

Lecture notes DESIGN OF RCC STRUCTURE



Faculty name: Tapaswini Mohanty
PTGF(AA Department)

1.0. Introduction

Design of RCC structure

Aim, objectives & scope of the subject

2.0. Analyze the single reinforced section

2.1. Define the terms related to single RCC section.

2.2. State & explain the properties of reinforced concrete, grades of concrete

2.3. Discuss the basic assumptions.

2.4. Discuss the modular ratio.

2.5. Discuss the distribution of stress in steel & concrete.

2.6. Discuss the equivalent concrete area.

2.7. Discuss the stress & strain diagrams.

2.8. Discuss the neutral axis & its location.

2.9. Discuss the balance, under reinforced & over reinforced.

2.10. State with examples of problems
- Moment of resistance calculation

Design of RCC structure

1.0 Introduction :-

Aim, objectives & scope of the subject-

Aim :- The aim of RCC design is to create safe, serviceable, durable, economical & constructible reinforced concrete structures that can resist expected loads throughout their lifespan while adhering to relevant codes & standards.

Objectives :- The objective of RCC design is to develop structural members that are strong, stable, durable, serviceable, economical & easy to construct, while complying with codal provisions to ensure safety & reliability throughout their service life.

Scope :- The scope of RCC design covers the analysis & design of basic structural elements (beams, slabs, columns & footings) under gravity loads, using

working stress and limit state methods,
as per IS codes.

Plain Cement Concrete :-

Plain cement concrete is a hardened mass obtained from a mixture of cement, sand, gravel & water in definite proportions. These ingredients are mixed together to form a plastic mass which is poured into desired shape moulds called as forms. This plastic mass hardens on setting & we get plain cement concrete. The hardening of this mixture is caused by a chemical reaction between cement & water.

Plain cement concrete has good compressive strength but very little tensile strength, thus limiting its use in construction. Plain concrete is used where good compressive strength & weight are the main requirements & tensile stresses are very low.

Reinforced Cement Concrete :-

Plain cement concrete has very low tensile strength. To improve the tensile strength of concrete, some sort of reinforcement is needed which can take up the tensile stresses developed in the structure. The most common type of reinforcement is in the form of steel bars which are quite strong in tension.

The reinforcing steel is placed in the forms & fresh concrete is poured around it. This solidified composite mass is called as Reinforced cement-concrete & is abbreviated as RCC.

Thus, Reinforced cement concrete is a composite material which is made up of concrete & steel reinforcement.

The steel reinforcement, generally in the form of steel bars, are placed in the tensile zone of the structure

and take up the tensile stresses. RCC is a versatile construction material which is strong in compression as well as tension. The use of reinforcement in concrete not only increases its strength but also helps in preventing the temperature & shrinkage stresses.

Uses of RCC :-

- | | |
|----------------|------------------------|
| 1. Buildings | 4. Road & Rail bridges |
| 2. Flyovers | 5. Chimneys & towers. |
| 3. Watertanks. | |

Properties of RCC

1. Strength :- RCC has very good strength in tension as well as compression.
2. durability :- RCC structures are durable if designed & laid properly. They can last up to 100 years.
3. Mouldability :- RCC sections can be given any shape easily by properly.

designing the formwork. Thus, it is more suitable for architectural requirements.

1. Ductility :- The steel reinforcement imparts ductility to the RCC structures.

2. Economy :- RCC is cheaper as compared to steel. ~~There~~ There is an overall economy by using RCC because its maintenance cost is low.

3. Transportation :- The raw materials which are required for RCC, i.e., cement, sand, aggregate, water & steel are easily available & can be transported easily.

4. Fire resistance :- RCC structures are more fire resistance than other commonly used construction materials like steel & wood.

5. Permeability : RCC is almost impermeable to moisture.

9. Seismic (Earthquake) Resistance :-

Properly designed RCC structures are extremely resistant to earthquakes.

Grades of Concrete :-

Concrete is graded or designated on the basis of its compressive strength.

As per IS 456:2000 concrete is graded into fifteen types as given in the table 2 (IS 456:2000).

Group	Designation	Characteristic Compressive strength, f_{ck} (N/mm ²)
Ordinary Concrete	M10	10
	M15	15
	M20	20
Standard Concrete	M25	25
	M30	30
	M35	35
	M40	40
	M45	45
	M50	50
	M55	55
High strength Concrete	M60	60
	M65	65
	M70	70
	M75	75
	M80	80

Concrete grades are expressed by letter M followed by a number. The letter 'M' refers to the mix & the no. refers to the characteristic compressive strength of concrete in N/mm^2 .

The specified characteristic strength is determined for 150mm size cube at 28 days.

The characteristic compressive strength of concrete is defined as that strength below which not more than 5 percent of the test results are expected to fall.

Concrete of grades lower than M20 may be used for plain concrete works, lean concrete, simple foundation, masonry walls & other simple construction works.

Reinforcing Material :-

Concrete is weak in tension & it is to be reinforced properly with suitable material. The purpose of providing reinforcement in RCC is :

- To take up all the tensile stresses developed in the structure.
- To increase the strength of concrete sections.
- To prevent the propagation of cracks developed due to temperature & shrinkage stresses.
- To make the sections thinner as compare to plain concrete section.

Suitability of steel as reinforcing material :-

- ① Steel is very strong in compression, tension, shear & torsion.
- ② Concrete develops very good bond with steel.
- ③ Steel is ductile in behaviour. More ductility means more elongation of steel before failure. This results in sufficient warning time before failure.
- ④ The steel bars can be cut, bent, lifted & welded easily with commonly

available tools & machines .

- ⑤ Steel has longer life .
- ⑥ Steel is easily available .

Types of Steel Reinforcement :-

- ① Mild steel plain bar (Fe 250)
- ② High yield strength deformed bars (HYSD) bars or Tor steel (Fe 415 & Fe 500)
- ③ Hard drawn steel wire fabrics (Wire mesh)
- ④ Rolled steel members (Angles, T sections, channels etc.)
- ⑤ TMT bars (Thermo-mechanically treated bars) and Corrosion resistant steel (CRS) bars .

Characteristic strength of steel :-

It means that value below which not more than 5% of the test results are expected to fall . As per IS 456 : 2000, the characteristic strength of steel is equal to the minimum yield stress. ~~or~~

Types of steel	Grade	Yield stress / Characteristic strength (N/mm^2)
Mild Steel	Fe 250	250
HYSD	Fe 550 (Tor 55)	550
High Strength	Fe 415 (Tor 40)	415
Deformed Steel	Fe 500 (Tor 50)	500

Types of loads on RCC structures ;—

- (i) Dead load (self weight of the structure)
- (ii) Live load or imposed load (temporarily placed on the structure)
- (iii) Wind load
- (iv) Snow load
- (v) Earthquake load

Unit weights of common building materials

- 1. PCC — 24 kN/m^3
- 2. RCC — 25 kN/m^3
- 3. Steel — 78.5 kN/m^3

Methods of RCC design :-

Design of any RCC member comprises of the following :-

- (i) To decide the size (dimensions) of the member & the amount of reinforcement required.
- (ii) To check whether the adopted section will perform safely & satisfactorily during the life time of the structure.

