

GOVT. POLYTECHNIC, BHUBANESWAR

CIVIL ENGINEERING DEPARTMENT

LECTURE NOTES OF GEOTECHNICAL

ENGINEERING

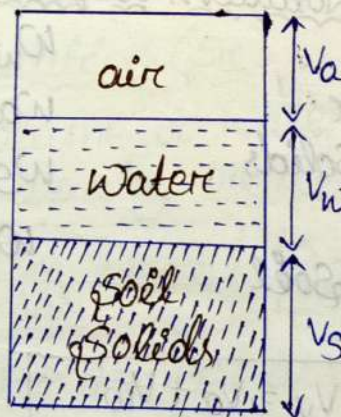
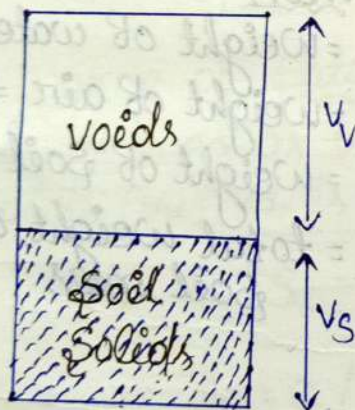
3RD SEMESTER

FACULTY NAME – AMRIT GAJENDRA

Basic Terminology & Interrelations

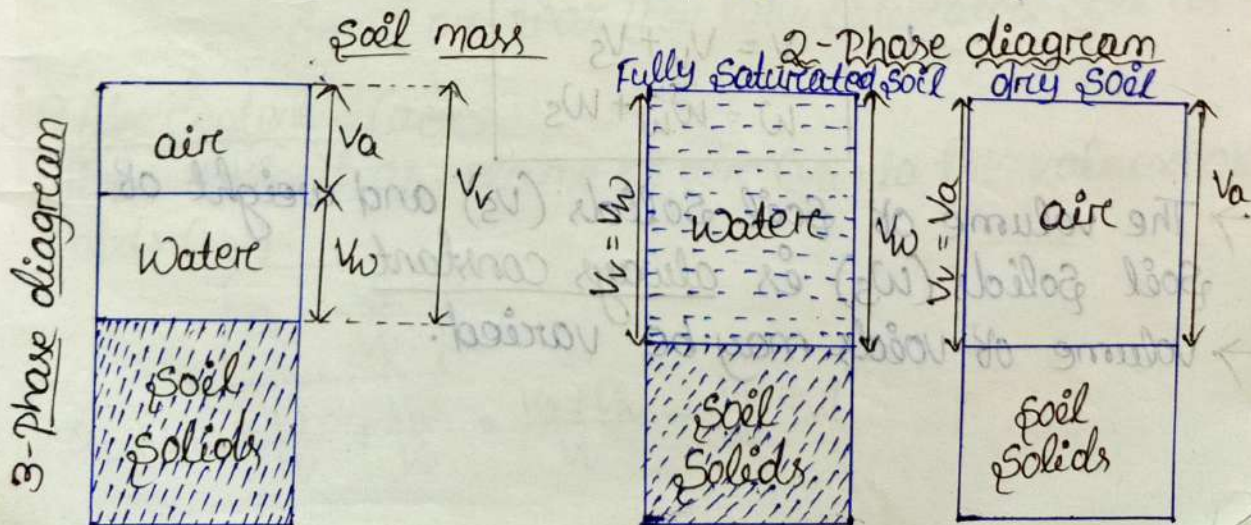
Soil mass:

- A soil mass is a combination of soil particles forming a porous structure.
- The soil particles in a soil mass is called as soil solids.
- The soil mass has free spaces or pores which is called as voids. which can be filled by air or water or both.
- If we separate the soil mass into different component, we will get phase diagram.
- Different components of soil mass are solids, water & air.



$$V_v = V_a + V_w$$

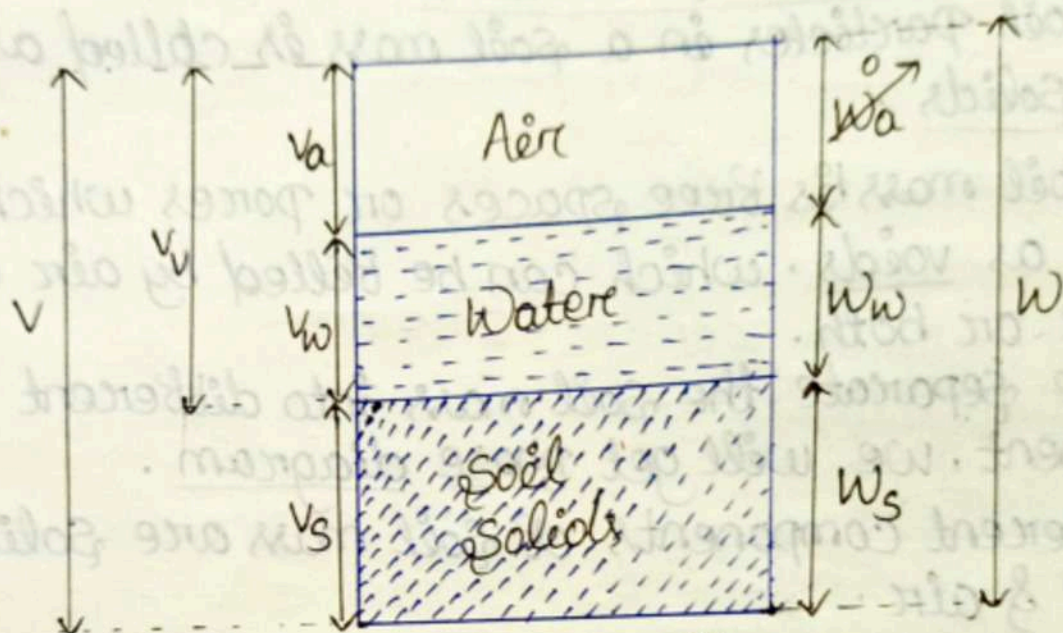
V_v = volume of voids
 V_a = volume of air
 V_w = volume of water
 V_s = volume of soil solids



→ In Bully Saturated Soil, the voids are Bully filled with water.

