

Numerical:-

The time period of a satellite in a circular orbit of radius R is T . What will be the period of another satellite in a circular orbit of radius $4R$?

Ans: Given - that

Satellite 1 (Circular) Satellite 2 (Circular)

$$R_1 = R \quad R_2 = 4R$$

$$T_1 = T \quad T_2 = ?$$

According to Kepler's 3rd law of planetary motion, $T^2 \propto R^3$

$$\Rightarrow \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

$$\Rightarrow \frac{T^2}{T_2^2} = \frac{R^3}{(4R)^3} = \frac{1}{64}$$

$$\Rightarrow T_2^2 = 64 T^2 \Rightarrow T_2 = 8T$$

A man can jump 6 times as high on the moon as on the earth. Justify.

$$g_{\text{moon}} = \frac{g_{\text{earth}}}{6}$$

Because the gravitational force of moon is one sixth of gravitational force of earth.

If the moon attracts the earth, why does earth not move?

The moon attracts the earth with the same gravitational force as the earth attracts the moon. Since the mass of earth is very large, the acceleration produced is negligible. Therefore, earth does not move towards the moon.

Why does Earth pull people towards itself but not vice versa?

Because the mass of the earth is quite larger than yours, the centre of mass is very close to the centre of the earth, but rather far away from us. Thus as we both fall to the common center, the earth hardly moves while we fly until we hit the ground.

A body is lifted from the centre of the earth to a point situated at infinite distance away. Where will it possess maximum weight?

On the surface of earth, the body will possess maximum weight. Because 'g' decreases with height as well as with depth.

If the radius of the earth were to shrink by 1%, its mass remaining the same, what would happen to value of 'g'?

Since 'g' varies inversely as R^2 , it increases due to a decrease in R.

M.C.Q.

- ① Value of G in M.K.S units is —
- ② Square of time period of rotation of planet around the sun varies as the cube of its —
- ③ Weight of the body at the centre of earth is —
- ④ M is the mass of the earth & R its radius. The ratio of the gravitational acceleration to gravitational constant is :
(a) $\frac{R^2}{M}$ (b) $\frac{M}{R^2}$ (c) MR^2 (d) M/R
- ⑤ If g_0, g_h & g_d be the acceleration due to gravity at earth's surface, at height ' h ' & at a depth ' d ' respectively, then :
a) $g_0 > g_h$ & $g_0 > g_d$ b) $g_0 < g_h$ & $g_0 < g_d$
c) $g_0 > g_h$ & $g_0 < g_d$ (d) $g_0 < g_h$ & $g_0 < g_d$
- ⑥ A body of mass m is orbiting the earth at a radius r from the centre of earth. Another body of mass $2m$ is orbiting at a radius $2r$. What is the ratio of their time period.
a) $1:2^{2/3}$ (b) $1:2$ (c) $1:\sqrt{2}$ (d) $1:2\sqrt{2}$

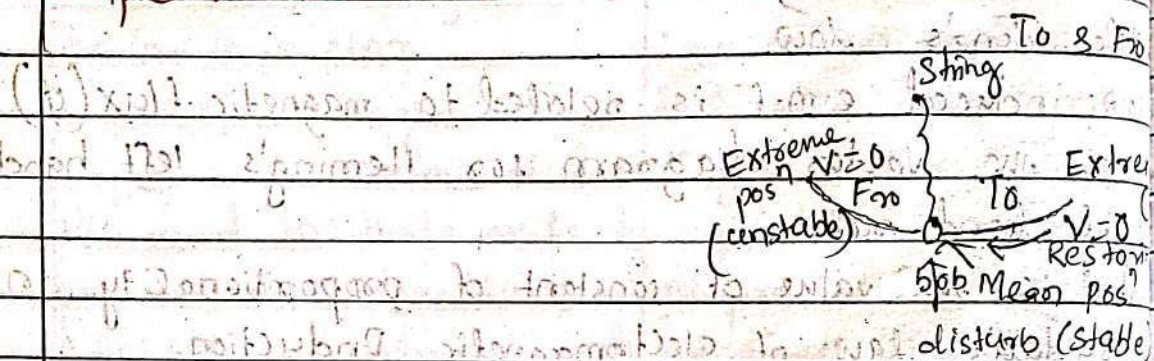
Unit-VI

Oscillations & Waves

Oscillation :-

Oscillation refers to any Periodic motion moving at a distance about the equilibrium position & repeat itself over & over for a period of time.

Simple Harmonic Motion (S.H.M.)



Simple Harmonic Motion is the motion in which restoring force is proportional to displacement from the mean position & opposes its increase.

i.e., Restoring Force $F \propto -y$

y → displacement from position

∴ $F = -ky$, 've' sign is due to Restoring force where k → Force const. the increment

of displacement $F = ma$ ∴ $ma = -ky$

Acceleration 'a' of a body vibrating in S.H.M is $a = \frac{dv}{dt} = \frac{d}{dt} \left(\frac{dy}{dt} \right) = \frac{d^2y}{dt^2}$

∴ $m \frac{d^2y}{dt^2} = -ky$

∴ $\frac{d^2y}{dt^2} = \left(\frac{-k}{m} \right) y$ Differential equⁿ of S.H.M

Another definition of S.H.M

S.H.M also can be defined as the projection of uniform circular motion along the vertical diameter of the

Ex:- of S.H.M

① Vibration of a simple Pendulum

② Vibration of a stretched string

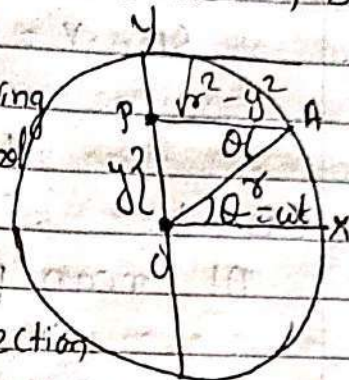
Characteristics:-

Q. calculate the displacement (y), velocity (v) & acceleration of a particle which vibrates in S.H.M

Ans:- Displacement (y)

Displacement of a particle vibrating in S.H.M. at any instant is defined as the distance from the mean position at that instant.

Let 'P' be the position of projection of 'A' in time 't'.



$OA = r$, $OP = y$, $AP = \sqrt{r^2 - y^2}$ (Using Pythagoras thⁿ)
In $\triangle OAP$, $\sin \theta = \frac{OP}{OA} = \frac{y}{r}$ in $\triangle OAP$

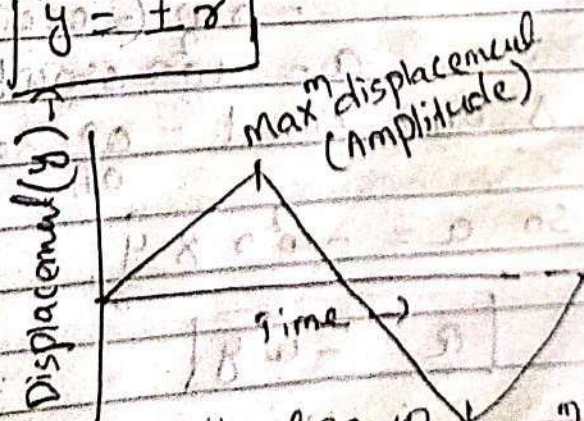
$y = r \sin \theta$

$y = r \sin \omega t$ ($\because \omega = \frac{\theta}{t} \Rightarrow \theta = \omega t$)
 $\omega \rightarrow$ Angular vel. of A

The displacement (y) of a particle vibrating in S.H.M is given by $y = r \sin \omega t$

Maximum displacement or Amplitude of vibration is $y = \pm r$

Displacement - Time graph



Amplitude of a particle vibrating in S.H.M is defined as its maximum displacement on either side of the mean position.

Velocity (V) :-

Differentiating 'y' w.r.t time.

$$V = \frac{dy}{dt} = \frac{d}{dt} (\tau \sin \omega t) = \tau \omega \cos \omega t$$

From ΔOAP , $\cos \omega t = \frac{AP}{OA} = \frac{\sqrt{\tau^2 - y^2}}{\tau}$

$$\text{So } V = \tau \omega \times \frac{\sqrt{\tau^2 - y^2}}{\tau} = \omega \sqrt{\tau^2 - y^2}$$

$$V = \omega \sqrt{\tau^2 - y^2}$$

Case-1

At mean position, displacement $y = 0$

$$\text{So } V = \omega \sqrt{\tau^2} = \tau \omega$$

$$\Rightarrow \boxed{V = \tau \omega} \text{ (Maximum Velocity)}$$

Case-2 At extreme position, $y = \pm \tau$

$$\text{So } V = \omega \sqrt{\tau^2 - \tau^2} = 0$$

$$\Rightarrow \boxed{V = 0}$$

A particle vibrating in S.H.M. passes with maximum velocity through mean position & rest ($V=0$) at the extreme position.

Acceleration (a) :-

Differentiating $V = \tau \omega \cos \omega t$ w.r.t time.

$$a = \frac{dV}{dt} = \frac{d}{dt} (\tau \omega \cos \omega t) = \tau \omega (-\omega \sin \omega t)$$

$$a = -\omega^2 \tau \sin \omega t$$

From ΔOAP , $\sin \omega t = \frac{OP}{OA} = \frac{y}{\tau}$

$$\text{So } a = -\omega^2 \tau \times \frac{y}{\tau}$$

$$\boxed{a = -\omega^2 y}$$

Case-1 At mean position, $y=0$

$$\text{So } a = -\omega^2 \times 0 = 0$$

at extreme position (ie. at y or y') $y = \pm r$

$$a = \mp \omega^2 r$$

So a particle vibrating in S.H.M. has zero acceleration while passing through mean position & has max^m acceleration while at extreme position.

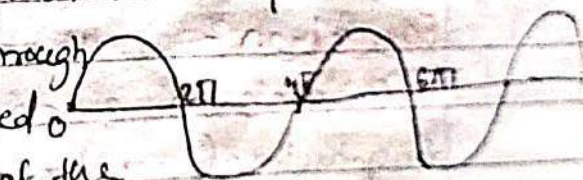
So acceleration of the particle which vibrates in S.H.M. is proportional to its displacement & is always directed towards mean position.

Waves

Wave-motion

It is the disturbance that travels in the medium which is due to repeated periodic motion of the particles in the medium. The motion being handed over from particle to particle.

* The disturbance that travels through the medium is due to repeated periodic motion of the particles of the medium



repeated periodic motion
mean positions but keep

* Particles do not leave their mean positions but keep on vibrating in S.H.M.

* Wave velocity & particle velocity are different from each other. Wave velocity is uniform whereas particle velocity is variable. It is maximum when the particle passes through the mean position & is zero at extreme position.

* Energy is always carried in the direction of propagation of wave.

Types of wave-motion

Two types of wave-motion

1. Transverse wave-motion
2. Longitudinal wave-motion

Transverse wave-motion

It is the type of wave-motion in which particles of the medium are vibrating in the direction at right angles to the direction of propagation of wave.

Ex:- A stone thrown in a pond of water, Light wave

Formation

Crest & Trough are formed during Transverse wave propagation.

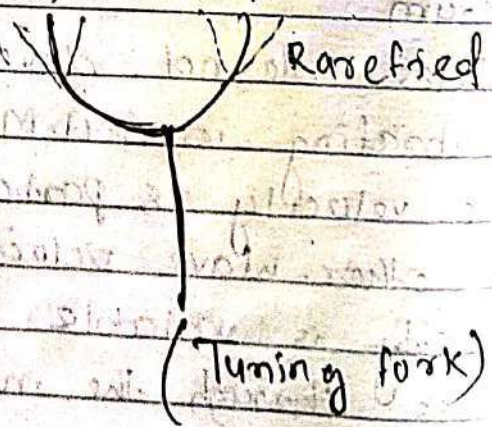
Longitudinal wave-motion

It is a type of wave-motion in which particles of the medium vibrate along the direction of propagation of wave.