Various methods used for the design of RCC structures are as follows :-

- (i) Working stress method
- (ii) Limit state method
- (iii) Ultimate load method

(i) Working stress method :-

This method is based on the elastic theory & assumes that both steel & concrete are elastic & obey Hooke's law. It means that stresses are directly proportional to strain up to the point of

collapse. Based on the elastic theory, & assuming that the bond between steel & concrete is perfect, permissible stresses of the materials are obtained. The basis of this method is that the permissible stresses are not exceeded anywhere in the structure when it is subjected to worst combination of working loads.

(iii) Ultimate load Method :-

In this method, ultimate load or collapse load is used as design load. This method takes into account the plastic behaviour of materials. This gives very thin sections which result in excessive deformation & cracking. Thus, making the structure almost unserviceable. This method is not at all used by designers.

(iv) Limit state method :-

The aim of this method is that the structure should be able to withstand safely all the load that are liable to act on it throughout

its life & it should also satisfy the serviceability requirements of limiting deflection & cracking. Limit state is defined as the acceptable limit of safety & serviceability requirements before failure.

There are of two types.

- (i) limit state of collapse
- (ii) limit state of serviceability

(i) limit state of collapse:- It corresponds to the maximum load carrying capacity i.e., the safety requirements of the structure. The limit state of collapse is assessed from collapse of the whole or part of the structure. As per limit state, the resistance to bending, shear, torsion & axial loads at every section shall not be less than that produced by the most unfavourable combination of loads on that structure.

types of limit state of collapse are

- (i) limit state of collapse in flexure (bending)

(iv) Limit state of collapse in compression

(v) Limit state of collapse in shear

(vi) Limit state of collapse in torsion

(ii) Limit state of serviceability :-

A structure is of no use if it is not serviceable. Thus, ^{this} limit state is introduced to prevent excessive deflection & cracking. This is of two types.

(i) Limit state of deflection

(ii) Limit state of cracking

Characteristic Load :- It is that value of load which has a 95 percent probability of not being exceeded during the life time of the structure. In absence of any data, loads given in various standards shall be assumed as the characteristic loads.

IS 875 (part 1) - for dead loads

IS 875 (part 2) - for ~~the~~ live load

IS 875 (part 3) - for W.L.

IS 875 (part 1) - for snow load

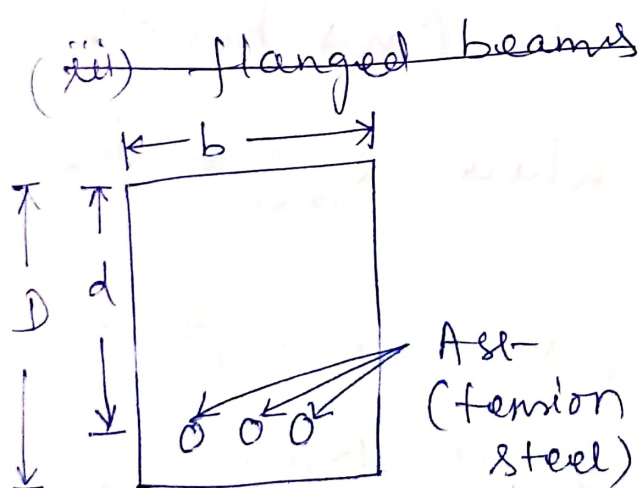
~~IS 875 (part 5)~~

IS 1893 (part 1) - for earthquake loads

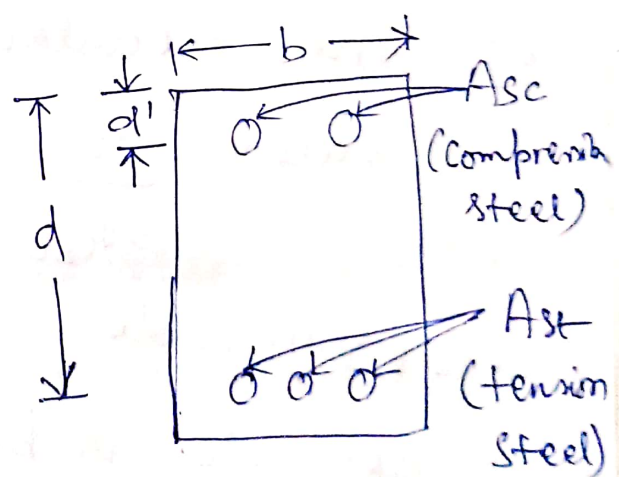
Types of RCC beams :-

(i) Singly Reinforced Beams :- The beams in which steel reinforcement is placed in the tensile zone only are called as singly reinforced beams.

(ii) Doubly reinforced beams :- The beams in which reinforcement is placed in the tensile as well as compression zone are called as doubly reinforced beam.



(i) Singly
reinforced



(ii) doubly
reinforced

Assumptions in the working stress method

- 1) A section which is plane before bending remains plane after bending.
- 2) The concrete & steel reinforcement are perfectly bonded.
- 3) All tensile stresses are taken up by steel & none by concrete.
- 4) The stress-strain relationship of steel & concrete under working loads is a straight line.
- 5) The moduli of elasticity of steel (E_s) & concrete (E_c) are constant.
- 6) The modular ratio (m) has the value $\frac{280}{3\sigma_{cbc}}$, where σ_{cbc} is the permissible compressive strength of concrete in bending in N/mm^2 .
- 7) There are no initial stresses in steel & concrete.

Factor of Safety :- It is the ratio between the ultimate (actual) strength of a material / structure & the allowable or design stress / load used in calculations.

It ensures that the structure has a margin of safety against unexpected loads, variations in material properties, workmanship errors & long-term effects.

The FOS used in working stress method are :

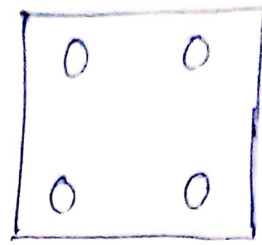
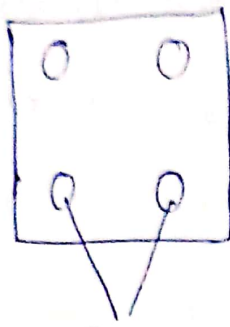
- (i) for concrete -
 - (a) in bending compression - 3.0
 - (b) in direct compression - 4.0
- (ii) for steel - 1.78

Modular ratio :- It is denoted by letter 'm'.
$$m = \frac{E_s}{E_c} = \frac{\text{Elastic modulus of steel}}{\text{Elastic modulus of concrete}}$$

Concept of Equivalent Section :-

Consider an RCC section

subjected to a compressive load P .