→ In dry soil, the voids are Bully filled with air.

$$V_v = V_a$$



Notations of Soil mass

V_a = volume of air
 V_w = volume of water
 V_s = volume of Soil Solids
 V_v = volume of voids
 V = volume of total Soil mass

W_w = weight of water
 W_a = weight of air = 0
 W_s = weight of Soil Solids
 W = total weight of Soil mass

$$\begin{aligned}
 V_v &= V_a + V_w \\
 V &= V_v + V_s \\
 W &= W_w + W_s
 \end{aligned}$$

→ The volume of Soil Solids (V_s) and weight of Soil Solids (W_s) is always constant.

→ volume of voids may be varied.

Sourmyakanta Sahoo

Definitions

(i) void ratio (e):

It is defined as the ratio of volume of voids (V_v) to the volume of soil solids (V_s)

$$e = \frac{V_v}{V_s}$$

(Range of e is 0 to ∞) [Range $\rightarrow 0 < e < \infty$]

(ii) porosity (n):

It is defined as the ratio of volume of voids (V_v) to the total volume of soil mass (V)

$$n = \frac{V_v}{V}$$

It is usually expressed as percentage (%)

$$n = \frac{V_v}{V} \times 100\% \quad (\text{Range} \rightarrow 0 < n < 100\%)$$

(iii) Degree of Saturation (S_r)

It is defined as volume of water (V_w) to the volume of voids (V_v). It is usually expressed as percentage.

$$S_r = \frac{V_w}{V_v} \times 100\% \quad \left\{ \text{Range} \rightarrow 0 \leq S_r \leq 100\% \right\}$$

$S_r = 0$ [For dry soil as $V_w = 0$]

$S_r = 1$ or 100% [For fully saturated soil as $V_v = V_w$]

(iv) Air content: (a_c)

It is defined as volume of air (V_a) to the volume of voids (V_v).

$$a_c = \frac{V_a}{V_v}$$

$$a_c + S_r = \frac{V_a}{V_v} + \frac{V_w}{V_v} = \frac{V_a + V_w}{V_v} = \frac{V_v}{V_v} = 1$$

Sourmyakanta Sahoo

(V) Percentage of air void (n_a):

It is defined as the ratio of volume of air (V_a) to the total volume of soil mass (V). It is expressed in %.

$$n_a = \frac{V_a}{V} \times 100\%$$

(Vi) Water content (w)

It is defined as the ratio of weight of water (W_w) present in soil mass to the weight of soil solids (W_s). It is also called as moisture content.

It is usually expressed in percentage (%).

$$w = \frac{W_w}{W_s} \times 100\%$$

$$\text{also } w = \frac{M_w}{M_s} \times 100\%$$

M_w = mass of water

M_s = mass of soil solids

Unit weights & densities

(1) Unit weight of water (γ_w)

It is the ratio of weight of a given volume of water, (W_w) to the volume of water (V_w).

$$\gamma_w = \frac{W_w}{V_w}$$

$$\rho_w = \frac{M_w}{V_w}$$

density of water

$$\rho_w = 1 \text{ g/cc} = \frac{1 \text{ g}}{\text{cm}^3}$$

$$\rho_w = 1000 \text{ kg/m}^3$$

$$\gamma_w = 9810 \text{ N/m}^3$$

$$\rho = \rho_{00}$$

$$I = \frac{W}{W} - \frac{W_w + W_s}{W} = \frac{W_w}{W} + \frac{W_s}{W} = 2 + 50$$

- (2) Bulk unit weight or bulk density (γ)
It is defined as the total weight or mass of soil mass to the total volume of soil mass.

$$\gamma = \frac{W}{V} \quad \text{expressed as } \frac{\text{KN}}{\text{m}^3}, \frac{\text{kg}}{\text{m}^3} \text{ or } \frac{\text{gram}}{\text{cm}^3}$$

$$\rho = \frac{M}{V}$$

Bulk density

- (3) Dry unit weight or dry density (γ_d or ρ_d)
It is defined as the ratio of weight of soil solids (W_s) or (M_s) to the volume of soil mass (V).

$$\gamma_d = \frac{W_s}{V} \quad \text{in } \frac{\text{KN}}{\text{m}^3} \text{ or } \frac{\text{N}}{\text{m}^3}$$

$$\rho_d = \frac{M_s}{V} \quad \text{in } \frac{\text{kg}}{\text{m}^3} \text{ or } \frac{\text{gram}}{\text{m}^3}$$

dry density

- (4) Saturated unit weight or saturated density (γ_{sat} or ρ_{sat})
It is defined as the ratio of weight or mass of fully saturated soil mass [W_{sat} or M_{sat}] to the volume of soil mass [V].