Ex:- Sound wave, Tuning fork vibration

Formation

Compression & rarefaction are formed during longitudinal wave propagation.



Comparison between Transverse wave & longitudinal wave-motion.

Transverse wave-motion

1. Definition

Particles in the medium are vibrating at right angles to the direction of propagation of wave.

2. Example - Light wave

3. Formation

crest & Trough are formed

4. There is a temporary change in shape of the medium

5.

They can be propagated through solids & surface of liquids but not through gases.

6. There is no change in the density of medium.

Longitudinal wave-motion

Definition -

Particles in the medium vibrate along the direction of propagation of wave

Ex - Sound wave

Formation - Compression & rarefactions are formed.

4. There is a temporary change in size of the medium.

5. They can be propagated through solids, liquids & gases.

6. There is a change in the density in medium

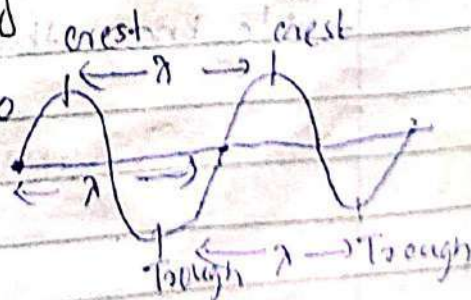
Relation between different wave parameters

Wavelength (λ) :- It is the distance travelled by a wave during the time particle executing S.H.M. completes one vibration.

It is the distance between two consecutive crests

OR

It is the distance between two consecutive troughs.



(2) Wave number ($\bar{n}$):-

It is the reciprocal of the wavelength of

$$\text{ie, } \boxed{\bar{n} = \frac{1}{\lambda}}$$

(3) Velocity of wave (v)

velocity v of the wave is given by

$$\text{velocity } (v) = \frac{\text{Distance}}{\text{Time}}$$

$$\boxed{v = \frac{\lambda}{T}}$$

but $\frac{1}{T} = n$ (Frequency)

$$\text{So } v = \lambda \times \frac{1}{T} = \lambda \times n$$

$$\Rightarrow \boxed{v = n \lambda}$$

Relation between v
Velocity of wave = frequency $\times$ wavelength.

Related numerical

A broadcasting station radiates at a 710 kHz . What is the wavelength.

Given the velocity of wave = $3 \times 10^8 \text{ m/s}$

Solⁿ

$$\text{Given } v = 3 \times 10^8 \text{ m/s}$$

$$n = 710 \text{ kHz} = 710 \times 10^3 \text{ Hz}$$

$$\lambda = ?$$

We know that $v = n \lambda$

$$\text{so } \lambda = \frac{v}{n} = \frac{3 \times 10^8}{710 \times 10^3} = 422.54$$

Ultrasonics:-

The sound of frequency greater than the upper limit of audible range ($> 20\text{ kHz}$) is called Ultrasonic. Frequency range of ultrasonic wave

$$20\text{ kHz} - 10^9\text{ Hz} \\ = 2 \times 10^4\text{ Hz} - 10^9\text{ Hz}$$

Upper range of ultrasonics have wave lengths nearly equal to wave length of visible light.

Properties of Ultrasonics:-

1. Ultrasonic waves are longitudinal in nature.
2. Propagation of ultrasonics through a medium results in the formation of compression & rarefaction.
3. These are waves of very high frequency range having $2 \times 10^4\text{ Hz} - 10^9\text{ Hz}$.
4. They travel with the speed of sound.
5. Since the energy of sound waves is proportional to the square of their frequency ($E \propto f^2$), Ultrasonics are highly energetic waves.
6. Due to their much smaller wavelength, ultrasonics do not spread as much as audible waves.

Application of Ultrasonics:-

1. Echo sounding - To measure the depth of sea.
2. Thickness gauging - The echo sounding process can also be used to measure thickness of rolled sheets in rolling mills.
3. Drilling holes
4. Determination of elastic constants:- velocity of ultrasonics is $v = \sqrt{Y/\rho}$, 'Y' being Young's modulus of elasticity & 'ρ' being the density of the material. Velocity is (knowning), value of 'Y' can be determined.

Q. Equation of motion of the particle vibrating in S.H.M is given by

$$y = a \sin \omega t$$

$$y = r \sin \omega t$$

$$T = \frac{2\pi}{\omega} \text{ sec.}$$

Find its Time period.

Solⁿ Given $r = 2 \text{ cm}$ (General eqⁿ of S.H.M)
 $\omega = 5$ is $y = r \sin \omega t$

We know that $T = \frac{2\pi}{\omega}$

$$T = \frac{2\pi}{5} \text{ sec}$$

Maximum displacement / Amplitude of the particle which vibrates in S.H.M is $\pm 2 \text{ cm}$
 $= \pm 2 \text{ cm}$

2. If a particle executes S.H.M of period 8 sec of amplitude 0.40 m. Find the maximum velocity & acceleration.

Ans: Given $T = 8 \text{ sec}$

$$r = 0.40 \text{ m}$$

$$\text{Max vel. } V_{\text{max}} = ? r \omega$$

$$a_{\text{max}} = ? \omega^2 r$$

We know that $\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4} \text{ sec}^{-1}$

$$\text{Max}^n \text{ velocity } (V_{\text{max}}) = r \omega = 0.40 \times \frac{\pi}{4} = 0.314 \text{ m/s (Ans)}$$

$$\text{Max}^n \text{ acceleration } a_{\text{max}} = \omega^2 \cdot r = \left(\frac{\pi}{4}\right)^2 \times 0.40 \text{ m} = 0.247 \text{ m/s}^2 \text{ (Ans)}$$

A particle executing S.H.M has maximum velocity of 1 m/s & a max^m acceleration of 1.57 m/s^2 . Calculate its time period.

Solⁿ

$$\text{Maximum velocity } (V_{\max}) = r\omega = 1 \text{ m/s}$$

$$\text{Maximum acceleration } a_{\max} = \omega^2 r = 1.57 \text{ m/s}^2$$

$$\text{Dividing, } \frac{\omega^2 r}{\omega r} = \frac{1.57}{1.00}$$

$$\Rightarrow \omega = 1.57 \text{ rad/sec}$$

$$\therefore \text{Time period } T = \frac{2\pi}{\omega} = \frac{2\pi}{1.57}$$

$$T = 4 \text{ sec (Ans)}$$

Eqⁿ of a particle vibrating in S.H.M is $y = 8 \sin \omega t$. Calculate the amplitude of vibration.

Solⁿ Eqⁿ of a particle vibrating in S.H.M is given by

$$y = 8 \sin \omega t$$

$$\Rightarrow y = 4 \sin \omega t$$

So the amplitude of vibration = $\pm 4 \text{ cm}$

Write the eqⁿ of S.H.M if amplitude sum a period 0.5 sec.

$$\text{Ans: } r = 5 \text{ cm}$$

$$T = 0.5 \text{ sec}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi \text{ rad/sec}$$

$$y = r \sin \omega t$$

$$r = 5 \text{ cm}$$

$$T = 0.5 \text{ sec}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5}$$

The eqⁿ of motion of S.H.M is $y = r \sin \omega t = 5 \sin 4\pi t$

$$\omega = 4\pi \text{ rad/sec}$$

$$r = 5 \text{ cm}$$

$$y = r \sin \omega t$$

$$y = 5 \sin 4\pi t$$

b. A broadcasting station radiates at a Frequency (710 KHz) with a velocity $3 \times 10^8 \text{ m/s}$. What is the wavelength.

Ans:- Given $\eta = 710 \text{ KHz} = 710 \times 10^3 \text{ Hz}$

$v = 3 \times 10^8 \text{ m/s}$

$$v = \eta \lambda \quad \lambda = ?$$

Using the relation, $v = \eta \lambda$

$$\lambda = \frac{v}{\eta}$$

$$\lambda = \frac{v}{\eta}$$

$$\lambda = \frac{3 \times 10^8}{710 \times 10^3}$$

$$\Rightarrow \lambda = 422.5 \text{ m} \quad (\text{Ans})$$

Unit - 7 (Heat & Thermodynamics)

Thermal Expansion:-

Every substance (solid, liquid & gas) expands on heating.

Application to Thermal Expansion:-

A gap is left in between the two consecutive pieces of rail track. In summer, the rails expand. If the gap is not there the rails may bend, thereby causing derailment of trains.

The telephone & telegraph wires get slackened in summer & become tight in winter.

Chimney of a burning lamp cracks when a drop of water is put over it.

Concrete floors are not laid in one piece. They are laid in the form of small sections.

Types of expansion coefficient:-

Expansion along 1-dimension - (Linear expansion)
(Increase/change in length)

Expansion along 2-Dimension - Superficial expansion
(Increase/change in area)

Expansion along 3-dimension - Cubical expansion
(Increase/change in volume)

Expansion along one dimension - (Linear expansion)

Consider a rod of length ' l_0 ' at 0°C . On heating the rod expands. Let ' l_t ' be the length of rod at $t^\circ\text{C}$.

So Increase/change in length = $l_t - l_0$ due to $t^\circ\text{C}$ rise in temp

' $l_t - l_0$ ' depends upon

(i) Original length ' l_0 ' of the rod. i.e. $l_t - l_0 \propto l_0$

(ii) Rise in temp. $t^\circ\text{C}$. i.e., $l_t - l_0 \propto t$

$$l_t - l_0 \propto l_0 t \quad \text{--- (1)}$$

$\Rightarrow l_t - l_0 = \alpha l_0 t$, where $\alpha \rightarrow$ coefficient of linear expansion of rod

$$\Rightarrow \alpha = \frac{l_t - l_0}{l_0 t} \quad \text{--- (2)}$$

If $l_0 = 1$, $t = 1^\circ\text{C}$, then $\alpha = l_t - l_0$.

So coefficient of linear expansion of rod is defined as the change in length per unit length, at 0°C , per degree centigrade rise of temp.

From equⁿ (1), $l_t - l_0 = \alpha l_0 t$

$$\Rightarrow l_t = l_0 + \alpha l_0 t$$

$$l_t = l_0 (1 + \alpha t) \quad \text{--- (3)}$$

Unit of ' α '

$$\alpha = \frac{l_t - l_0}{l_0 t} = \frac{\text{m}}{\text{m} \cdot ^\circ\text{C}} = ^\circ\text{C}^{-1} \quad \frac{\text{SI}}{\text{K}^{-1}}$$

~~Unit of~~

Dimension of ' α '

$$\alpha = \frac{l_t - l_0}{l_0 t} = \frac{[L]}{[L][K]} = [K^{-1}]$$

Fig-1

Rod of l_0 at 0°C
length l_0 at 0°C l_t at $t^\circ\text{C}$

After heating, length expands to l_t as the temp. increases on heating.

Superficial expansion:- (Expansion in 2-dimension)

Consider a thin sheet having original area S_0 at 0°C , & S_t at $t^\circ\text{C}$.

Increase/change in area due to $t^\circ\text{C}$ rise in temp.

$$= S_t - S_0$$

$S_t - S_0$ depends upon

(i) original area S_0 at 0°C , i.e.

$$S_t - S_0 \propto S_0 \quad - (1)$$

(ii) Rise in temp at $t^\circ\text{C}$,

$$\text{i.e. } S_t - S_0 \propto t \quad - (2)$$

From (1) & (2), $S_t - S_0 \propto S_0 t$

$$\Rightarrow S_t - S_0 = \beta S_0 t \quad - (3)$$

where $\beta \rightarrow$ coefficient of superficial expansion.