(a) RCC section

(b) transformed section
in terms of concrete

Let $A \rightarrow$ Area of cross section

$A_c \rightarrow$ Area of concrete

$A_{st} \rightarrow$ Area of steel

$m \rightarrow$ modular ratio

$\sigma_s \rightarrow$ stress in steel

$\sigma_c \rightarrow$ stress in concrete

$\epsilon_s \rightarrow$ strain in steel

$\epsilon_c \rightarrow$ strain in concrete

$P_s \rightarrow$ load carried by steel

$P_c \rightarrow$ load carried by concrete

$A_{eqc} \rightarrow$ Equivalent area of section
in terms of concrete

$E_s \rightarrow$ Young's modulus of elasticity of steel

$E_c =$ Young's modulus of elasticity of concrete

$$P = P_s + P_c$$

$$\Rightarrow P = \sigma_s A_{st} + \sigma_c A_c \quad \text{--- (i)}$$

Assuming perfect bond between steel & concrete, strains in steel & concrete will be equal.

$$\epsilon_s = \epsilon_c \left(\text{Strain} = \frac{\text{Stress}}{\text{Young's modulus}} \right)$$

$$\Rightarrow \frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\Rightarrow \sigma_s = \frac{E_s}{E_c} \cdot \sigma_c = (m) (\sigma_c) \quad \text{--- (i)}$$

$$\text{Hence } P = \sigma_s A_{st} + \sigma_c A_c$$

$$= (m \sigma_c) A_{st} + \sigma_c A_c$$

$$= \sigma_c (m A_{st} + A_c)$$

$$\Rightarrow \sigma_c = \frac{P}{A_c + m A_{st}} = \frac{P}{A_{eqc}}$$

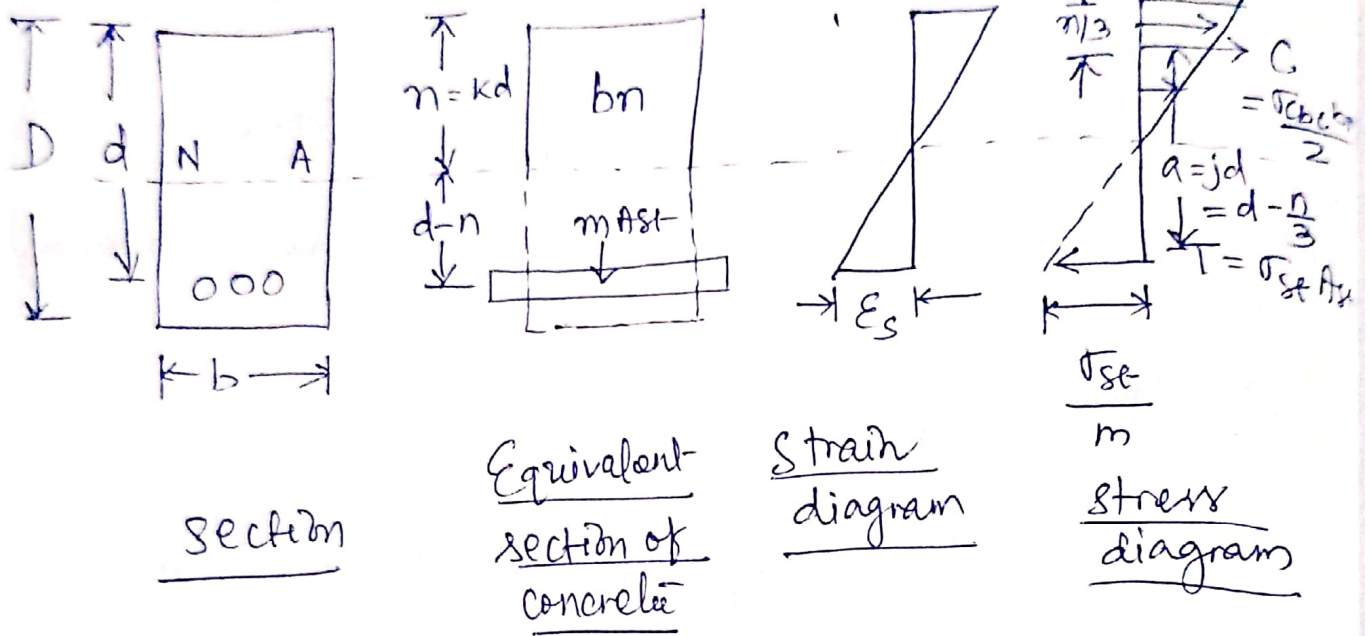
A_{eqc} = Equivalent area of section in terms of concrete

$$= A_c + m A_{st}$$

$$= (A - A_{st}) + m A_{st}$$

$$= A + (m - 1) A_{st}$$

Analysis of Singly Reinforced Beam :- (Working Stress Method)



(i) Equivalent section :- The equivalent area is equal to the area of concrete in the compression zone & an additional concrete area $m A_{st}$ of concrete corresponding to steel area A_{st} .

σ_{cbc} = Maximum permissible stress at the top most fibre in concrete

σ_{st} = Maximum permissible stress on steel.

$\frac{\sigma_{st}}{m}$ = Maximum stress in equivalent

concrete area at the level of steel

cbc $\rightarrow$ concrete in bending compression

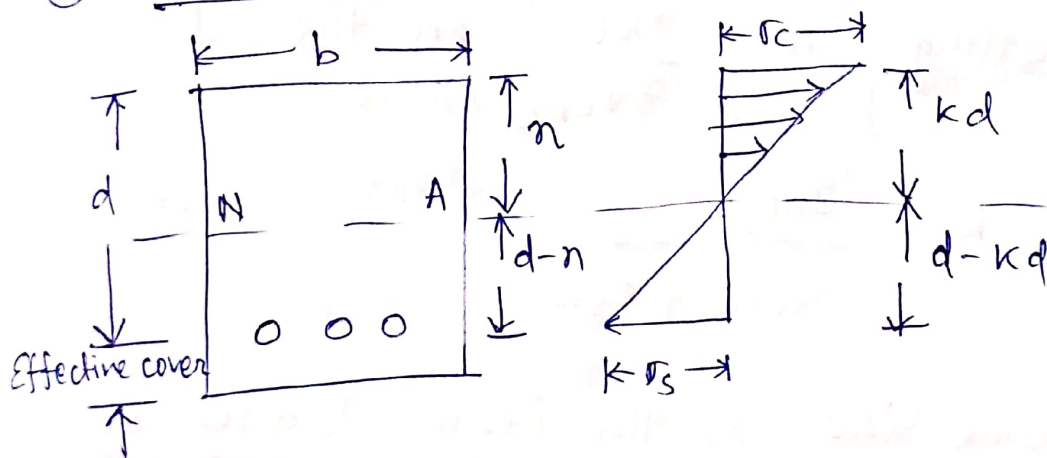
st $\rightarrow$ steel in tension.

(ii) Neutral axis (n) :-

It lies at the centre of gravity of the section. It is defined as that axis at which the stresses are zero. It divides the section into tension & compression zone.

The position of neutral axis of any rectangular section can be found by the following two methods :-

(a) Stresses in concrete & steel are known



$$\frac{\sigma_c}{n} = \frac{\sigma_s/s}{d-n} \quad (\text{law of similarity of triangles})$$

$$\Rightarrow \frac{m \sigma_c}{\sigma_s} = \frac{n}{d-n}$$

$$\Rightarrow \frac{m \sigma_{cbc}}{\sigma_{st}} = \frac{n}{d-n}$$

This neutral axis, corresponding to permissible values of stresses of concrete & steel is called as critical neutral axis (n_c).