$$\gamma_{\text{sat}} = \frac{W_{\text{sat}}}{V} = \frac{W_w + W_s}{V} \quad \text{in } \frac{\text{KN}}{\text{m}^3} \text{ or } \frac{\text{N}}{\text{m}^3}$$

$$\rho_{\text{sat}} = \frac{M_{\text{sat}}}{V} = \frac{M_w + M_s}{V} \quad \text{in } \frac{\text{kg}}{\text{m}^3} \text{ or } \frac{\text{g}}{\text{m}^3}$$

Saturated density

- (5) Submerged unit weight or submerged density (γ' or ρ'): It is the ratio of submerged weight of soil mass (W_{sub} or M_{sub}) to the total volume of soil mass (V).

$$\gamma' = \frac{W_{\text{sub}}}{V} = \gamma_{\text{sat}} - \gamma_w \quad (\text{KN/m}^3)$$

$$\rho' = \rho_{\text{sat}} - \rho_w \quad \left(\frac{\text{kg}}{\text{m}^3} \text{ or } \frac{\text{g}}{\text{cm}^3} \right)$$

(6) Unit weight of soil solids (γ_s):

It is ratio of weight of soil solids (W_s) to the volume of soil solids (V_s) in a given soil mass.

$$\boxed{\gamma_s = \frac{W_s}{V_s}} \text{ in } \frac{\text{KN}}{\text{m}^3}$$

$$\boxed{\rho_s = \frac{M_s}{V_s}} \frac{\text{Kg}}{\text{m}^3} \text{ or } \frac{\text{g}}{\text{cm}^3}$$

density of soil solids

Specific gravity:

Specific gravity of soil particles (G_r):

It is defined as the ratio of weight of a given volume of soil particles to the weight of an equivalent volume of water. or

It is defined as density or unit wt. of soil solids to the unit wt (or) density of water.

$$\boxed{G_r = \frac{\gamma_s}{\gamma_w} \text{ or } \frac{\rho_s}{\rho_w}}$$

Specific gravity of soil mass (G_m):

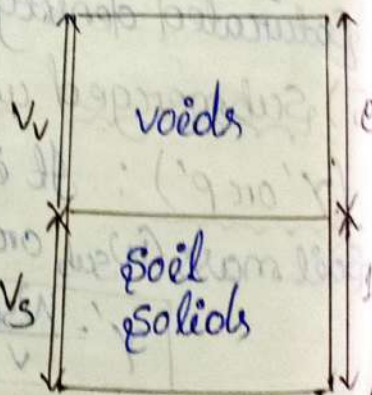
It is defined as the ratio of bulk unit weight of soil mass to the unit weight of water.

$$\boxed{G_m = \frac{\gamma}{\gamma_w} \text{ or } \frac{\rho}{\rho_w}}$$

Unit Phase diagram:

Here, volume of soil solids = 1

volume of voids = e



Relation b/w 'e' & 'n' :

$$n = \frac{V_v}{V} = \frac{V_v}{V_v + V_s} = \frac{\frac{V_v}{V_s}}{\frac{V_v}{V_s} + 1} = \frac{e}{e+1}$$

$$\boxed{n = \frac{e}{1+e}}$$

n = porosity
e = void ratio

Important Formulas :

① $e \cdot S_r = wG$

② $\gamma_d = \frac{\gamma}{1+w}$

③ $\gamma = \frac{(G + e \cdot S_r) \gamma_w}{1+e}$

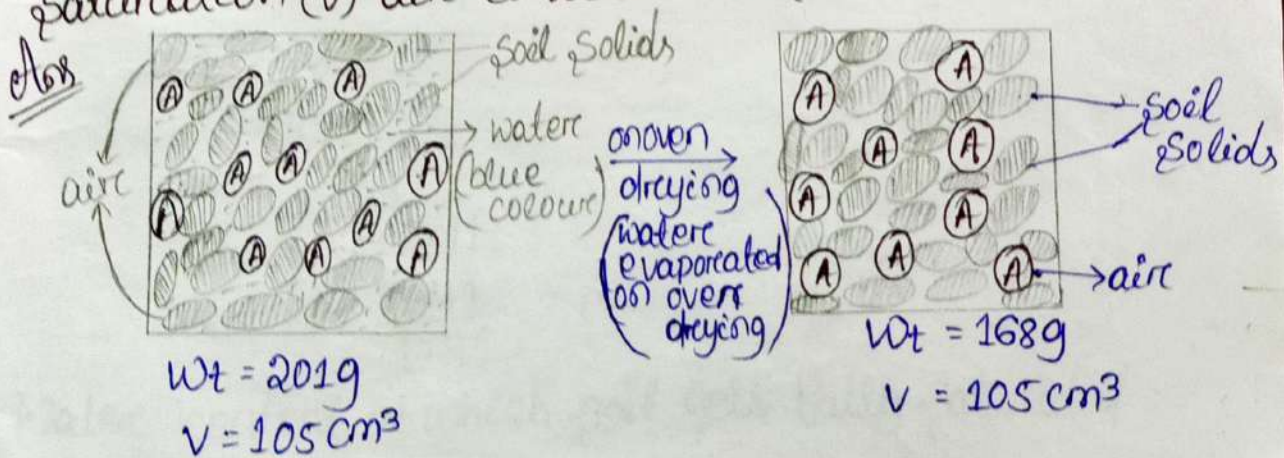
④ $\gamma_d = \frac{G \gamma_w}{1+e}$ [For γ_d
 $S_r = 0$]

⑤ $\gamma_{sat} = \frac{(G+e) \gamma_w}{1+e}$ [For γ_{sat}
 $S_r = 1$]