From equⁿ (3),

$$S_0 \left[\beta = \frac{S_t - S_0}{S_0 t} \right]$$

$$S_t = S_0 + \beta S_0 t$$

$$S_t = S_0 (1 + \beta t)$$

if $S_0 = 1 \text{ m}^2$ & $t = 1^\circ\text{C}$, then $\beta = S_t - S_0$

So coefficient of superficial expansion is defined as the increase/change in area per unit original area at 0°C , per deg centigrade rise in temperature.

Unit of β

$$\beta = \frac{S_t - S_0}{S_0 t}$$

$$= \frac{\text{m}^2}{\text{m}^2 \cdot ^\circ\text{C}} = ^\circ\text{C}^{-1}$$

$$\frac{\text{SI Unit}}{\text{K}}$$

Dimension of β

$$\beta = \frac{S_t - S_0}{S_0 t} = \frac{[L^2]}{[L^2][K]} = [K^{-1}]$$

Cubical Expansion - (Expansion along 3-D)

Consider a cube having original volume V_0 at 0°C & V_t be the volume at $t^\circ\text{C}$.

Increase/change in volume

= $V_t - V_0$ due to $t^\circ\text{C}$ rise in temp

$V_t - V_0$ depends upon

(i) Original volume ' V_0 ' at 0°C , $V_t - V_0 \propto V_0$

(ii) Rise in temp. at $t^\circ\text{C}$, $V_t - V_0 \propto t$

So $V_t - V_0 \propto V_0 t$

$$\Rightarrow V_t - V_0 = \gamma V_0 t \quad \text{--- (1)}$$

where $\gamma \rightarrow$ coefficient of cubical expansion.

From eqn (1),

$$\gamma = \frac{V_t - V_0}{V_0 t} \quad \text{--- (2)}$$

$$V_t = V_0 + \gamma V_0 t$$

$$V_t = V_0 (1 + \gamma t) \quad \text{--- (3)}$$

If $V_0 = 1 \text{ m}^3$, $t = 1^\circ\text{C}$,

$$\text{then } \gamma = V_t - V_0 \quad \text{--- (4)}$$

Coefficient of cubical expansion is defined as the increase/change in volume per unit original volume at 0°C , per deg centigrade rise of temp.

Unit of ' γ '

$$\gamma = \frac{V_t - V_0}{V_0 t} = \frac{\text{m}^3}{\text{m}^3 \cdot ^\circ\text{C}} = ^\circ\text{C}^{-1} = \frac{\text{SI}}{\text{K}^{-1}}$$

dimension of ' γ '

$$\gamma = \frac{V_t - V_0}{V_0 \cdot t} = \frac{[L^3]}{[L^3][K]} = [K^{-1}]$$

Note Three expansion coefficients are

$$\alpha = \frac{L_t - L_0}{L_0 t} \rightarrow \text{coefficient of Linear expansion}$$

$$\beta = \frac{S_t - S_0}{S_0 t} \rightarrow \text{coefficient of Superficial expansion}$$

$$\gamma = \frac{V_t - V_0}{V_0 t} \rightarrow \text{coefficient of Cubical expansion}$$

Relation between expansion coefficients (Relation between α , β & γ)

Relation between α & β

Consider a square sheet having each side ' l_0 ' at 0°C .
Coefficient of superficial expansion is written as

$$\beta = \frac{S_t - S_0}{S_0 \cdot t}$$

where S_0 = Original area of square sheet at 0°C

$$= l_0 \times l_0 = l_0^2$$

S_t = Area at $t^\circ\text{C}$ after heating

$$= l_t \times l_t = l_t^2$$

$$\text{so } \beta = \frac{l_t^2 - l_0^2}{l_0^2 \cdot t} \quad \text{but } l_t = l_0(1 + \alpha t)$$

where α = coefficient of linear expansion.

$$= \frac{l_0^2 (1 + \alpha t)^2 - l_0^2}{l_0^2 \cdot t}$$

$$= \frac{l_0^2 t}{(1 + \alpha t)^2 - 1}$$

$$= \frac{t}{1 + \alpha^2 t^2 + 2\alpha t - 1}$$

$$= \frac{t}{\alpha^2 t^2 + 2\alpha t}$$

$$\beta = \frac{t}{\alpha^2 t^2 + 2\alpha t}$$

Since α is very small, higher orders of ' α ' can be neglected.

$$\text{So } \boxed{\beta = 2\alpha}$$

$$\text{or } \boxed{\alpha = \frac{\beta}{2}} \quad \text{--- (1)}$$

Relation between α & γ

Consider a cuboid having each side l_0 .
Original volume V_0 at 0°C & V_t at $t^\circ\text{C}$ respectively.

Coefficient of cubical expansion is given by

$$\gamma = \frac{V_t - V_0}{V_0 t}$$

$$\text{but } V_0 = l_0 \times l_0 \times l_0 = l_0^3$$

$$V_t = l_t \times l_t \times l_t = l_t^3$$

$$\text{So } \gamma = \frac{l_t^3 - l_0^3}{l_0^3 t} \quad \text{but } l_t = l_0(1 + \alpha t)$$

$$= \frac{l_0^3 (1 + \alpha t)^3 - l_0^3}{l_0^3 t}$$

$$= \frac{(1 + \alpha t)^3 - 1}{t}$$

$$= \frac{1 + \alpha^3 t^3 + 3\alpha t + 3\alpha^2 t^2 - 1}{t}$$

$$= \frac{\alpha^3 t^3 + 3\alpha t + 3\alpha^2 t^2}{t}$$

$$\gamma = \alpha^3 t^2 + 3\alpha + 3\alpha^2 t$$

Since α is very small, higher order of ' α ' can be neglected.

So neglecting α^3 & α^2 ,

$$\boxed{\gamma = 3\alpha} \Rightarrow \boxed{\alpha = \frac{\gamma}{3}} \quad \text{--- (2)}$$

Relation between α , β & γ

$$\text{From eqn (1), } \alpha = \frac{\beta}{2}$$

$$\text{or } \boxed{\alpha : \beta : \gamma = 1 : 2 : 3}$$

$$\text{From eqn (2), } \alpha = \frac{\gamma}{3}$$

$$\text{From eqn (1) & (2), } \boxed{\alpha = \frac{\beta}{2} = \frac{\gamma}{3}}$$

or $\alpha : \beta : \gamma$

Heat :-

Heat is a form of energy that flows between a system & its environment by virtue of a temperature difference between them.

Heat can be measured in 'Cal' unit.

Temperature

It is the degree of hotness & coldness of a body. It rises when heated & falls when cooled down.

It is measured in the unit of Kelvin 'K' in SI

Heat

It is the form of energy that can transfer from hot body to cold body.

It is the total K.E & P.E. obtained by molecules in an object.

Heat flows from hot body to cold body.

Temperature

It is the degree of hotness & coldness of a body.

It rises when heated & falls down when an object is cooled down.

In the transmission of heat total amount of heat remain unchanged.

In the transmission of heat temp. does not remain the same.

Its SI unit is 'Joule' 'J'

Its SI unit is Kelvin 'K'

It is measured by Calorimeter

It is measured by Thermometer

Specific heat Capacity (c)

Different bodies of same mass required different amount of heat to raise their temp to same level.
2) greater the mass 'm' of body, greater heat is required to raise its temp,

So quantity of heat $Q \propto m$ - (1)

greater heat is required to raise the temp higher

so $Q \propto \Delta T$, where $\Delta T \rightarrow$ Rise in temp

- (2)

$= T_2 - T_1$

From eqn (1) & (2),

$$Q \propto m \Delta T$$
$$\Rightarrow \boxed{Q = C m \Delta T}$$

where 'C' is called 'specific heat' or specific heat capacity of the body.

So $C = \frac{Q}{m \Delta T}$, if $m = 1$, $\Delta T = 1^\circ\text{C}$ then $\boxed{C = Q}$

specific heat capacity of a material is defined as the amount of heat required to raise the temp. of a unit mass of material through 1°C .
Unit of specific heat 'C'

$$C = \frac{Q}{m \Delta T} = \frac{\text{kcal}}{\text{kg } ^\circ\text{C}} \quad \text{or} \quad \frac{\text{J}}{\text{kg K}}$$

Dimensional formula for 'C'

$$C = \frac{Q}{m \Delta T} = \frac{[M^1 L^2 T^{-2}]}{[M^1][K^1]} = [L^2 T^{-2} K^{-1}]$$

Ex

Specific heat capacity of ice = $0.5 \text{ cal/g}^\circ\text{C}$
Specific heat capacity of water = $1 \text{ cal/gm}^\circ\text{C}$

Numerical

$$\approx 4.186 \text{ J/gm}^\circ\text{C}$$

A body with mass 2 kg absorbs heat 100 cal when its temp raises from 20°C to 70°C . Find the specific heat of the body.

Sol?

$$m = 2 \text{ kg} = 2000 \text{ gm}$$

$$Q = 100 \text{ cal}$$

$$\Delta T = 70^\circ\text{C} - 20^\circ\text{C} = 50^\circ\text{C}$$

$$C = ? \quad \frac{Q}{m \Delta T}$$

$$\Rightarrow C = \frac{100}{2 \times 10^3 \times 50} = 10^{-3} \text{ cal/gm}^\circ\text{C}$$

A piece of copper 125gm has a heat capacity of 19687.6 J also it is heated from 150 to 250°C heat. Find the specific heat.

Ans:- Given $m = 125\text{gm}$

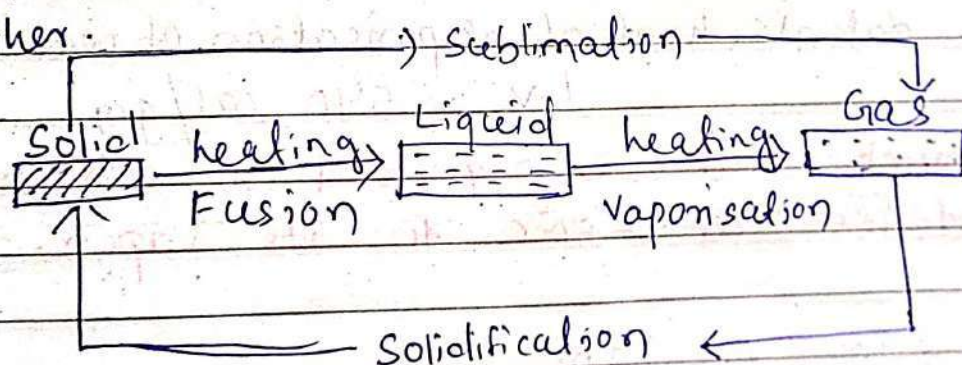
$$Q = 19687.6 \text{ J}$$

$$\Delta T = 250^\circ\text{C} - 150^\circ\text{C} = 100^\circ\text{C}$$

$$C = \frac{Q}{m \Delta T} = \frac{19687.6}{125 \times 100} = 1.575 \text{ J/gm}^\circ\text{C}$$

Change of state

Three states of matter can be converted into one another.



Fusion:- Conversion of solid into liquid state.

Vaporization:- Conversion of liquid into gaseous state at its boiling point.

Latent heat:- (Hidden heat)

It is the amount of heat required/absorbed to change the state from solid to liquid & liquid to gaseous state without rising the temp. of the substance.

So heat supplied is not utilized to increase the temp & has been utilized for changing the state of matter from solid to liquid & liquid to gas.

Specific Latent heat:-

① Specific latent heat of fusion:- (L_f)

The amount of heat required to convert 1g of substance from solid to liquid state at its melting point, without any rise of temp.

N.B

Specific latent heat of fusion of ice $L_f = 80 \text{ cal/gm}$
 $\approx 336 \text{ J/kg}$

② Specific latent heat of vaporisation (L_v):

The amount of heat required to convert 1g of liquid into gaseous state at its boiling point without rising the temperature.