$n_c = kd$, where k = neutral axis depth factor

$$\Rightarrow \frac{m \sigma_{cbc}}{\sigma_{st}} = \frac{kd}{d - kd}$$

$$\Rightarrow k = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}}$$

Putting $m = \frac{280}{3 \sigma_{cbc}}$ in the above equation

$$k = \frac{280/3}{280/3 + \sigma_{st}}$$

⑥ dimensions of the beam & area of steel are known :-

Moment of compressive area

= Area of compression $\times$ distance between c.g. of compressive area & N.A.

$$= b \cdot n \cdot \frac{n}{2} = \frac{bn^2}{2}$$

Moment of tensile area

= Equivalent tensile area $\times$ distance of centroid of steel reinforcement from NA.

$$= m A_{st} (d - n)$$

Moment of compressive area

= Moment of tensile area

$$\Rightarrow \frac{bn^2}{2} = m A_{st} (d - n)$$

(c) Percentage of steel (P_t) :-

It is the area of steel (A_{st}) expressed as percentage of total area of concrete.

$$P_t = \frac{A_{st}}{bd} \times 100$$

We know $\frac{bn^2}{2} = m A_{st} (d - n)$

$$\Rightarrow A_{st} = \frac{bn^2}{2m(d - n)}$$

$$\Rightarrow P_t = \frac{50n^2}{m(d - n)}$$

Putting $n = kd$, $P_t = \frac{50k^2}{m(1 - k)}$

(d) Lever arm :- It is the distance between the resultant compressive force & the resultant tensile force. As the compressive area is triangular, the resultant compressive force (C) will act at $\frac{n}{3}$ from the top compressive fibre. The resultant tensile force (T) will act at the centroid of the steel reinforcement.

$$\text{Lever arm} = a = d - \frac{n}{3} = jd$$

where j = lever arm depth factor

$$jd = d - \frac{kd}{3}$$

$$\Rightarrow j = 1 - \frac{k}{3}$$

(e) Moment of Resistance (M_r) :-

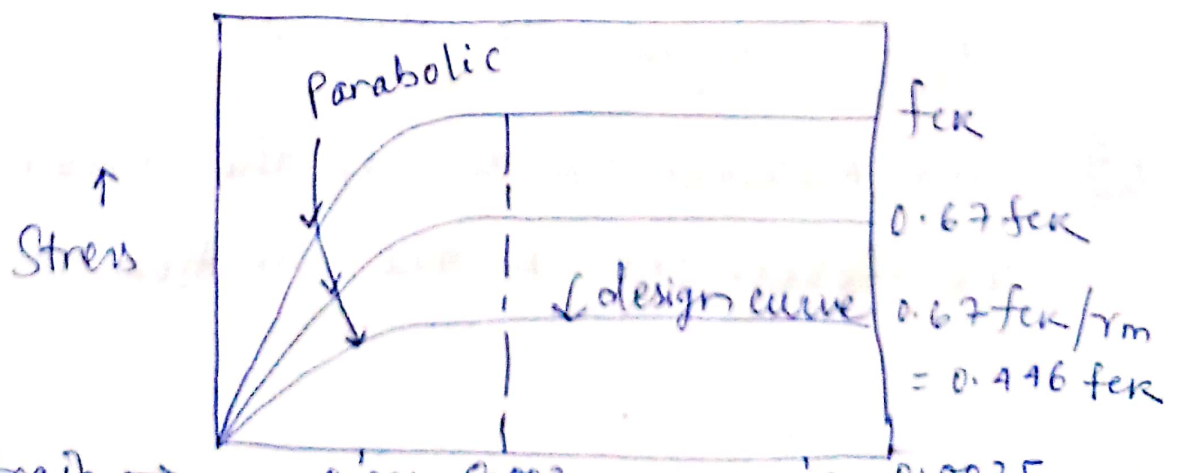
It is the resistance offered by the beam against external loads.

As there is no resultant force acting on the beam & the section is in equilibrium, the total compressive force is equal to the total tensile force.

There two forces () will form a couple & the moment of this couple is equal to the resisting moment or moment of resistance of the section.

Assumptions in limit state of collapse in flexure (Bending) :- (Refer cl. 38, IS 456)

- (a) Plane sections normal to the axis remain plane after bending.
- (b) The maximum strain in concrete at the outermost compression fiber is taken as 0.0035 in bending.
- (c) The relationship between the stress-strain distribution in concrete is assumed to be parabolic as shown in fig. below.



For design purpose, the compressive strength of concrete is assumed to be 0.67 times the characteristic strength of concrete. The partial safety factor (γ_{mc}) = 1.5 shall be applied in addition to this

$$\begin{aligned} \text{Max}^m \text{ compressive stress in concrete} \\ = \frac{0.67 f_{ck}}{1.5} \end{aligned}$$

where f_{ck} = Characteristic strength of concrete

- (d) The tensile strength of the concrete is ignored.
- (e) The stresses in the reinforcement are taken from the stress-strain curve for the type of steel used. For design purposes, the partial safety factor (γ_{ms}) equal to 1.15 shall be applied.
- (f) The maximum strain in the tension reinforcement in the section at failure

shall not be less than

$$\frac{f_y}{1.15 E_s} + 0.002$$

where f_y = characteristic strength of steel

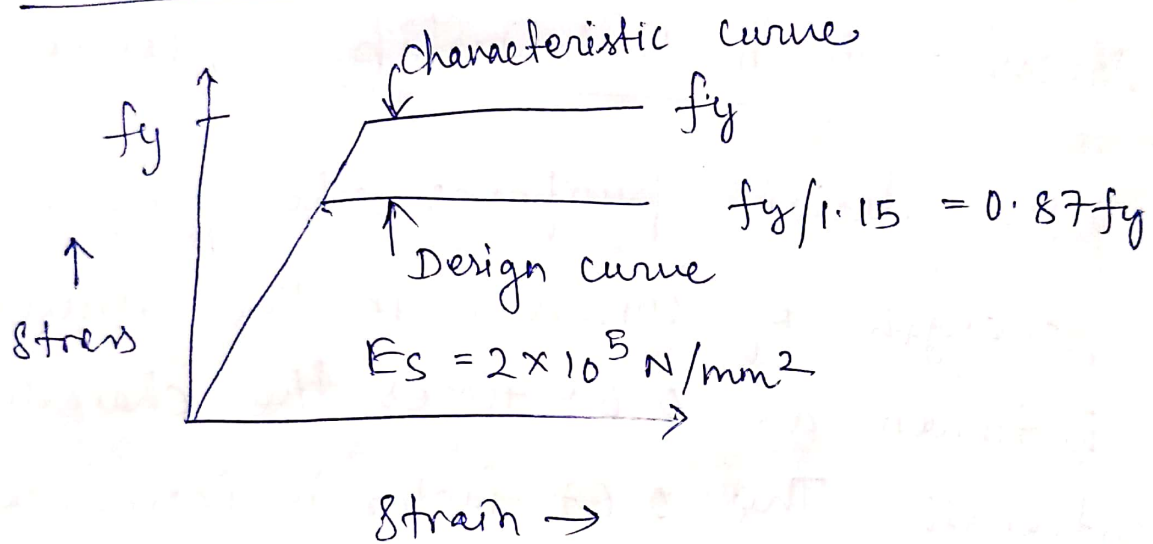
E_s = Modulus of elasticity of steel.