⑥ $\gamma_{ol} = \frac{(1-n_a) G \gamma_w}{1+wG}$

Q.1

A soil sample in its undisturbed state was found to have volume of 105 cm^3 & mass 2019 g . After oven drying the mass got reduced to 1689 g . Compute (i) water content (ii) void ratio (iii) porosity (iv) degree of saturation (v) air content. Take $G = 2.7$



Volume of soil mass = $105 \text{ cm}^3 = (V)$

Mass of soil mass = $201 \text{ g} = (M)$

Mass of dry soil, mass of soil solids = $168 \text{ g} = (M_d)$

$M_w = \text{mass of water} = M - M_d = 201 - 168 = 33 \text{ g}$

(i) water content 'w' = $\frac{M_w}{M_s} = \frac{33}{168} = 0.196\%$

(ii) dry density, ρ_d or $\gamma_d = \frac{M_s}{V} = \frac{168}{105} = 1.6 \text{ g/cm}^3$

$$\boxed{\gamma_d = \frac{G \gamma_w}{1+e}} \Rightarrow e = \frac{G \gamma_w}{\gamma_d} - 1 = \frac{2.7(1)}{1.6} - 1 = 0.69$$

$$e = 0.69$$

$$n = \frac{e}{1+e} = \frac{0.69}{1+0.69} = \frac{0.69}{1.69} = 0.408$$

$$\boxed{e S_r = w G}$$

$$(0.69) (S_r) = (0.196) (2.7)$$

$$\Rightarrow S_r = \frac{(0.196) (2.7)}{(0.69)} = 0.767$$

$$\boxed{a_c = 1 - S_r}$$

$$a_c = 1 - 0.767$$

$$a_c = 0.233$$

Diagram showing soil mass components:
Total mass = 201 g
Volume = 105 cm^3

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Q. A moist soil has a mass of 633g & a volume of 300cc & at a water content of 11% assuming $G = 2.68$. Determine e , S_r , n also determine the water content at which the soil gets fully saturated without any increase in volume.

Sol: $M = 633g$, $V = 300cm^3$

$w = 11\% = 0.11$, $G = 2.68$

$\rho = \frac{M}{V} = \frac{633}{300} = 2.11 g/cc$

$\rho_d = \frac{\rho}{1+w} = \frac{2.11}{1+0.11} = \frac{2.11}{1.11} = 1.90 g/cc$

$\rho_w = \frac{G \rho_w}{1+e}$ $\left[\gamma_d = \frac{G \gamma_w}{1+e} \right]$

$1.90 = \frac{(2.68)(1)}{1+e} \Rightarrow 1+e = \frac{(2.68)(1)}{1.90}$

$\Rightarrow e = \frac{(2.68)(1)}{1.90} - 1$

$\Rightarrow e = 0.41$

$\Rightarrow \boxed{e \cdot S_r = w G}$

$0.41 (S_r) = 0.11 \times 2.68$

$\Rightarrow S_r = \frac{0.11 \times 2.68}{0.41}$

$= 0.719$

$n = \frac{e}{1+e}$

$= \frac{0.41}{1+0.41}$

$= \frac{0.41}{1.41} = 0.29$

Water content at which soil gets fully saturated

$$e \cdot S_r = w G$$

$$\Rightarrow (0.41 \times 1) = w \times 2.68$$

$$\Rightarrow w = \frac{0.41}{2.68} = 0.152 \text{ or } 15.2\%$$

Q: A compacted sample of soil with bulk unit weight of 19.62 kN/m^3 , with water content of 20%. Determine dry density, degree of saturation. $G=2.66$

$$\gamma = 19.62 \text{ kN/m}^3$$

$$\gamma_w = 9.81 \text{ kN/m}^3$$

$$w = 20\% = 0.2$$

$$G = 2.66$$

$$\boxed{\gamma_d = \frac{\gamma}{1+w}} \Rightarrow \frac{19.62}{1+0.2} = 16.35 \text{ kN/m}^3$$

$$\boxed{\gamma_d = \frac{G \gamma_w}{1+e}} \Rightarrow \frac{2.66 \times 9.81}{1+e} = 16.35$$

$$\Rightarrow e = 0.596$$

Q. A sample of sand has a Porosity of 40%.
 Taking $G = 2.7$, compute (i) dry unit weight (ii) Saturated unit weight (iii) Submerged unit weight (iv) bulk unit weight when degree of saturation is 60%.