N.B

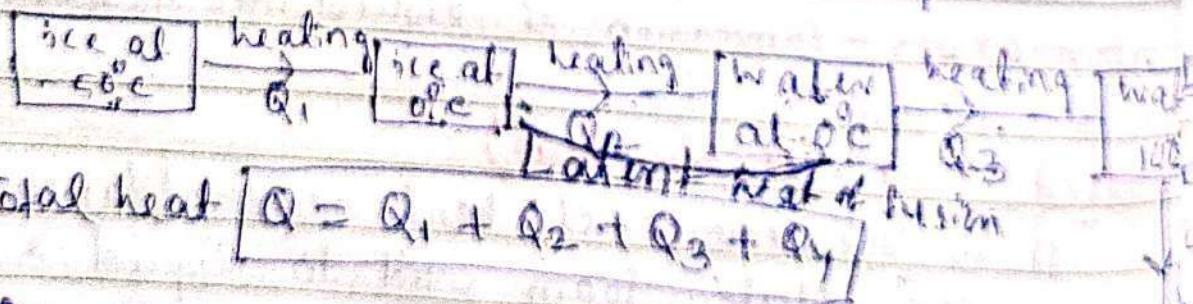
Specific latent heat of vaporisation of water
 $L_v = 540 \text{ cal/gm}$

Q. How much heat is required to convert 1gm of ice at -50°C to its vapor.

Sol

Given data $m = 1 \text{ gm}$

ice at -50°C to its vapour



Q1

ice at -50°C to ice at 0°C

$$\text{So } \Delta T = 0^\circ\text{C} - (-50^\circ\text{C}) = 50^\circ\text{C}$$

$$Q_1 = mc\Delta T = 1 \times 0.5 \times 50 \quad c = \text{specific heat of ice}$$

$$[Q_1 = 25 \text{ cal}] \quad (C_{\text{ice}} = 0.5 \text{ cal/gm})$$

= Amount of heat required to change the state from ice at 0°C to water at 0°C . This amount of heat supplied does changing the state from ice to water without rising the temp. which is called latent heat of fusion.

$$\begin{aligned}\text{So } Q_2 &= m L_f & L_f &= \text{latent heat of fusion of ice} \\ &= 1 \times 80 & (L_f)_{\text{ice}} &= 80 \text{ cal/gm} \\ \boxed{Q_2} &= 80 \text{ cal}\end{aligned}$$

= Amount of heat required to raise the temp from water at 0°C to water at 100°C

$$\begin{aligned}\text{So } \Delta T &= T_2 - T_1 \\ &= 100^\circ\text{C} - 0^\circ\text{C} = 100^\circ\text{C}\end{aligned}$$

$$\begin{aligned}Q_3 &= m c \Delta T & c &= \text{specific heat of water} \\ &= 1 \times 1 \times 100 \\ \boxed{Q_3} &= 100 \text{ cal} & (c_{\text{water}} &= 1 \text{ cal/gm}^\circ\text{C})\end{aligned}$$

4 :- Amount of heat required to convert the state from water to vapor without rising the temp which is called latent heat of vaporisation.

$$\begin{aligned}\text{So } Q_4 &= m L_v & L_v &= \text{latent heat of vaporisation} \\ &= 1 \times 540 & (L_v &= 540 \text{ cal/gm}) \\ Q_4 &= 540 \text{ cal}\end{aligned}$$

The total heat required to convert 1 gm of ice from 0°C to its vapor is $Q = Q_1 + Q_2 + Q_3 + Q_4$

$$\begin{aligned}&= 25 + 80 + 100 + 540 \\ &= 745 \text{ cal}\end{aligned}$$

Joule's mechanical equivalent of heat:-

A.T. Joule, there is equivalence between work & heat.

"Whenever heat is converted into work or work into heat, the quantity of energy disappearing in one form is equivalent to the quantity of energy appearing in the other."

If an amount of heat work 'W' results in production of an amount 'H' of heat

$$W \propto H$$

$$\Rightarrow W = J H$$

where 'J' is called 'Joule's mechanical equivalent of heat'.

$$\text{If } H = 1, \text{ then } W = J$$

Joule's mechanical equivalent of heat is defined as the amount of work required to produce a unit quantity of heat.

$$\text{So } J = \frac{W}{H} = 4.2 \times 10^7 \text{ erg/cal}$$

$$= 4.2 \text{ J/cal}$$

First law of Thermodynamics:-

Statement:-

"If the quantity of heat supplied to a system is capable of doing work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system & the external work done by it."

$$\text{ie } dQ = dU + dW$$

dQ = Infinitesimal am of heat added to

Assignment Questions:-

2-marks question:-

Why there is a gap maintained in between the two railway tracks.

Write the unit of linear expansion coefficient (α)

Specific heat of ice is — & specific heat of water —
Value of latent heat of fusion of ice is — & latent heat of vaporisation of water is —

What are the factors upon which specific latent heat depends.

Calculate the latent heat of fusion for 10 gm of ice.

How much amount of heat required to raise the temp. of 5 kg of water through 20°C .

Define mechanical equivalent of heat (J).

State First law of Thermodynamics.

The length of a rod at 0°C is 1.0000 m. Calculate its length at 100°C . Coefficient of linear expansion of rod is $1.5 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$.

Define Co-efficient of superficial expansion (β). Write its SI unit & dimension.

Write the relation between α , β & γ .

Define specific heat capacity of material. Write the factors upon which it depends. Write its dimensional formula.

The process of conversion of solid into liquid is — & the conversion of liquid into vapour is —.

A metal rod is 64.522 cm long at 12°C & 64.576 cm at 90°C . Find the coefficient of linear expansion of its material.

Long type Question:-

- (1) Establish the relation between expansion coefficients
- (2) establish the relation $\alpha: \beta: \gamma = 1: 2: 3$
how much heat is required to convert 5 kg of ice at -10°C to its vapour.
- (3) Derive the expression for α, β & γ . Write unit & dimension.

Numericals

- (1) A metal bar measures 60 cm at 10°C . What would be its length at 110°C .
Given $\alpha = 1.5 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$
- (2) A brass rod & an iron rod are each of length at 0°C . Find the difference in their length at 110°C . Find the difference.
 $\alpha_{\text{brass}} = 19 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$
 $\alpha_{\text{iron}} = 10 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$
- (3) Volume of a lead ball at 0°C is 100 cm^3 . Find its volume at 100°C if $\gamma_{\text{lead}} = 2.83 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$.
- (4) How much amount of heat is required to convert 1 kg of ice at -4°C to water at 60°C .
- (5) How much heat is required to convert 10 gm of ice at 0°C to water at 50°C .
- (6)

prepared by Sthitipragyan paramita Nath

guest faculty in math and science dept

Unit-8 Optics

Optics:-

It is the branch of physics that studies the behaviour & properties of light, including its interaction with matter.

Medium:-

Substance through which light propagates. It is of following three kinds:

1) Transparent:-

It is a medium through which light can be propagated easily, i.e., glass, water etc.

2) Translucent:-

It is a medium through which light is propagated partially. e.g., paper, ground glass etc.

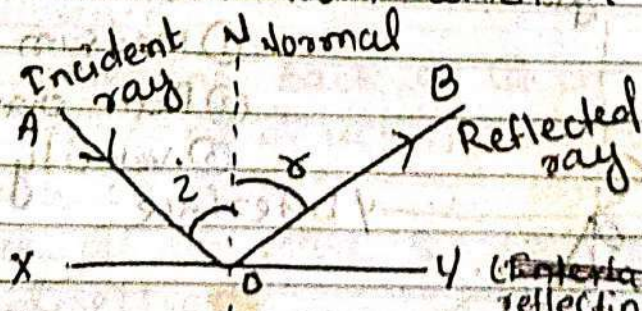
3) Opaque:-

It is a medium through which light can't be propagated. e.g., wood, iron, etc.

Reflection:-

It is the property of light by virtue of which, light is sent back into the same medium after being obstructed from which it is coming a surface.

Fig-1



(Reflection)

Ray AO incident on a shining surface XY.
The ray gets reflected along OB

- ① Medium does not change
- ② Path does not change
- ③ Velocity does not change

$AO \rightarrow$ Incident ray, $OB \rightarrow$ Reflected ray
 ON - Normal to the interface
 $i \rightarrow$ Angle of incidence (Angle between normal & Incident ray)
 $r \rightarrow$ Angle of reflection (Angle between normal & reflected ray)

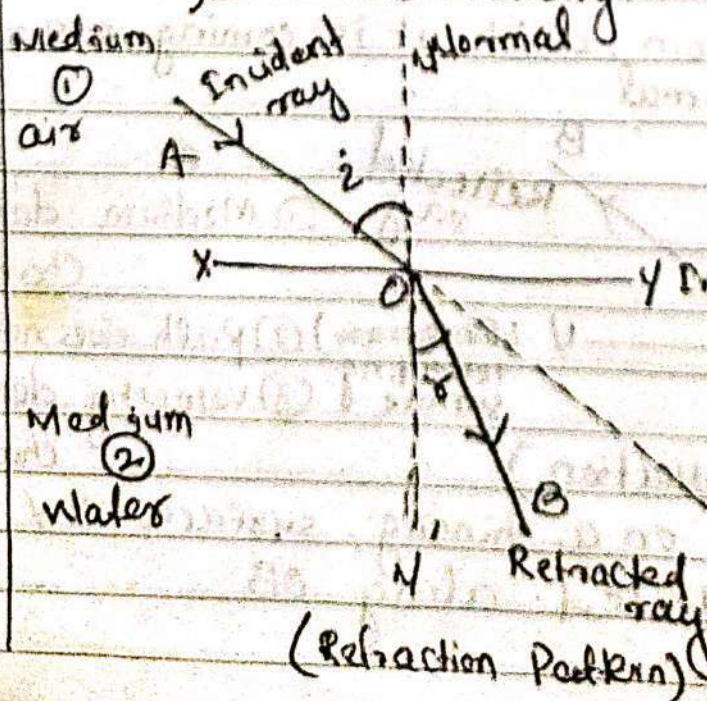
The ray which suffers reflection has to satisfy the laws of Reflection.

Laws of Reflection

- (i) The incident ray, the reflected ray & the normal to the reflecting surface at the point of incidence, all lie in one plane & that plane is perpendicular to the reflecting surface.
- (ii) The angle of incidence is equal to angle of reflection. i.e., $i = r$.

Refraction

It is the phenomenon by virtue of which a ray of light travelling from one medium to the other undergoes a change in its velocity.



In case of Refraction

- ① Medium changes
- ② Path changes
- ③ Velocity changes

(Refraction Pathkin)

The light ray while travelling from one medium to other medium changes its path thereby changing its velocity in the medium.

Incident ray (AO) :- The ray which approaches the interface

Refracted ray (OB) :- The ray which goes into the second medium

(i) $\rightarrow$ Angle of incidence (Angle between normal & Incident ray)

(r) $\rightarrow$ Angle of Refraction (Angle between normal & Refracted ray)

The ray which suffers refraction has to satisfy the laws of Refraction.

Laws of Refraction :-

1) The sine of angle of Incidence bears a constant ratio with the sine of the angle of refraction.

ie, $\frac{\sin i}{\sin r} = \text{constant}$ Snell's Law.

2) The incident ray, the reflected ray & the normal to the interface at the point of incidence all lie in one plane & that plane is perpendicular to the interface separating the two media.

Reflection

1) It is the reverting back of light in the same direction when it falls on plane.