Stress - Strain Relationship for concrete

- (i) for design purposes, the compressive strength of concrete in the structure is taken as 0.67 times the characteristic strength. The 0.67 factor is introduced to account for the difference in the strength indicated by a cube test & the strength of concrete in actual structure.
- ii) The partial safety factor (γ_{mc}) equal to 1.5 is applied in addition to this 0.67 factor.

(iii) The initial portion of the curve is parabolic. After a strain of 0.002, (0.2 %), the stress becomes constant with increasing load, until a strain of 0.0035 is reached & here the concrete is assumed to have failed.

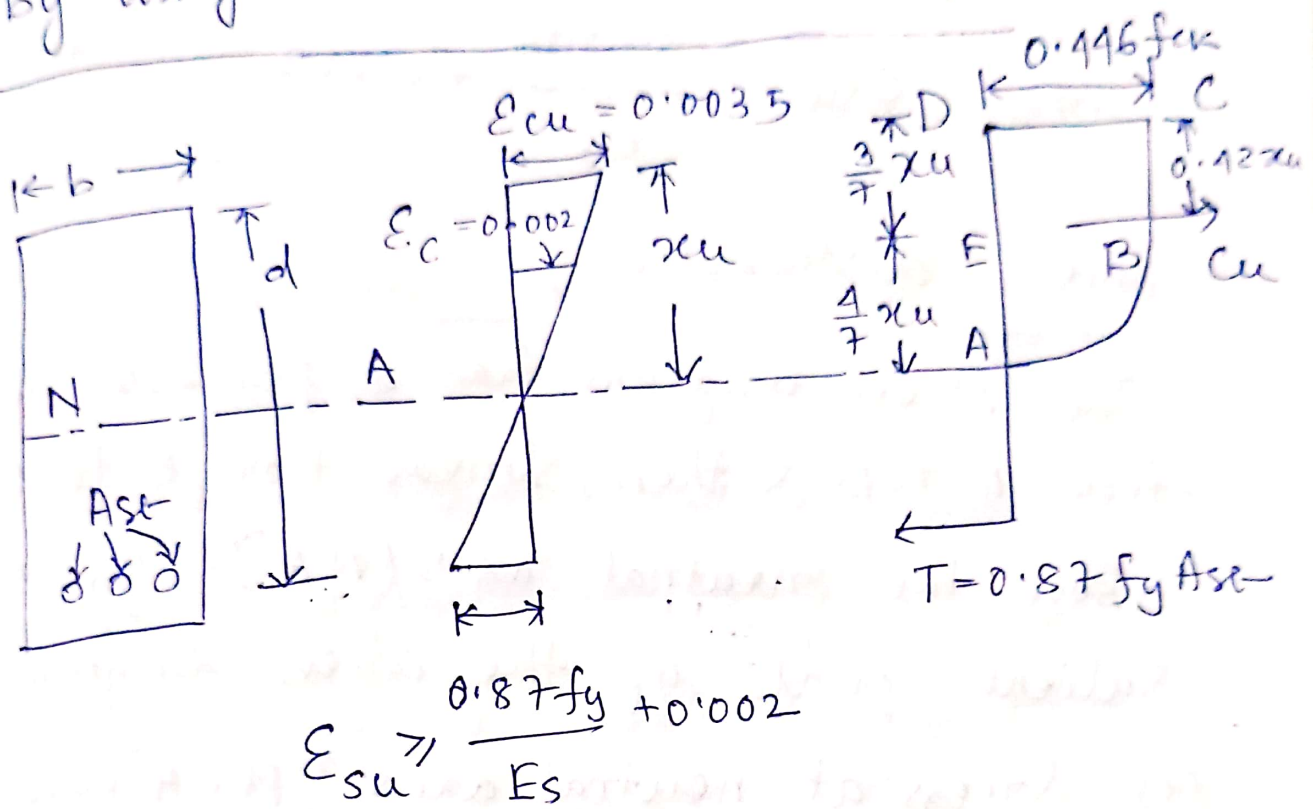
stress strain relationship for steel



Stress-strain curve for mild steel

For mild steel, the value of characteristic stress is taken as yield stress & the design curve is obtained after applying a factor of safety of 1.15 to yield stress. The design stress for steel is equal to $\frac{f_y}{1.15}$, i.e., $0.87f_y$.

Analysis of a singly reinforced beam
by using limit state method :-



(a) Section (b) Strain diagram (c) Stress diagram

Stress & strain distribution in a singly reinforced beam as per IS 456:2000

Strain distribution :-

- Strain at NA = 0
- Maximum or ultimate strain in concrete at extreme fibre, $\epsilon_{cu} = 0.0035$
- Strain at constant stress of $0.67 f_{ck}$, $\epsilon_c = 0.002$

(iv) Ultimate strain in steel corresponding to maximum stress at failure,

$$\epsilon_{su} = \frac{0.87 f_y}{E_s} + 0.002$$

Stress distribution :-

The stress diagram has a parabolic shape from A to B & then linear from B to C above the neutral axis (NA). The various salient points of the stress diagram are

(i) Stress at neutral axis (Pt. A) = 0

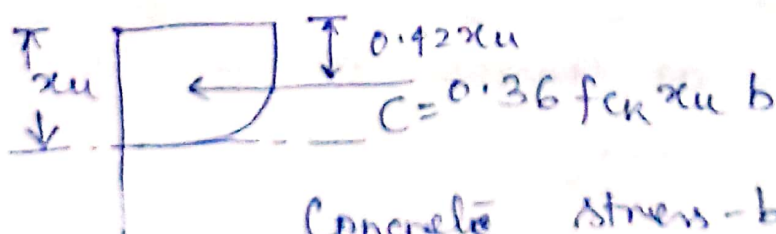
(ii) Stress at 0.002 strain (Pt. B)

$$= \frac{0.67 f_{ck}}{1.5} = 0.446 f_{ck}$$

(iii) Stress at extreme fibre (Pt. C) = 0.446 f_{ck}

(iv) Below the neutral axis, the concrete is assumed to be cracked & maximum stress in steel = $\frac{f_y}{1.15} = 0.87 f_y$