Ans $n = 40\% = 0.4$

$$n = \frac{e}{1+e}$$

$$\Rightarrow n(1+e) = e$$

$$\Rightarrow n + ne = e$$

$$\Rightarrow n = e - ne$$

$$\Rightarrow n = e(1-n)$$

$$\frac{n}{1-n} = e$$

$$e = \frac{0.4}{1-0.4} = 0.67$$

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

$$= \frac{2.7 \times 1 \text{ g/cc}}{1+0.67}$$

$$= 1.62 \text{ g/cc}$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

$$= \frac{(2.7+0.67) \times 1 \text{ g/cc}}{1+0.67}$$

$$= 2.018 \text{ g/cc}$$

$$\gamma = \frac{(G+e S_r)\gamma_w}{1+e}$$

[degree of saturation]
 $= 60\% = 0.6$

$$= \frac{(2.7+0.67 \times 0.6) \times 1 \text{ g/cc}}{1+0.67}$$

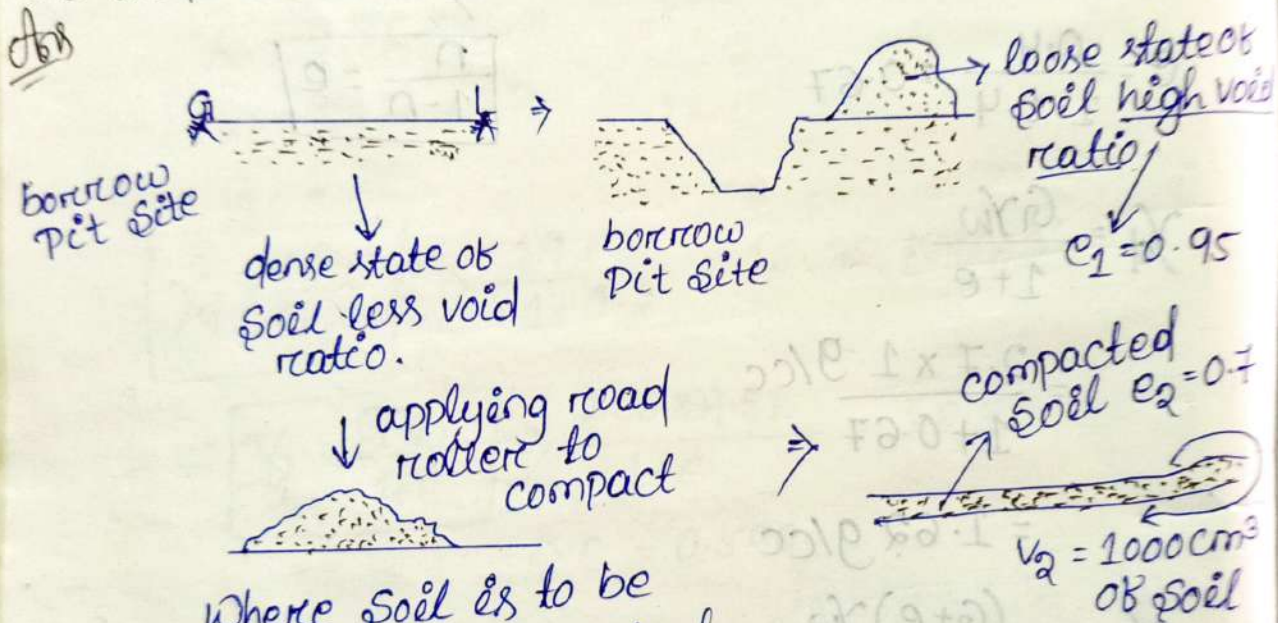
$$= 1.86 \text{ g/cc}$$

$$\gamma' = \gamma_{sat} - \gamma_w$$

$$= (2.018 - 1) \text{ g/cc}$$

$$= 1.018 \text{ g/cc}$$

Q: 1000 cm^3 of earthfill is to be constructed. How many cubic metres of soil is to be excavated from borrow pit in which the void ratio is 0.95, if the void ratio of earthfill is to be 0.7?



Where soil is to be deposited & compacted

$$\Rightarrow e = \frac{V_v}{V_s} \Rightarrow 1+e = 1 + \frac{V_v}{V_s}$$

$$\Rightarrow 1+e = \frac{V_v + V_s}{V_s} = \frac{V}{V_s}$$

$$\Rightarrow \boxed{(1+e) V_s = V}$$

$$(1+e_1) V_s = V_1 \rightarrow \text{at borrow pit}$$

$$(1+e_2) V_s = V_2 \rightarrow \text{at earthfill site}$$

$$e_1 = 0.95, e_2 = 0.7, V_2 = 1000 \text{ cm}^3$$

$$\boxed{\frac{(1+e_1)}{(1+e_2)} = \frac{V_1}{V_2}}$$

$$\Rightarrow V_1 = \frac{(1+e_1)}{(1+e_2)} V_2$$

$$\Rightarrow V_1 = \frac{(1 + 0.95)}{(1 + 0.7)} \times 1000 \text{ cm}^3$$

$$= 1147 \text{ cm}^3$$

Q: A soil deposit is being considered as a fill for a building site. In its original state in the borrow pit the void ratio is 0.95. Based on laboratory tests, the desired void ratio in its compacted state at the building site is to be 0.65. Determine percentage decrease of volume of the deposit from original state.

Ans

1. Particle size distribution
2. Consistency limits & Shrinkage
3. Density index
4. Specific Gravity (G_s)

Practically specific gravity (G_s) can be computed by density bottle and pycnometer methods.

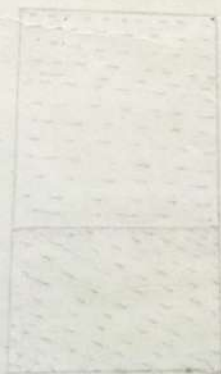
Density bottle method is more accurate & it is suitable for all types of soil.

Pycnometer method is used in case of coarse grained soils only.

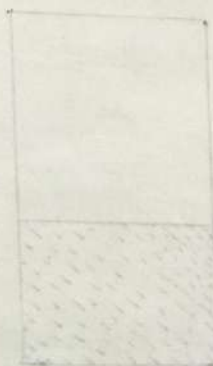
Both test follows same observation.



(i) Density bottle (v) with water (M_w = M₁)



(ii) Density bottle (ii) with soil & water (M_w = M₂)



(iii) Density bottle (iii) with soil (M_s = M₃)



(iv) Empty bottle (iv) (M_b = M₄)

Index Properties of Soils

• Index properties of the soils are those properties which are mainly used in identification & classification of soil.