2) It Bounce off the plane & changes direction

3) Angle of incidence (i) is equal to the angle of reflection (r)

4) It takes place in mirrors

Refraction

1) It is the shift in the direction of light ray when it enters medium

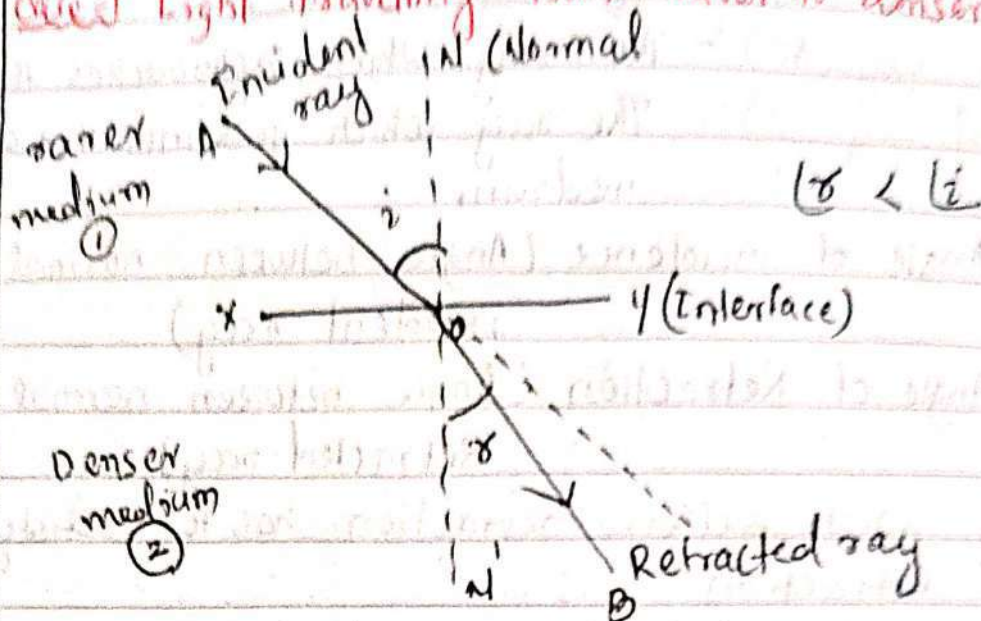
(2) Passes through the surface that changes their speed

(3) Angle of incidence (i) is not equal to the angle of reflection (r).

(4) It occurs in lenses.

Relative magnitudes of i & r

Case-1 Light travelling from rarer to denser medium

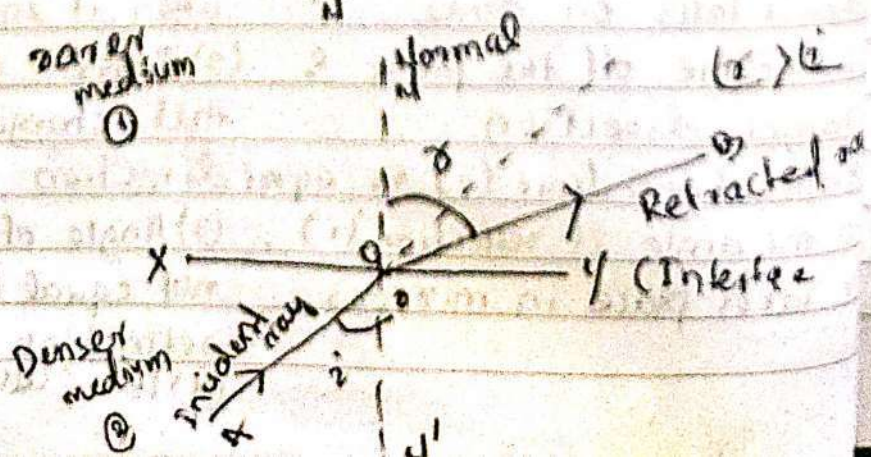
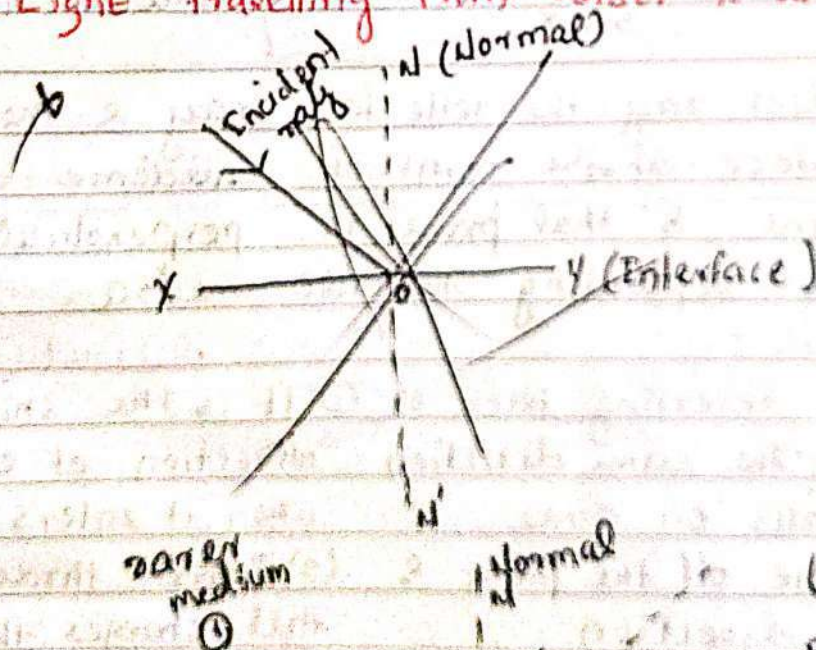


When the light ray travels from rarer to denser medium, the refracted ray bends towards the normal.

So i is less than r &

& Angle of deviation = $(i - r)$

Case-2 Light travelling from denser to rarer medium



When the light goes from denser medium to rarer medium the refracted ray bend away from the normal.

So r is greater than i .

Angle of deviation = $r - i$

Refractive Index (μ)

It is a measure of Optical density of the medium.

* A medium having a greater value of Refractive Index is said to be optically denser than that having a lower value.

$$\mu_a = 1, \mu_w = 1.33, \mu_g = 1.5$$

(1) Definition of Refractive Index

From Snell's law, $\frac{\sin i}{\sin r} = \text{constant}$
 $= \mu_2$

where $\mu_2 \rightarrow$ Refractive Index of 2nd medium w.r.t 1st medium

If the light ray travels from air medium to water medium, then it can be written as

$$\mu_{w/a}$$

(2) $\mu_2 = \frac{v_1}{v_2}$, where $v_1 \rightarrow$ velocity of light in 1st medium
 $v_2 \rightarrow$ vel. of light in 2nd medium

Ex:-

If the 1st medium is air/vacuum, the R.I. is written as Absolute Refractive Index & written as μ_2 or μ

$$\text{So } \mu = \frac{c}{v}, \text{ where } c \rightarrow \text{vel. of light in air}$$

$v \rightarrow$ vel. of light in 2nd medium

$c \rightarrow 3 \times 10^8 \text{ m/s}$ in air/vacuum

③ $\boxed{\mu_2 = \frac{\lambda_1}{\lambda_2}}$, where $\lambda_1 \rightarrow$ wavelength of light in 1st medium
 $\lambda_2 \rightarrow$ wavelength of light in 2nd medium

④ $\mu_2 = \frac{\mu_2}{\mu_1}$, in terms of absolute R.I.

e.g. $\mu_w = \frac{\mu_w}{\mu_a} = \frac{1.3}{1} = 1.3$

Numerical

- ① R.I. of water w.r.t air is $\frac{4}{3}$ while that of glass is $\frac{3}{2}$; what will be the R.I. of glass w.r.t water?

Solⁿ given

R.I. of water w.r.t air is $\frac{4}{3}$

$$\mu_w = \frac{4}{3}$$

$$\mu_g = \frac{3}{2}$$

$$\mu_g = ? = \frac{\mu_g}{\mu_w} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8}$$

- ② A ray of light travelling in water is incident at an angle of 30° on a water glass interface. Calculate the angle of Refraction in glass if R.I. of water is $\frac{4}{3}$ & that of glass is $\frac{3}{2}$.

Solⁿ Given $i = 30^\circ$, $\mu_w = \frac{4}{3}$, $\mu_g = \frac{3}{2}$
 $r = ?$

From Snell's law, $\mu_g = \frac{\sin i}{\sin r} \Rightarrow \frac{9}{8} = \frac{1}{\sin r}$

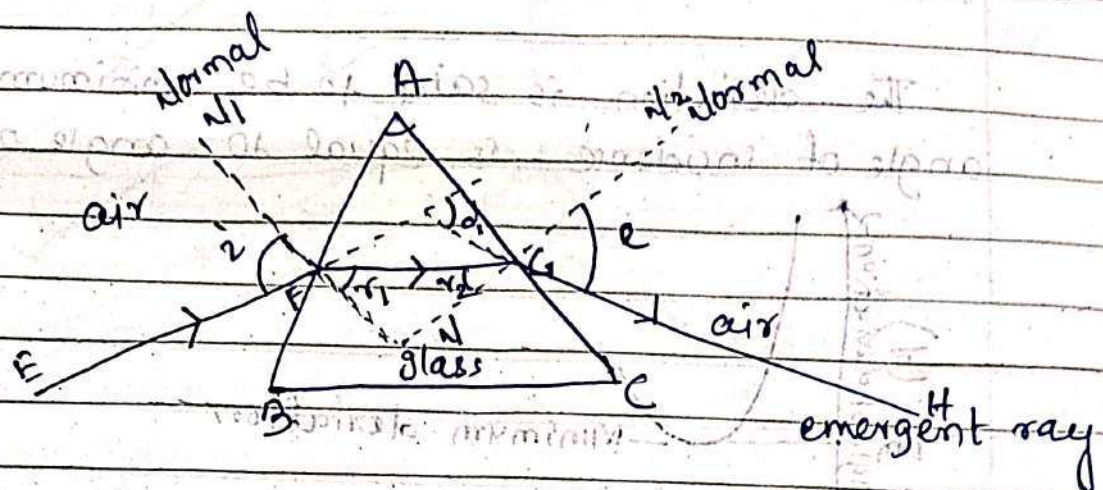
$$\Rightarrow \frac{\mu_g}{\mu_w} = \frac{\sin 30}{\sin r}$$

$$\Rightarrow \sin r = \frac{4}{9}$$

$$\Rightarrow \frac{3}{2} \times \frac{3}{4} = \frac{1}{\sin r}$$

$$\Rightarrow r = 26.4^\circ$$

Refractive Index of material of Prism:-



ABC represents principal section of a glass-prism
A is the refracting angle.

AB $\rightarrow$ Interface separating air medium & glass medium

AC $\rightarrow$ Interface separating glass & air medium.

EF $\rightarrow$ Incident ray incident on the interface AB.

i $\rightarrow$ Angle of Incidence.

FG $\rightarrow$ Refracted ray

r $\rightarrow$ Angle of refraction

For AC face, FG ray becomes incident ray.

So $r = i$

GH $\rightarrow$ Final emergent ray.

e $\rightarrow$ Angle of emergence.

N_1N $\rightarrow$ Normal to the interface AB.

N_2N $\rightarrow$ Normal to the interface AC.

d $\rightarrow$ Angle of deviation.