Stress block Parameters :-



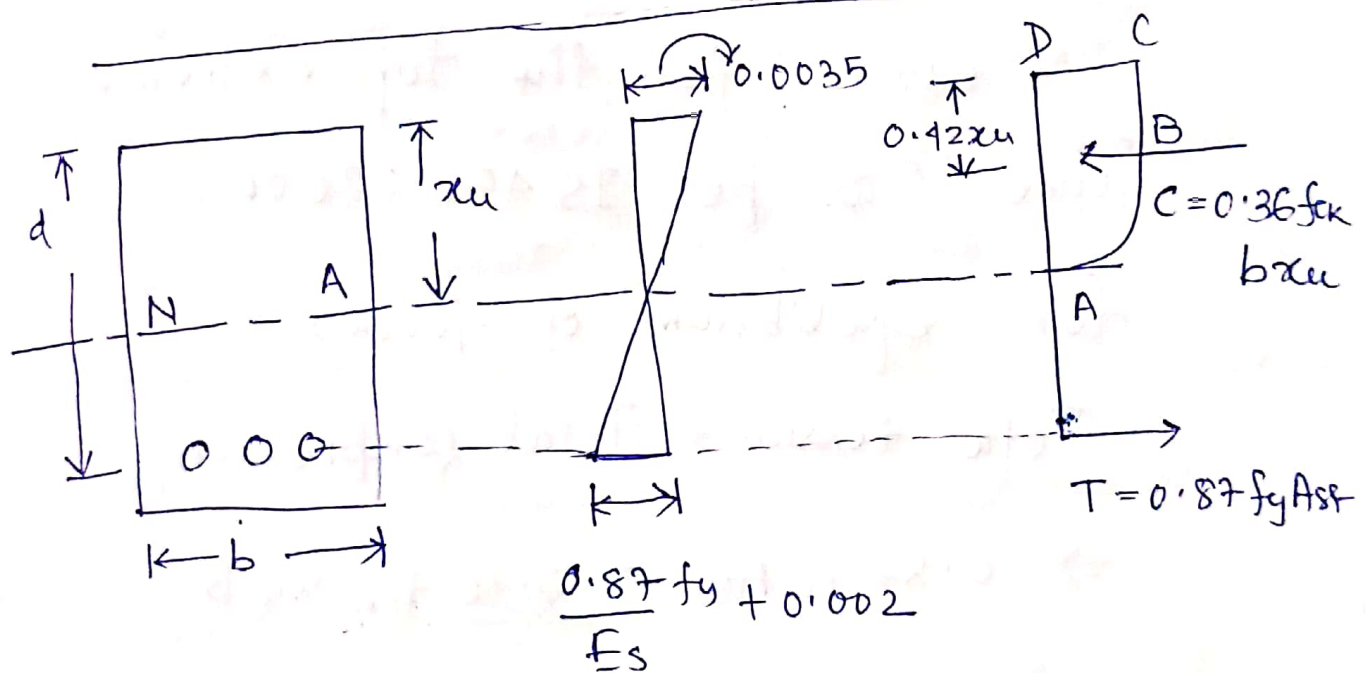
Concrete stress-block parameters in compression

for the stress strain curve of concrete the design stress block parameters are taken as following as per IS 456:2000

$$\text{Area of stress block} = 0.36 f_{ck} x_u$$

Depth of centre of compressive force from the extreme fibre in compression
 $= 0.42 x_u$

Neutral axis Depth (x_u) :-



Neutral axis is the axis at which the stresses are zero & it is situated at the centre of gravity of the section. The depth of neutral axis for a singly reinforced beam is calculated by taking equilibrium of tensile & compressive forces.

Let b = width of section

d = Effective depth of beam

A_{st} = Area of steel reinforcement

x_u = depth of neutral axis

C = Total compression

T = Total tension

$$\text{Total tension } (T) = 0.87 f_y \cdot A_{st}$$

$$\text{Total compression } (C) = 0.36 f_{ck} \cdot b \cdot x_u$$

at $0.42 x_u$ from the top extreme fibre (as per IS 456 : 2000)

for equilibrium of forces

$$\text{Total tension} = \text{Total compression}$$

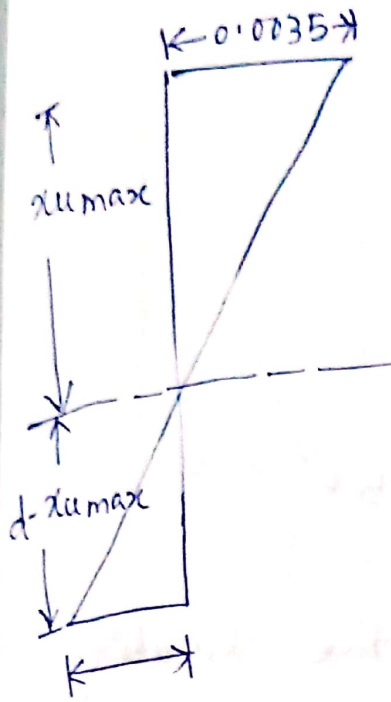
$$\Rightarrow 0.87 f_y A_{st} = 0.36 f_{ck} x_u b$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \quad \text{----- (i)}$$

which is given in Clause G 1.1 of IS code.

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

Limiting depth of Neutral axis ($x_{u\max}$)



$$\frac{0.87f_y}{E_s} + 0.002$$

strain diagram

The strain distribution for the singly reinforced beam is shown in the figure. The maximum strain in concrete is 0.0035. As per code, the strain in steel at failure should not be less than $\frac{0.87f_y}{E_s} + 0.002$.

This limits the depth of neutral axis to its maximum or limiting value.

from the diagram :

$$\frac{0.0035}{x_{u\max}} = \frac{\frac{0.87f_y}{E_s} + 0.002}{d - x_{u\max}}$$

[from similar triangles]

$$\Rightarrow \frac{x_{u\max}}{d - x_{u\max}} = \frac{0.0035}{\frac{0.87f_y}{E_s} + 0.002}$$

$$\Rightarrow x_{\max} \left(\frac{0.87 f_y}{E_s} + 0.002 + 0.0035 \right)$$

$$= 0.0035 d$$

$$\Rightarrow \frac{x_{\max}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

The above equation gives the limiting or maximum values of depth of neutral axis for different grades of steel.