→ These properties helps in predicting the suitability of soil as foundation/construction material.

→ Following are the list of index properties :-

1. Specific gravity of soil particles

2. Particle size distribution

3. consistency limits & indices

4. Density Index

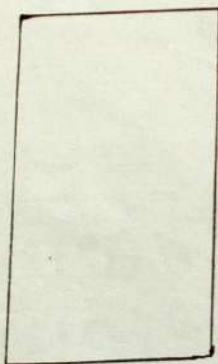
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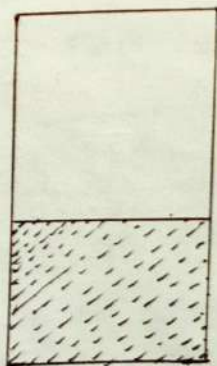
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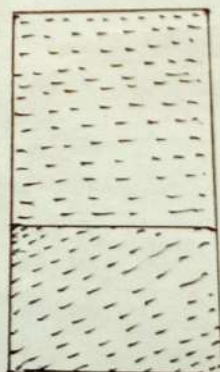
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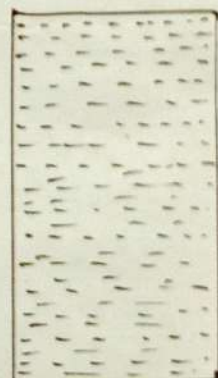
(i) Empty bottle
(Mass = M_1)



(ii) with dry soil
(Mass = M_2)



(iii) with soil & water
(Mass = M_3)



(iv) with water
(Mass = M_4)

The specific gravity of soil solids is computed as :-

$$G_s = \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)}$$

Water content

The water content of a soil can be determined by the following methods :-

- (i) oven drying method
- (ii) Pycnometer method
- (iii) Rapid methods.

(i) oven-drying method:

→ This is most accurate method & saturated method for computing water content in laboratory.

- A cup with tight fitting lid, is used. The mass M_1 of the cup with lid is found, suitable quantity of wet soil sample whose water content is to be determined is put inside the cup & lid replaced. The mass M_2 of the cup with soil & lid is found. The lid is removed & the soil is kept inside an oven & soil is dried for 24 hrs @ $105 - 110^\circ\text{C}$. The cup is taken out of oven & lid is replaced & cooled. The mass M_3 of cup with dry soil & lid is found.

$$\text{Water content} = \frac{\text{Weight of water}}{\text{Weight of solids}} = \frac{(M_2 - M_3)}{(M_3 - M_1)}$$

(ii) Pycnometer method:

→ This method can be used for quick determination of water content.

→ This method is conducted when specific gravity of soil solids (G_s) is known.

the mass (M_1) of Pycnometer with cone fitted to it is found.

→ The cone is removed & suitable amount of soil sample is put inside Pycnometer. The cone is refitted & the mass M_2 of Pycnometer with soil is found.

→ Water is added to the soil inside Pycnometer until excess water oozes out of the hole in cone. Outer surface is wiped & mass M_3 of Pycnometer with soil & water is found.

→ The Pycnometer is emptied & clean & filled with water only. Mass M_4 of Pycnometer with water is found.

→ Water content is calculated as

$$w = \left[\frac{M_2 - M_1}{M_3 - M_4} \left(\frac{G - 1}{G} \right) - 1 \right] \times 100\%$$

(iii) Rapid Methods :-

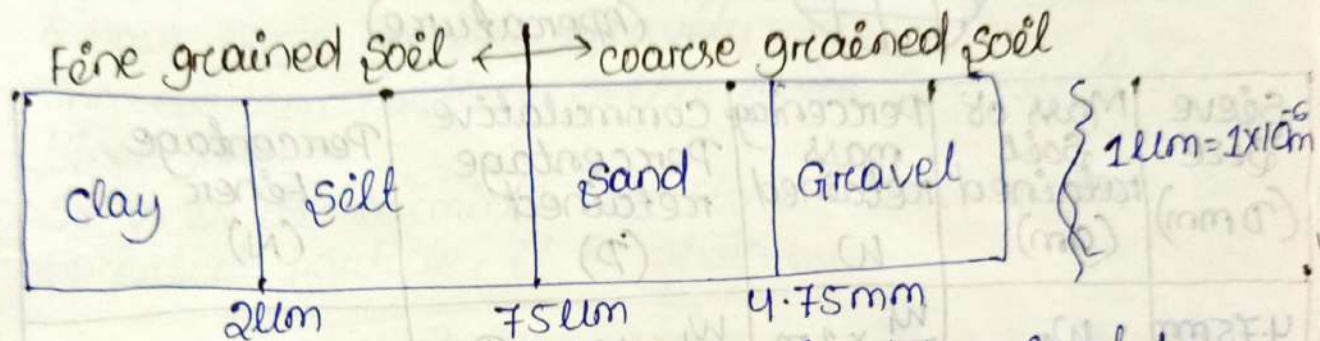
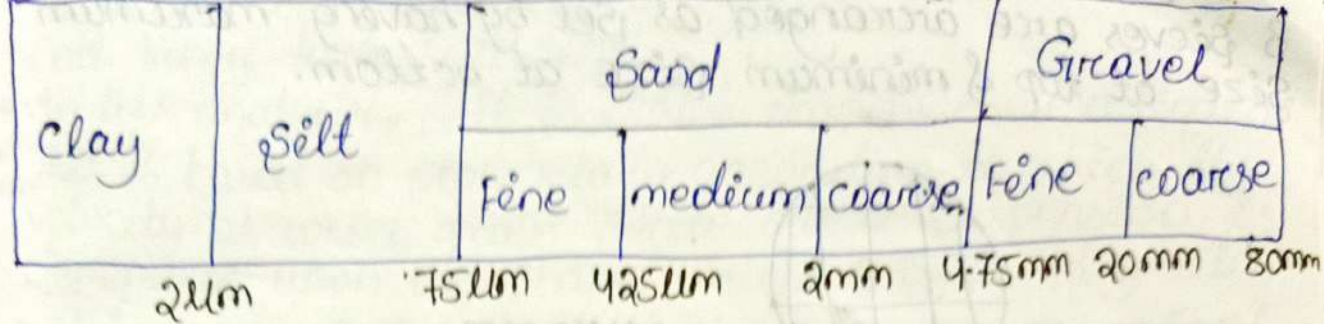
a. Infra-red lamp & torsion balance method.