Refractive Index (μ) of Prism is given by

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \left(\frac{A + D_m}{2} \right)}{\sin \left(\frac{A}{2} \right)}$$

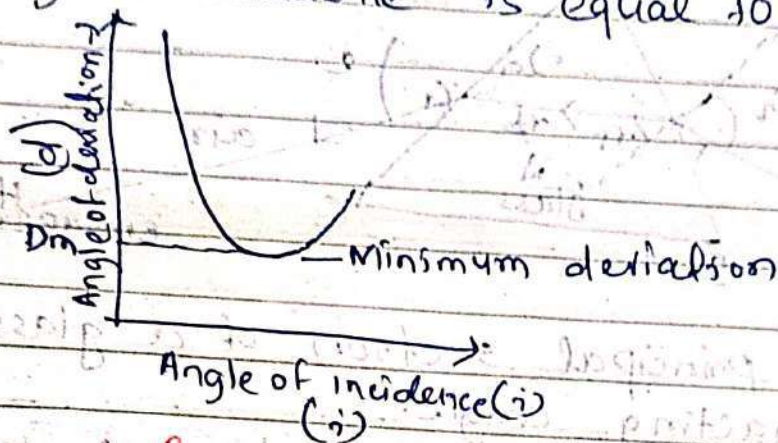
where $i = (A + D_m)/2$ $A \rightarrow$ Angle of Prism

$r = A/2$

$D_m \rightarrow$ Angle of minimum deviation.

Condition for Angle of minimum deviation:-

The deviation is said to be minimum when the angle of incidence is equal to angle of refraction.



Numerical

The refractive index of material of a prism is 1.5. When the prism is placed in minimum deviation position the angle of incidence is 51° . Calculate the angle of prism & the angle of minimum deviation.

Solⁿ Given $\mu = 1.5$, $i = 51^\circ$, $A = ?$, $D_m = ?$

For Prism $\mu = \frac{\sin i}{\sin r}$, $r = \frac{A + D_m}{2}$

$$1.5 = \frac{\sin 51^\circ}{\sin\left(\frac{A}{2}\right)}$$

$$1.5 = \frac{0.7771}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \sin\left(\frac{A}{2}\right) = \frac{0.7771}{1.5} = 0.5180$$

$$\Rightarrow \frac{A}{2} = \sin^{-1}(0.5180)$$

$$\Rightarrow A = 2\sin^{-1}(0.5180) = 62^\circ 24'$$

$$\Rightarrow A = 62^\circ 24'$$

$$D_m = 39^\circ 36'$$

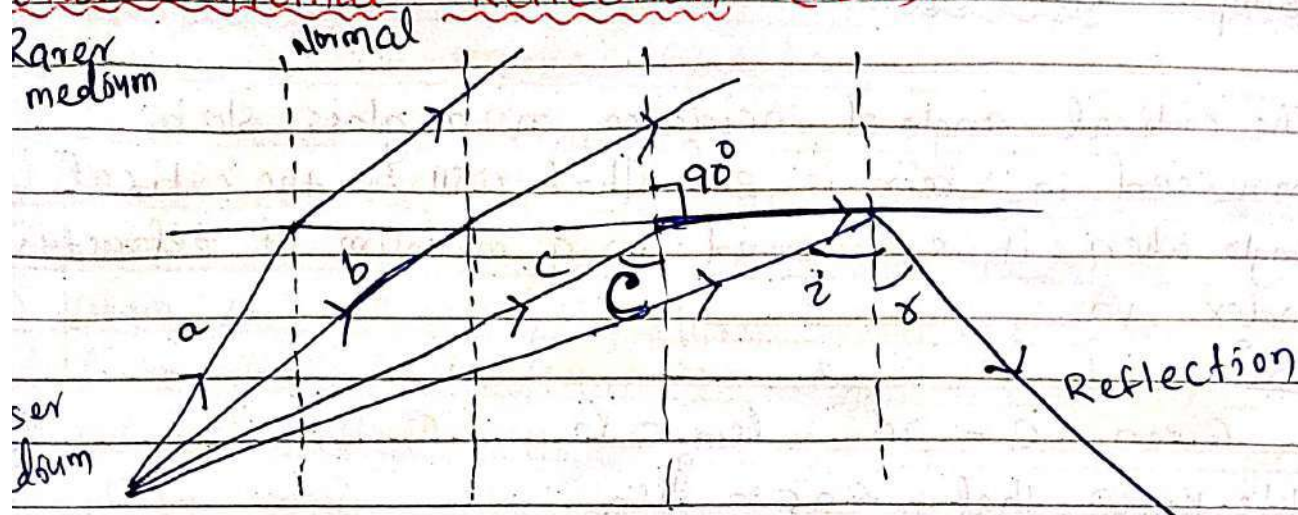
since $i = \frac{A + D_m}{2}$

$$D_m = 2i - A$$

$$= 102^\circ - 62^\circ 24'$$

$$= 39^\circ 36'$$

Total Internal Reflection (TIR)



Critical Angle:- (c)

It is the angle of incidence of ray of light denser medium such that its angle of refraction in rarer medium is 90° .

Total Internal Reflection:-

It is the phenomenon by virtue of which a ray of light travelling from a denser to rarer medium is sent back to the same medium provided it is incident on the surface at an angle greater than critical angle.

Condition for Total Internal Reflection

- 1) Light ray should be travel from denser medium to rarer medium.
- 2) The angle of Incidence should be greater than critical angle.

For ray 'c'

$$\mu_1 = \frac{\sin i}{\sin r} = \frac{\sin C}{\sin 90^\circ} = \sin C$$

$$\Rightarrow \boxed{\mu_1 = \sin C}$$

Relation between Critical angle & Refractive Index

Simple Numerical :-

- ① The critical angle of incidence in a glass slab immersed in air is 30° . What will be the critical angle when it is immersed in a medium of refractive index $\sqrt{2}$.

Sol:

Given $C = 30^\circ$ $\mu_m = \sqrt{2}$

We know that $\sin C = \frac{\mu_a}{\mu_g}$

$$\Rightarrow \frac{\mu_a}{\mu_g} = \sin 30^\circ = \frac{1}{2}$$

$$\Rightarrow \mu_a = \frac{1}{2} \Rightarrow \mu_g = 2$$

When the glass is immersed in a medium whose refractive index $\sqrt{2}$, then A.T relation

$$\sin C = \frac{\mu_m}{\mu_g} = \frac{\mu_m}{2}$$

$$\Rightarrow \sin C = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow C = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

$$\boxed{C = 45^\circ}$$

- ② What is the critical angle for a ray going from glass to water. The refractive indices of glass & water are 1.62 & 1.32 respectively.

Ans:- $\mu_g = 1.62$, $\mu_w = 1.32$

$$C = ?$$

$$\sin C = \frac{\mu_w}{\mu_g}$$

$$\Rightarrow \frac{\mu_w}{\mu_g} = \sin C$$

$$\Rightarrow \frac{1.32}{1.62} = \sin C$$

$$\Rightarrow \frac{1.32}{1.62} = \sin C$$

$$\Rightarrow 0.8148 = \sin C$$

$$\Rightarrow C = \sin^{-1}(0.8148) = 54^\circ 37'$$

Assignment:-

2-marks Question:-

- 1) State laws of Refraction.
- 2) Draw the refraction pattern when the light ray travel from denser medium to rarer medium.
- 3) Draw the refraction pattern when the light ray travel from rarer medium to denser medium.
- 4) Define Critical angle.
- 5) Write any two application of Optical fiber.
- 6) Write the condition for minimum deviation.
- 7) Draw IVD curve for a prism.
- 8) Write the condition for Total Internal Reflection.
- 9) Define Total Internal Reflection.
- 10) A ray of light, while travelling from denser to rarer medium, is incident at critical angle on the interface. What will be the angle of refraction in rarer medium.
- 11) Velocity of light in a medium is found to be $2.25 \times 10^8 \text{ m/s}$. Calculate its absolute Refractive Index.
- 12) How do you define refractive index of a medium w.r.t another? Give two definitions.

5-marks question

- 1) Explain Critical angle & Total Internal Reflection with diagram.
- 2) Draw the refraction pattern through prism.
- 3) Draw the refraction pattern when the light ray travel from a) denser to rarer medium
b) Air to water medium
- 4) Establish the relation between critical angle & Refractive index of medium

Some numericals

- ① The refractive indices of glycerine & water are 1.46 & 1.33 respectively. What is the critical angle when the ray passes from glycerine to water.
- ② The critical angle of incidence in a glass slab immersed in air is 30° . What will be the critical angle when it is immersed in a medium of refractive index $\sqrt{2}$.
- ③ Velocity of light in a medium is found to be 2.25×10^8 m/s. Calculate its refractive index (absolute).
- ④ A ray of light while passing through a prism suffers minimum deviation. Calculate the angle of minimum deviation & the angle of incidence if the refractive index of the material of prism is 1.5 & the angle of prism is 60° .
- ⑤ A ray of light passing through a glass prism of refracting angle 60° , undergoes a minimum deviation of 30° . Calculate the vel. of light in glass if the velocity of light in air is 3×10^8 m/s.
- ⑥ What is the critical angle for a ray going from glass to water.

prepared by Sthitipragyan paramita Nath

Guest faculty in math nd science dept

Magnetostatics

Magnetostatics $\div$ The study of magnet which are at rest.

Magnet Static (Rest)

Properties of a magnet:-

- a) Two poles of a magnet $\div$ North pole & South pole
 Geometric length = Face to face length of the magnet
 Magnetic length = Pole to pole length
 Magnetic length is slightly lesser than the geometric length.

$$\text{Magnetic length, } 2l = \frac{7}{8} \times \text{Geometric length}$$
- b) Attracting property of a magnet $\div$ A magnet is capable of attracting small pieces of iron towards it.
- c) Directional Property $\div$ From North to South pole.
- d) No existence of isolated magnetic poles.
- e) Nature of force between two poles $\div$ Attractive for opposite force & Repulsive for like poles.

Coulomb's law in magnetism:-

Coulomb's force is responsible for force of attraction or repulsion between two poles.

Statement:-

The magnitude of the force between two magnetic poles varies directly as the product of the strength of their poles & inversely as the square of the distance between them.

Consider two magnetic poles of similar nature (n) of pole strengths m_1 & m_2 separated by a distance 'r' from each other.

$\begin{array}{ccc} & m_1 & m_2 \\ & \leftarrow & \rightarrow \\ \text{Force of repulsion between them} & & \end{array}$

$F \propto m_1$, $F \propto m_2$

$F \propto \frac{1}{r^2}$ (Inverse-square law)

$$2) F = K \frac{m_1 m_2}{r^2}$$

where $K \rightarrow$ constant of proportionality.

In C.G.S. system, $K = 1$ (in CGS system)

So Coulomb's force where $\mu_0 = 4\pi \times 10^{-7} \frac{Wb}{Am}$

$F = \frac{m_1 m_2}{r^2}$ $\mu_0 \rightarrow$ Absolute magnetic permeability of free space

So Coulomb's force in SI

$$F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$$

$$= 10^{-7} \frac{m_1 m_2}{r^2}$$

Unit pole

(Pole strength to be unity)

In C.G.S. System

Coulomb's force in CGS is given by

$$F = \frac{m_1 m_2}{r^2}$$

Let $m_1 = m_2 = m$ & $r = 1 \text{ cm}$

Then $F = \frac{m^2}{1} = m^2$

$$2) F = m^2$$

In order to get unit pole strength

let $F = 1 \text{ dyne}$

$$\text{so } m^2 = 1 \Rightarrow m = \pm 1$$

A unit pole in CGS, is that pole which when placed in air at a distance of 1 cm from a similar pole repels it with a force 1 dyne.

Magnetic field (B) :-

Magnetic field of any magnetic pole is the region around it in which its magnetic influence can be realised.

Magnetic lines of force

It is the path along which a unit north pole would move.

- Lines of force is a straight line for isolated magnetic pole
- Lines of force is a curved line for combination of poles.

Properties of magnetic lines of force :-

- 1. Lines of force are directed away from a north pole & directed towards a south pole.
- Tangent at any point to the magnetic lines of force gives the direction of magnetic field intensity at that point.
- Two magnetic lines of force never intersect each other.