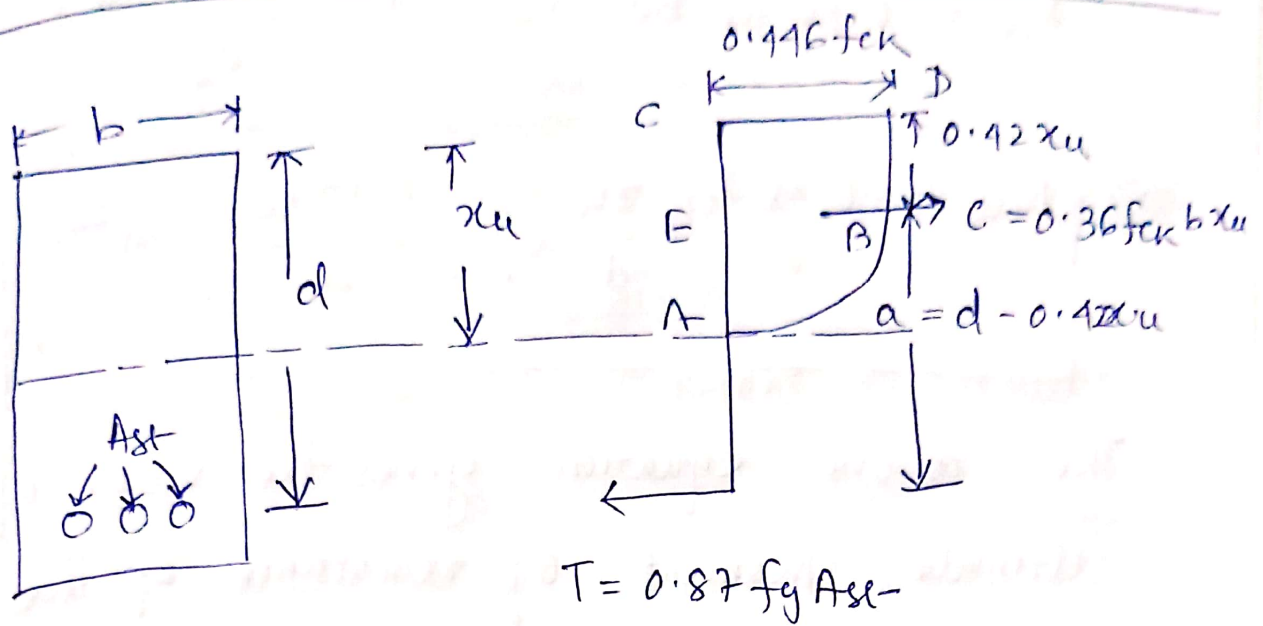
Values of $x_{\max}/d$

Grade of Steel	f_y (N/mm ²)	$\frac{x_{\max}}{d}$
Mild steel (Fe 250)	250	0.53
Fe 415	415	0.48
Fe 500	500	0.46

The value of E_s or modulus of elasticity of steel is taken as

$$2 \times 10^5 \text{ N/mm}^2.$$

Moment of Resistance (Clause G1.1, IS 456)



The moment of resistance is equal to the moment of the couple formed by two equal and opposite forces, i.e.

total compression & total tension (C & T)

Ultimate moment of resistance = M_u

$$M_u = C \times \text{lever arm} = T \times \text{lever arm}$$

$C = 0.36 f_{ck} b \cdot x_u$, which acts at $0.42 x_u$ from the top most fibre.

$$T = 0.87 f_y \cdot A_{st}$$

$$\therefore \text{Lever arm} = d - 0.42 x_u$$

$$M_u = 0.36 f_{ck} b \cdot x_u (d - 0.42 x_u)$$

$$M_u = 0.36 f_{ck} b d \cdot x_u \left(1 - \frac{0.42 x_u}{d} \right)$$

$$\Rightarrow M_u = 0.36 f_{ck} \frac{x_u}{d} \left(1 - \frac{0.42 x_u}{d} \right) b d^2 \quad \text{--- (i)}$$

The above equation gives the value of ultimate moment of resistance of the singly reinforced beam. The moment of resistance of this beam can also be obtained from tension side as follows:

$$\begin{aligned} M_u &= T \times \text{lever arm} \\ &= 0.87 f_y A_{st} (d - 0.42 x_u) \end{aligned}$$

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{0.42 x_u}{d} \right) \quad \text{--- (ii)}$$

By putting value of x_u in above equatⁿ, we get

$$M_u = 0.87 f_y A_{st} \cdot d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

Limited value of Moment of Resistance

The depth of neutral axis is limited to x_{max} . The maximum value of neutral axis depth gives the maximum or limiting value of moment of resistance.

The limiting or maximum value of moment of resistance is given by $M_{u\lim}$ which we get by substituting x_{max} for x_u in eqnⁿ (ii)

$$M_{u\lim} = 0.36 f_{ek} \cdot \frac{x_{max}}{d} \left(1 - \frac{0.42 x_{max}}{d} \right) b d^2$$

↑ as given in Para G.1.1. (c) of IS456

for example: $M_{u\lim}$ for mild steel is obtained as follows:

$$\frac{x_{max}}{d} = 0.53 \quad \left(\text{for mild steel by putting } f_y = 250 \text{ N/mm}^2 \text{ in expression for } \frac{x_{max}}{d} \right)$$

$$M_{u\lim} = 0.36 f_{ek} \times 0.53 (1 - 0.42 \times 0.53) b d^2 = 0.148 f_{ek} b d^2$$

Percentage of steel (P_t)

The percentage of tensile reinforcement can be found by equating the tensile & compressive forces

$$T = C$$

$$\Rightarrow 0.87 f_y \cdot A_{st} = 0.36 f_{ck} b x_u$$

$$\Rightarrow \frac{A_{st}}{b} = \frac{0.36 f_{ck} x_u}{0.87 f_y}$$

$$\Rightarrow \frac{A_{st}}{bd} = \frac{0.36 f_{ck} x_u}{0.87 f_y d}$$

$$\Rightarrow P_t = \frac{A_{st}}{bd} = \frac{0.36 f_{ck} x_u}{0.87 f_y d}$$

As the value of $\frac{x_u}{d}$ is limited, its maximum value gives the maximum percentage of steel.

$$P_{t \text{ lim}} (\%) = \frac{0.36 f_{ck}}{0.87 f_y} \cdot \frac{x_{u \text{ max}}}{d} \times 100$$

For mild steel, the value of $\frac{x_{u \text{ max}}}{d}$ is 0.53 & its yield stress (f_y) is 250 N/mm^2 .

Putting these values in the expression of maximum percentage of steel, we get :

$$P_{t \text{ lim}} = \frac{0.36 f_{ck}}{0.87 \times 250} \times 0.53 \times 100$$
$$= 0.88 f_{ck}$$

Thus, by putting various values of f_{ck} for different grades of concrete, we get limiting percentage of steel.

Balanced, Under-Reinforced & Over-Reinforced Sections :-

The value of $\frac{x_u}{d}$, calculated from the stress block, is equal to :

$$\frac{x_u}{d} = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b}$$

The limiting value of $\frac{x_u}{d}$, i.e.,

$\frac{x_{u \text{ max}}}{d}$, as calculated from the strain

distribution diagram is as follows :