b. calcium carbide method.

c. Proctor needle method.

$$\frac{(M_2 - M_1)}{(M_3 - M_4)} = \frac{\text{Weight of water}}{\text{Weight of solids}} = \text{Water content}$$

This method can be used for quick determination of water content. This method is conducted when specific gravity of soil solids (G) is known.



→ The particle size distribution is determined by conducting grain size analysis also known as mechanical analysis.

→ It consists of two parts:

(i) Sieve Analysis: It is conducted for Sand & gravel.

(ii) Sedimentation Analysis: It is conducted for silt & clay.

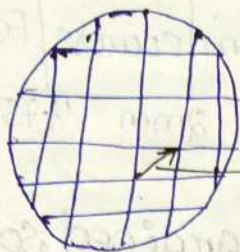
Sieve Analysis:

→ The soil retained on 4.75mm sieve is subjected to coarse sieve analysis which consist of sieving the soil through a nest of sieve consist of 40mm, 20mm & 10mm IS sieve.

→ The soil passing through 4.75mm sieve is subjected to fine sieve analysis which consist of sieving the soil through a nest of sieves which is consist of 2mm, 1mm, 600 μ m, 300 μ m, 210 μ m, 150 μ m, 75 μ m sieves.

Dry sieve analysis:

8 sieves are arranged as set by having maximum size at top & minimum size at bottom.



Sieve Size (D mm)	Mass of Soil retained (g)	Percentage mass retained (W)	Cumulative Percentage retained (P)	Percentage Finer (N)
4.75 mm	W_1	$\frac{W_1}{W} \times 100$	$\frac{W_1}{W} \times 100 = P_1$	$100 - P_1$
2.36 mm	W_2	$\frac{W_2}{W} \times 100$	$\frac{W_1}{W} \times 100 + \frac{W_2}{W} \times 100 = P_2$	$100 - P_2$
1.18 mm	W_3	$\frac{W_3}{W} \times 100$	$\frac{W_1}{W} \times 100 + \frac{W_2}{W} \times 100 + \frac{W_3}{W} \times 100 = P_3$	$100 - P_3$

Q. By conducting sieve analysis out of 1000g, 400g was retained on 600 μ m sieve size, 300g was retained on 425 μ m sieve & 200g was retained on 75 μ m sieve size.

Sieve Size (D mm)	Mass of Soil retained (g)	Percentage mass retained (W)	Cumulative % retained (P)	% Finer (N)
600 μ m	400	$\frac{400}{1000} \times 100 = 40\%$	40%	$100 - 40 = 60\%$
425 μ m	300	$\frac{300}{1000} \times 100 = 30\%$	$40 + 30 = 70\%$	$100 - 70 = 30\%$
75 μ m	200	$\frac{200}{1000} \times 100 = 20\%$	$70 + 20 = 90\%$	$100 - 90 = 10\%$

Sedimentation Analysis :- or (wet analysis)

- It is also wet mechanical analysis, it is conducted on Soil finer than 75 μ Sieve.
 - In this analysis, soil is kept in suspension in a liquid.
 - This is based on Stoke's law, according to which the velocity at which grain settle out of suspension, is dependent upon the shape, weight & size of grain.
- It is assumed that the soil particles are spherical & have same specific gravity.
- The coarser particle settle more quickly than finer ones.
- If V_t is the terminal velocity of sinking of a spherical particle, it is given by

$$V = \frac{gd^2}{18\mu} (\rho_s - \rho_w)$$

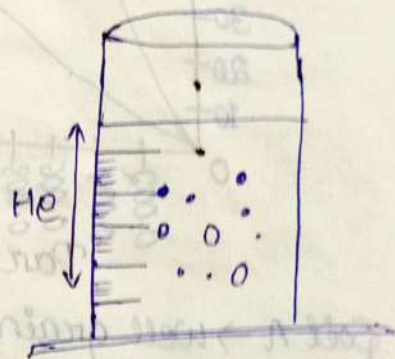
d = dia of spherical particle

V = terminal velocity

ρ_s = density of soil particle

ρ_w = density of water

μ = viscosity (in poise)



If a particle of dia ' d ' mm falls through a ht. ' H_e ' cm in ' t ' min.