Reason If two lines were to cross, two tangents could be drawn to the magnetic lines of force at the common point, thereby giving two direction of magnetic intensity at that point, which is obviously not possible.

The more concentration of magnetic lines of force represents stronger magnetic field.

* All lines of force start from a unit magnetic pole.

Unit of magnetic field (B)

CGS

Gauss (G)

MKS/SI

Tesla (T)

$$1 \text{ T} = 10^4 \text{ Gauss}$$

Magnetic flux (Φ_B)

It gives the information about the number of magnetic lines of force of magnetic field cross a certain area.

Magnetic flux Φ_B through area $\vec{A}$

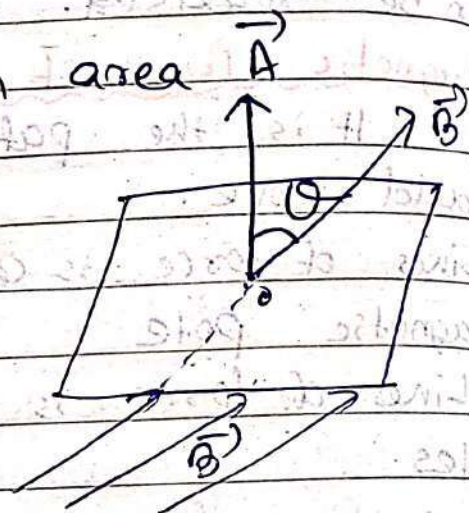
'a' is given by

$$\Phi_B = \vec{B} \cdot \vec{A} = |\vec{B}| |\vec{A}| \cos \theta$$

Case-1 when $\theta = 90^\circ$,

$$\cos \theta = 0$$

$$\Rightarrow \boxed{\Phi_B = 0}$$



No magnetic flux is linked with the surf when the field is parallel to the surf.

Case-2, if $\theta = 0^\circ$, $\cos \theta = 1$

$$\boxed{(\Phi_B)_{\max} = |\vec{B}| |\vec{A}|}$$

Unit of magnetic flux (Φ_B)

$$\text{SI / MKS } |\Phi_B|_{\max} = |\vec{B}| |\vec{A}|$$

CGS

$$\boxed{\text{Weber} = \text{Tesla} \times \text{m}^2}$$

$$\text{Gauss} \times \text{cm}^2$$

$$= \boxed{\text{Maxwell}}$$

SI unit of magnetic flux is

$$\boxed{\text{Weber (Wb)}}$$

CGS unit of

flux is $\boxed{\text{Maxwell}}$

$$1 \text{ Wb} = 1 \text{ T} \times 1 \text{ m}^2$$

Relation between Weber & Maxwell

$$1 \text{ Weber} = 1 \text{ Tesla} \times 1 \text{ m}^2$$

$$= 10^4 \text{ Gauss} \times (100)^2 \text{ cm}^2$$

$$= 10^4 \text{ G} \times 10^4 \text{ cm}^2$$

$$= 10^8 \text{ G} \times \text{cm}^2$$

$$1 \text{ kG} = 10^8 \text{ Maxwell}$$

Dim. Formula of Φ_B

$$(\Phi_B)_{\text{max}} = B A$$

$$= \frac{F}{q v} A$$

$$[\Phi_B] = \frac{[M L^2 T^{-2}] [L^2]}{[A^{-1} T^{-1}] [L T^{-1}]}$$

$$[\Phi_B] = [M L^2 T^{-2} A^{-1}]$$

prepared by Sthitipragyan paramita Nath
Guest faculty in math nd science dept

Electrostatics
The study of electric charges which are at rest.

Coulomb's laws of electrostatics:-

Like charges repel each other whereas unlike charges attract each other. The force between charges is Coulomb's Force.

The electrostatic force of attraction & repulsion between two charged bodies is proportional to the product of their charges & varies inversely as the square of the distance between the two bodies.

Consider two charges q_1 & q_2 are situated at a distance ' r ' from each other.

The electrostatic force F is given by

$$F \propto q_1, F \propto q_2$$

$$F \propto \frac{1}{r^2} \text{ (Inverse-square law)}$$

$$\Rightarrow F \propto \frac{q_1 q_2}{r^2} \quad \text{or} \quad \boxed{F = \beta \frac{q_1 q_2}{r^2}}$$

where ' β ' is called proportionality constant depends upon nature of the medium in which charges are situated & the system of unit. Coulomb's law is valid for point charges.

value of β

in CGS

$$\beta = \frac{1}{K}, \quad K \rightarrow \text{Dielectric constant of the medium}$$

So Coulomb's law is given by
$$\boxed{F = \frac{1}{K} \frac{q_1 q_2}{r^2}} \quad (\text{for any medium})$$

For air medium, $K = 1$

So Coulomb's law in air medium is
$$\boxed{F_{\text{air}} = \frac{1}{K} \frac{q_1 q_2}{r^2} = \frac{q_1 q_2}{r^2}}$$

In SI units:-

$$\beta = \frac{1}{4\pi\epsilon_0\epsilon_r}$$

where $\epsilon_0 \rightarrow$ Permittivity of free space $= 8.854 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

$\epsilon_r \rightarrow$ Relative permittivity of the given medium.

So Coulomb's law in SI is given by

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1 q_2}{r^2}$$

but $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

So $F = \frac{9 \times 10^9}{\epsilon_r} \frac{q_1 q_2}{r^2}$ For any medium

For air medium, $\epsilon_r = 1$

$$F_{\text{air}} = 9 \times 10^9 \frac{q_1 q_2}{r^2}$$

Note

$$\epsilon_r = K$$

Coulomb's law in different system of unit

SI

$$\beta = \frac{1}{4\pi\epsilon_0\epsilon_r}$$

Coulomb's law is given by

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \left(\frac{q_1 q_2}{r^2} \right) \text{ For any medium}$$

For air medium $\epsilon_r = 1$

So Coulomb's law in air medium is

$$F_{\text{air}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

but $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

$$F_{\text{air}} = 9 \times 10^9 \frac{q_1 q_2}{r^2}$$

C.G.S

$$\beta = \frac{1}{K}$$

Coulomb's law is given

$$F = \frac{1}{K} \left(\frac{q_1 q_2}{r^2} \right) \text{ For any medium}$$

For air medium

$$K = 1$$

Coulomb's law in air medium is given by

$$F_{\text{air}} = \frac{q_1 q_2}{r^2}$$

Relative Permittivity (ϵ_r):-

Relative permittivity of a medium is defined as the ratio between absolute permittivity (ϵ) of the medium & the absolute permittivity (ϵ_0) of the free space.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}, \text{ Unitless quantity.}$$

Unit of ϵ_0

$$\epsilon_0 = \frac{1}{4\pi} \frac{q_1 q_2}{F \times r^2} = \frac{C^2}{N \times m^2}$$

Dim. Formula for ϵ_0

$$[\epsilon_0] = \frac{[q_1][q_2]}{[F] \times [r]^2} = \frac{[A^1 T^1]^2}{[M^1 L^{-1} T^{-2}][L]^2}$$

$$[\epsilon_0] = [M^{-1} L^3 T^4 A^2]$$

Unit charge

In CGS system

In CGS system, Coulomb's law is given by

$$F = \frac{1}{K} \frac{q_1 q_2}{r^2}$$

For air, $K = 1$. So Coulomb's law in air is

$$F = \frac{q_1 q_2}{r^2}$$

Let $q_1 = q_2 = q$ (say), $r = 1 \text{ cm}$

$$\text{then } F = q^2$$

Let $F = 1 \text{ dyne}$, then $q^2 = 1 \Rightarrow q = \pm 1$

Unit charge in CGS is that amount of charge which when placed in air at a distance of 1 cm from a similar ~~point~~ charge repels it with force of 1 dyne.

In SI system:-

In SI, Coulomb's law is given by

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1 q_2}{r^2}$$

If the charges are placed in air medium, then $\epsilon_r = 1$

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \approx 9 \times 10^9 \frac{q_1 q_2}{r^2}$$

Let $q_1 = q_2 = q$ & $r = 1 \text{ m}$, then

$$F = \frac{1}{4\pi\epsilon_0} q^2$$

$$F \approx 9 \times 10^9 q^2$$

Let $F = 9 \times 10^9 \text{ N}$, then

$$9 \times 10^9 = 9 \times 10^9 q^2$$

$$\Rightarrow q^2 = 1$$

$$\Rightarrow q = \pm 1 \text{ C}$$

Unit charge in SI is that charge which when placed in air at a distance of 1m from a similar charge repels it with a force $9 \times 10^9 \text{ N}$.

Electric Potential:-

It determines the direction of flow of charge between two bodies.

Flow of electric charge always from higher potential to lower potential.

OR

Electric potential at any point in an electric field is defined as the amount of work done in moving a unit positive charge from infinity to that point.

$$\text{So } V = \frac{W}{q}$$

Unit of Potential or potential difference

SI unit = Volt

$$V = \frac{W}{q}$$

$$1 \text{ volt} = \frac{1 \text{ J}}{1 \text{ C}} = 1 \text{ J/C}$$

Electric field

When an electric charge is placed at a point, the properties of space around the charge get modified. This space around the electric charge is called electric field.

Electric field Intensity (E)

It is the force experienced by a unit charge placed at that point.

ie,

$$E = \frac{F}{+q}$$

If $q = 1 \text{ C}$

then

$$E = F$$

Capacitance

If 'V' is the potential of the conductor and a charge 'Q' is given to it, then

$$Q \propto V \Rightarrow Q = CV \quad \text{--- (1)}$$

where 'C' is known as the capacity of the conductor.

From eqn (1), $C = \frac{Q}{V}$

The capacity of a conductor is defined as the ratio between the charge on the conductor and its potential.

If $V = 1 \text{ volt}$, then $C = Q$

So capacity of a conductor is defined as the charge required to raise it through a unit potential.

Unit of Capacity

Since $C = Q/V$

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

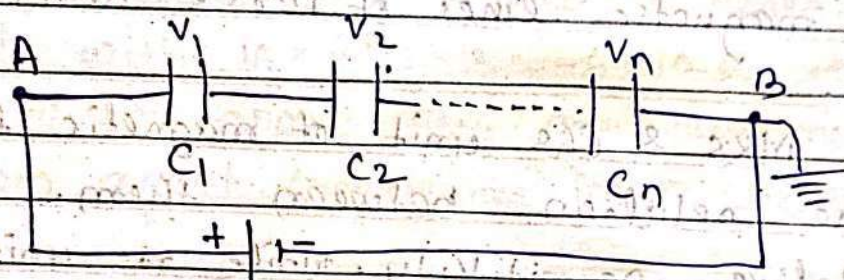
SI unit of capacity is Farad.