$$\frac{x_{u \text{ max}}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

$\frac{x_u}{d}$ & $\frac{x_{u\max}}{d}$ are compared & the

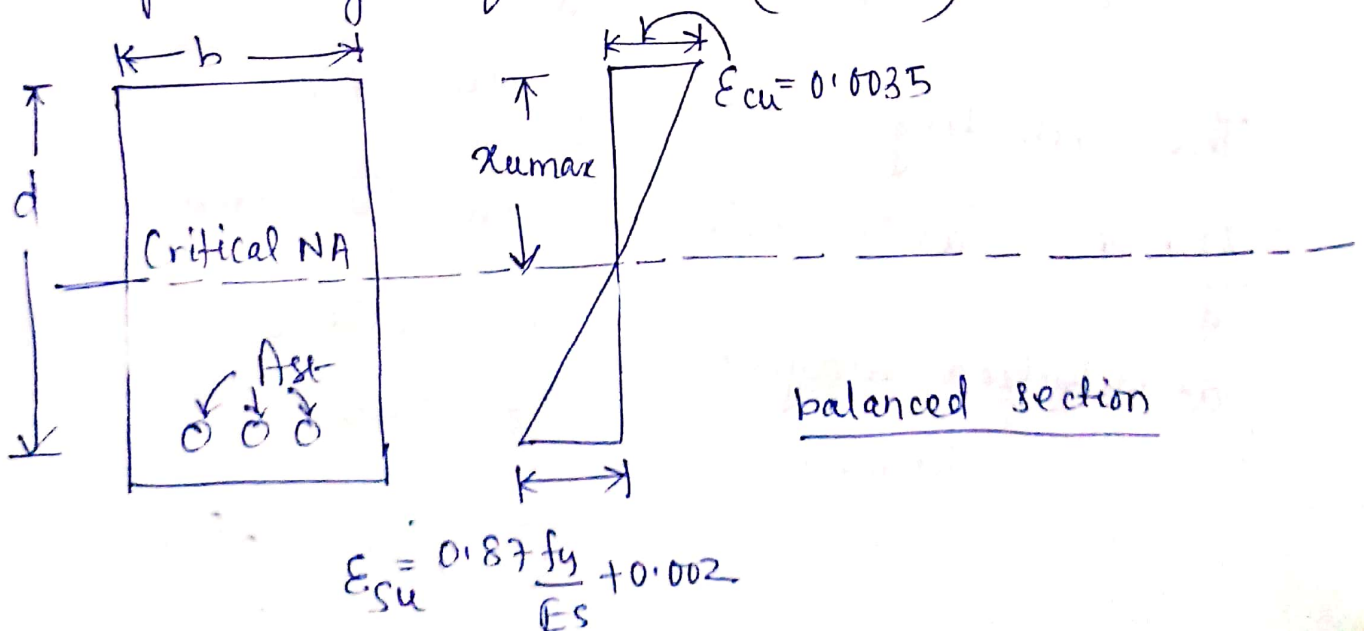
following three cases may arise

(Clause 41.1. of IS 456)

Case - 1 Balanced section $\left(\frac{x_u}{d} = \frac{x_{u\max}}{d} \right)$

In the balanced section the steel reinforcement reaches the yield strain, i.e., $\frac{0.87 f_y}{E_s} + 0.002$ at the same time

as the concrete reaches the ultimate strain value of 0.0035. The stress diagram for this case is shown in figure below. The percentage of steel in a balanced section is equal to the limiting percentage of steel ($P_{t\lim}$).



Case (i) $\frac{x_u}{d} = \frac{x_{u\max}}{d}$

(ii) $P_t = P_{t\lim}$

(iii) The yield strain in steel & ultimate strain in concrete reaches their maximum value at the same time.

(iv) The moment of resistance is equal to its limiting value $M_{u\lim}$ & can be calculated as:

$$M_{u\lim} = 0.36 f_{ck} \frac{x_{u\max}}{d} \left(1 - \frac{0.42 x_{u\max}}{d}\right) b d^2$$

$$\text{Or } M_{u\lim} = 0.87 f_y A_{st} d \left(1 - \frac{0.42 x_{u\max}}{d}\right)$$

Case II: Underreinforced Section $\left(\frac{x_u}{d} < \frac{x_{u\max}}{d}\right)$

In this case, the steel fails first by reaching its yield strain value, although in concrete the ultimate strain has not reached. Steel is ductile material & it gives sufficient warning before failure, hence the underreinforced

sections are preferred by designers. In underreinforced section, the percentage of steel is less than its maximum or limiting value.

In an under reinforced section :

- (i) The strain in steel reaches its yield value first, i.e., $\frac{0.87f_y}{E_s} + 0.002$ but at that time the strain in concrete is less than 0.0035.
- (ii) $\frac{x_u}{d} < \frac{x_{u\max}}{d}$, i.e., depth of neutral axis is less than the limiting value.
- (iii) $\rho < \rho_{\lim}$, i.e., the percentage of steel is less than the maximum or limiting value of percentage of steel required for balanced section. Hence the under reinforced sections are economical.
- (iv) The section fails in a ductile manner.

(v) The moment of resistance is calculated as follows:

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{0.42 x_u}{d} \right)$$

Case III : Over reinforced Section

$$\left(\frac{x_u}{d} > \frac{x_{u\max}}{d} \right) ; -$$

The overreinforced section is that in which strain in concrete reaches its ultimate value earlier than the steel. This means that overreinforced beam fails by crushing failure of concrete. Concrete being brittle, fails suddenly without warning. Therefore, code IS 456: 2000 recommends that over-reinforced section should be redesigned.

for over-reinforced section:

- (i) Strain in concrete reaches its ultimate value, i.e., 0.0035 first & the strain in steel at that time is less than

$$\frac{0.87 f_y}{E_s} + 0.002$$

(ii) $\frac{x_u}{d} > \frac{x_{u\max}}{d}$, i.e., depth of neutral axis is greater than its limiting or maximum value.

(iii) $P > P_{lim}$ the percentage of steel is greater than that in the balanced section.

Thus, this section is uneconomical.

(iv) failure is sudden, without warning.

(v) The moment of resistance of over-reinforced section is calculated as follows:

By putting $x_u = x_{u\max}$

$$M_{u\lim} = 0.36 f_{ck} \frac{x_{u\max}}{d} \left(1 - 0.42 \frac{x_{u\max}}{d} \right) b d^2$$

Finding Moment of Resistance of Singly Reinforced beam :-

The moment of resistance of a singly reinforced beam can be obtained as follows (IS 456: 2000, Annex G):

1. For the given grades of concrete & steel (f_{ck} & f_y known), find the depth of neutral axis of the given section:

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

Or
$$x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} b}$$

2. Find limiting value of neutral axis, i.e., $x_{u,max}$ by the following expression or refer Table 4.2 (Page 70 of IS 456)

$$\frac{x_{u,max}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

3. Compare x_u & $x_{u,max}$

- (i) If $\frac{x_u}{d} < \frac{x_{u,max}}{d}$, the beam is under reinforced & moment of resistance is calculated by the following equation:

$$M_u = 0.87 f_y \cdot A_{st} \cdot d \left(1 - \frac{0.12 x_u}{d} \right)$$

(i) If $\frac{x_u}{d} = \frac{x_{u\max}}{d}$, the beam is

designed as balanced section & moment of resistance of the section is given by following expression:

$$M_{u\lim} = 0.36 f_{ek} \frac{x_{u\max}}{d} \left(1 - \frac{0.42 x_{u\max}}{d}\right) b d$$

(ii) If $\frac{x_u}{d} > \frac{x_{u\max}}{d}$, the moment of resistance of the section is equal to $M_{u\lim}$ but the code recommends that the section is to be redesigned as it is a case of over reinforced section.

Example: Determine the moment of resistance of a beam of dimension $250\text{mm} \times 350\text{mm}$. The area of steel consists of 3 bars of 12mm diameter placed at a distance of 40mm from bottom of beam. Use M20 & Fe 415 steel.

Solution :-

Given $b = 250 \text{ mm}$

$$d = 350 - 40 = 310 \text{ mm}$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 12^2 = 339 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2 \text{ (for M20 concrete)}$$

$$f_y = 415 \text{ N/mm}^2 \text{ (for Fe 415 steel)}$$

Depth of neutral axis (x_u)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 339}{0.36 \times 20 \times 250} = 68 \text{ mm}$$

$x_{u \max}$ (limiting depth of neutral axis)

$$x_{u \max} = 0.48 d$$

$$= 0.48 \times 310$$

$$= 148.8 \text{ mm} > 68 \text{ mm}$$

$\therefore x_{u \max} > x_u$, hence, it is under-reinforced section.

Moment of resistance (M_u)

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 339 (310 - 0.42 \times 68)$$

$$= 34.44 \text{ kNm}.$$