Grouping of capacitors

The number of capacitors can be grouped in two ways

- Series connection
- Parallel connection

Series Connection



In series combination, each capacitor is charged with the same charge

so Effective capacitance is

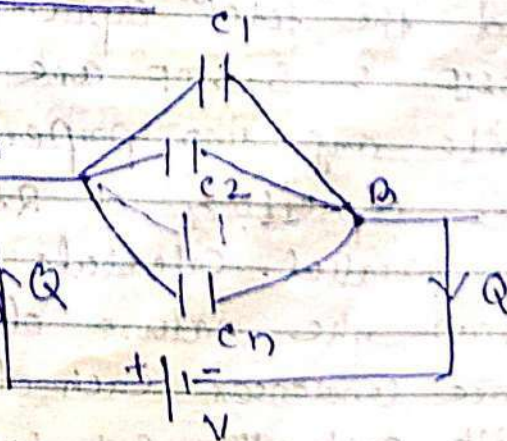
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

If there are 'n' number of identical capacitors, then effective capacitance

$$C_{eq} = \frac{C}{n}$$

Parallel Combination:-

Since all the capacitors are connected between two common points A & B, therefore across potential difference the potential difference across each of them is same



Effective capacitance for parallel combination,

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_n$$

If there are 'n' number of identical capacitors connected in parallel, then

$$C_{eq} = nC$$

Assignment Questions

2 marks question:-

1. a) Define electrostatics.
- b) Define unit charge.
- c) Why two magnetic lines of force never intersect each other.
- d) Write the MKS & CGS unit of magnetic flux. Establish the relation between them.
- e) Define Relative permeability. Write its unit.
- f) Two condensers of capacity $4\mu F$ & $6\mu F$ are connected in series. Find the equivalent capacity of the condenser.
- (g) What does a magnetic lines of force mean.
- (h) Write the unit & dimensional formula of ϵ_0 .
- (i) Two equal & similar charges $0.03m$ apart in a repel each other with a force of 4.5 kgf . Find charge in coulomb.
- (j) A capacitor with a capacity of $50\mu F$ is connected to a battery of 400 volt . Find the charge.
- (k) Find the total capacity when two capacitors of $4\mu F$ & $3.3\mu F$ are connected in series.
- (l) Write any two properties of magnetic lines of force.
- (m) Three capacitors of $2\mu F$, $3\mu F$ & $5\mu F$ are connected in parallel. Calculate the resultant capacity.
- (n) Draw the nature of Inverse square law.
- (o) Three condensers each of $2\mu F$ are connected in parallel with each other. Calculate the net capacity.

Long type Question:-

State & explain Coulomb's law in magnetism.

State & explain Coulomb's law in electrostatics.

Write the properties of magnetic lines of force.

Numericals:-

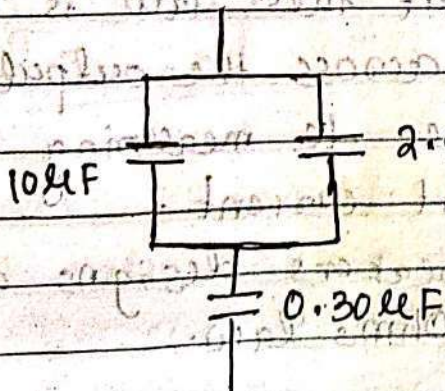
Two equal & similar charges 0.03m apart in air repel each other with a force of 3.5kgf . Find the charge in Coulomb.

Two equal conducting spheres of negligible size are charged with $16 \times 10^{-14}\text{C}$ & $-6 \times 10^{-14}\text{C}$ respectively & are placed 0.20m apart. They are then moved to a distance of 0.50m apart. Compare the forces between them in two position.

Find the total capacity when two capacitors of capacity $400\mu\text{F}$ & 3.3mF are a) connected in series
b) connected in parallel.

Two magnetic poles of strength 5N/T each are separated by a distance of 0.2m . Calculate the force between them.

Two similar magnetic poles repel each other with a force 10^{-5}N when separated 1m apart in air. Calculate the strength of each pole.



Find the total capacitance of the combination of capacitors.

Unit-11 : Electromagnetism & Electromagnetic Induction

Electromagnetism:-

Production of magnetic field due to a current flowing through a conductor.

Force on a conductor carrying current placed in a uniform magnetic field.

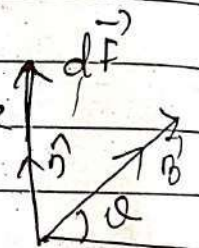
Consider a conductor xy placed in a uniform magnetic field $\vec{B}$. A charge q moves with a velocity $\vec{v}$ in such a way that the direction of motion of charge makes an angle θ with the direction of magnetic field $\vec{B}$. Current ' i ' flows through conductor from x to y .

Then the force $d\vec{F}$ experienced by this charge is given by

$$d\vec{F} = dq (\vec{v} \times \vec{B})$$

If the charge travels a small distance $d\vec{l}$ in time dt , then

$$\vec{v} = \frac{d\vec{l}}{dt}$$



$$\begin{aligned} \text{so } d\vec{F} &= dq \left(\frac{d\vec{l}}{dt} \times \vec{B} \right) \\ &= \frac{dq}{dt} (d\vec{l} \times \vec{B}) \end{aligned}$$

$$d\vec{F} = i (d\vec{l} \times \vec{B})$$

Net force acting on the conductor can be given by

$$F = \int dF = i \int d\vec{l} \times \vec{B}$$

$$\vec{F} = i (\vec{l} \times \vec{B})$$

$$F = i l B \sin \theta$$

2 Fleming's left hand rule:-

Fleming's left hand rule is used to determine the direction of force on a current carrying conductor.
statement:-

If we stretch first finger, central finger & the thumb of our left hand in mutually perpendicular directions, then the first finger points towards the magnetic field, central finger points towards the electric current, then the thumb will point towards the direction of force acting on the conductor.

Vector diagram

first finger $\rightarrow$ Magnetic field $\uparrow \vec{F}$ (Thumb)

central finger $\rightarrow$ Electric current

Thumb $\rightarrow$ Direction of force

Application:-

1. It is applied in DC Motor.

Electromagnetic Induction:-

The induction of current electricity due to ~~current~~ ~~electricity~~ changing magnetic field is called electromagnetic induction.

Electric current can be induced by changing magnetic field in two different ways.

a) Induction due to motion of magnet towards the coil.

b) Induction due to motion of coil towards the magnet

Electric current or e.m.f can be induced either by moving the magnet (keeping coil const.) or by moving the coil (keeping magnet constant). As the magnet approaches the coil, the strength of magnetic field around the coil increases which results in an increase in magnetic flux, thereby observing the deflection in the galvanometer.

indicating the induction of electric current. It also observed that the direction of deflection of galvanometer gets reversed if the magnet is away from the coil. So electric current is due to motion of magnet.

Faraday's laws of Electromagnetic Induction

Faraday's law deals with the induction of e.m.f in an electric circuit when magnetic flux linked with the circuit changes.

1. Whenever magnetic flux linked with a circuit changes, an e.m.f is induced in it.
2. The induced e.m.f exist in the circuit so as the change in magnetic flux linked with it is continuous.

imp 3. The induced e.m.f is directly proportional to negative rate of change of magnetic flux linked with the circuit.

If $\Delta\phi_B$ is the change in magnetic flux linked with a circuit.

Rate of change of magnetic flux = $\frac{d\phi_B}{dt}$

If 'E' is the e.m.f induced in the circuit

$$E \propto - \frac{d\phi_B}{dt} \Rightarrow E = -K \frac{d\phi_B}{dt}$$

By selecting units of 'E', ' ϕ_B ' & 't' in a proper way we have $K = 1$

$$\text{So } E = - \frac{d\phi_B}{dt}$$

'-ve' sign is due to direction of induced e.m.f

Lenz's law:-

It deals with the direction of e.m.f induced in the circuit due to a change in magnetic flux linked with it.

"It states that the direction of induced e.m.f is such that it tends to oppose the very cause which produces it."

Fleming's Right hand rule:-

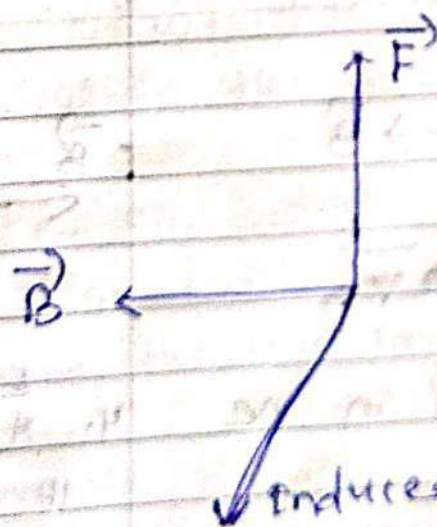
This rule is used to find the direction of induced current in a conductor.

Statement:-

"If we stretch our first finger, central finger & the thumb of our right hand in three mutually perpendicular directions, if the first finger points towards the magnetic field ($\vec{B}$), thumb points towards the direction of motion of conductor, then the central finger gives the direction of induced current set up in the conduct."

Application

Fleming's right hand rule is applied in ac generator.



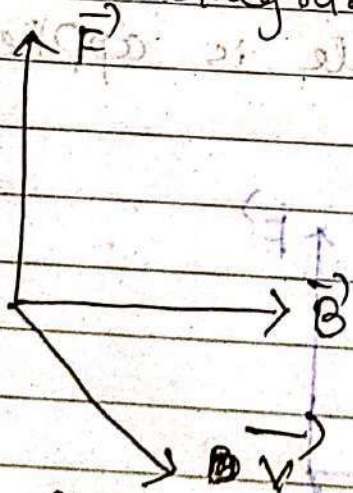
(Fleming's Right hand rule)

Comparison between Fleming's left hand rule & Fleming's right hand rule

Fleming's left hand rule

1. It is used to find the direction of magnetic force on a conductor
2. Stretch first finger, middle finger & the thumb of our left hand in three mutually perpendicular direction, then first finger points towards the direction of magnetic field, central finger points towards the direction of current & the thumb will point towards the direction of magnetic force

B. Vector diagram



4. Application

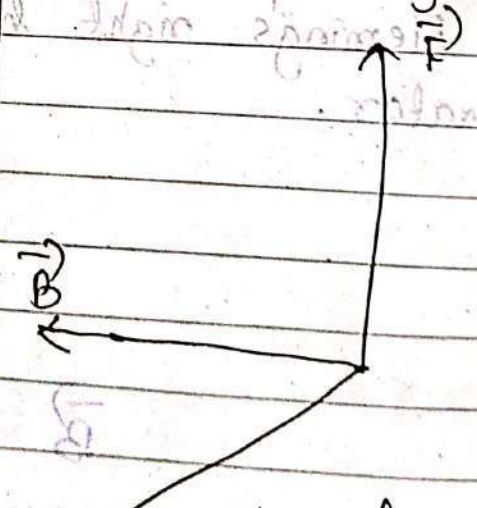
It is applied in DC Motor

5. Permanent magnetic field & current is provided

Fleming's right hand rule

1. It is used to find direction of induced current in a conductor.
2. Stretch first finger, middle finger & the thumb of our right hand in three mutually perpendicular direction, then first finger points towards the direction of magnetic field, middle finger points towards the direction of induced current & the thumb points towards the direction of force

B. Vector diagram



4. Application

It is applied in generator.

5. Permanent magnetic & force is provided

Assignment

1. Define electromagnetism
2. State Fleming's left hand rule.
3. State Fleming's right hand rule
4. Define electromagnetic Induction.
5. State Lenz's law
6. How induced e.m.f is related to magnetic flux (ϕ)
7. Draw the vector diagram for Fleming's left hand & right hand rule.
8. What is the value of constant of proportionality in Faraday's law of electromagnetic Induction.
9. Which rule, Fleming's left hand or Fleming's right hand rule, is applied in case of a dynamo.
10. Give the comparison statement between Electromagnetism & Electromagnetic Induction.

Long type question

- 1) State Faraday's laws of electromagnetic Induction.
- 2) Compare between Fleming's left hand & right hand rule.
- 3) Magnetic flux in a circuit changes from 1.5 kwb to 2 wb in 10^{-2} sec . Calculate the magnitude of induced e.m.f.
- 4) An induced e.m.f of 5 V is set up in a circuit when the magnetic flux linked with it changes from 3 kwb to 3.5 kwb . Calculate the time taken by the flux to change.
- 5) A field of 0.0125 T is at right angles to a coil of area $5 \times 10^{-3} \text{ m}^2$. It is removed from the field in $1/20 \text{ sec}$. Find the e.m.f induced.