$$V = \frac{H_e}{60t} \text{ cm/s} = \frac{gd^2}{18\mu} (\rho_s - \rho_w)$$

→ Sedimentation analysis is done by 2 methods :-

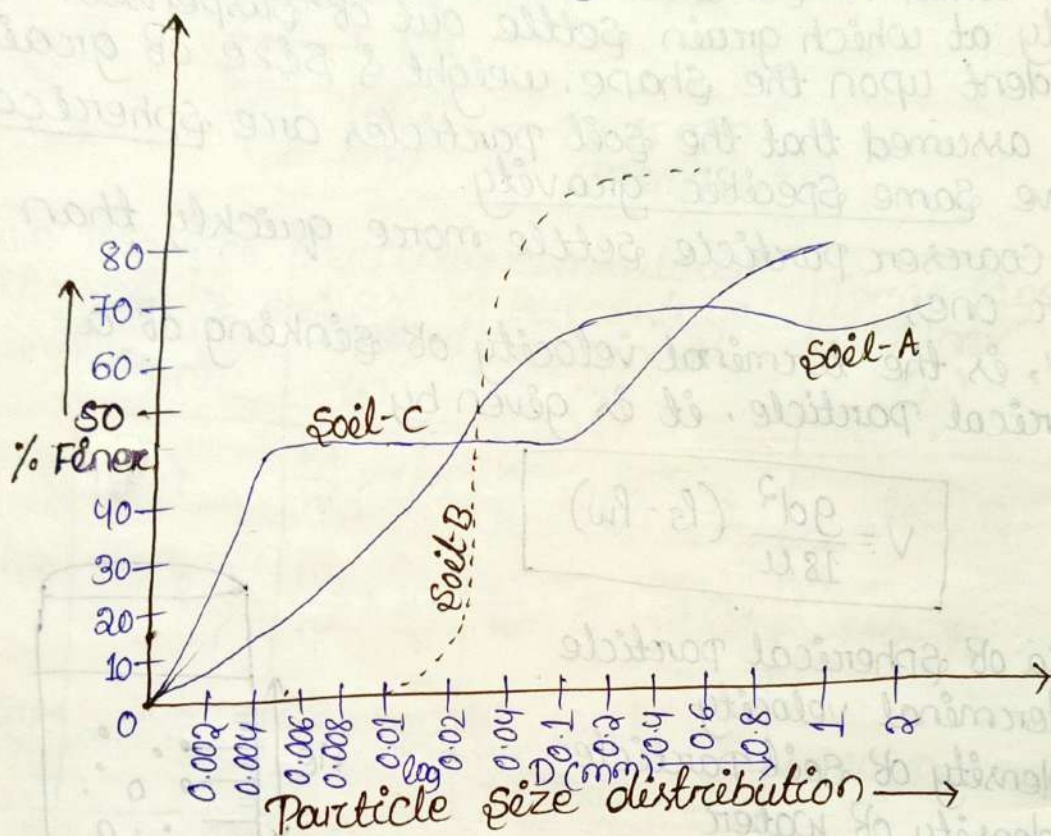
(i) Pipette method

(ii) Hydrometer method

$$\frac{\rho_s}{\rho_w} = G$$

Particle Size distribution curve

→ The Particle Size distribution curve is obtained by Plotting Percentage finer 'N' as ordinate on natural scale against Particle Size (Dmm) on abscissa (x-axis) on logarithmic scale.



Soil A → well graded soil (Soil has particles of all size)

Soil B → uniform graded soil (Soil has particles of similar type)

Soil C → Gap graded soil (Some diameter of soil are missing)

For coarse graded soil D_{10} , D_{30} & D_{60} are important.

D_{10} → 10% of the particle are finer than this size.

D_{10} → It is also called effective size or diameter.

D_{60} → 60% of mass of soil particle or finer than this size.

C_u (Uniformity coefficient) → It is a measure of Particle size range.

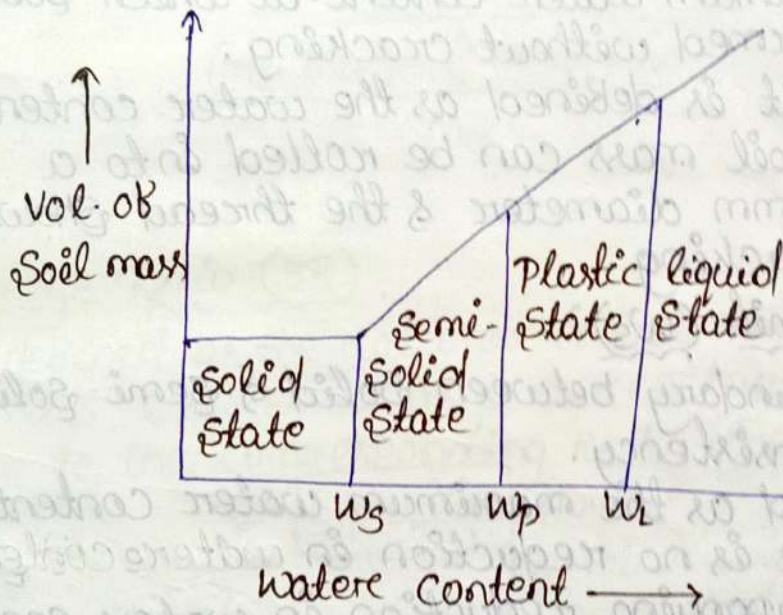
$$C_u = \frac{D_{60}}{D_{10}}$$

C_c (Coefficient of curvature) $\rightarrow$ It present shape of Particle size curve

$$C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}}$$

Consistency of Soil:

- $\rightarrow$ consistency is the relative ease with which soil can be deformed. This term is mostly used for fine grained soil.
- $\rightarrow$ consistency is directly related to water content.
- $\rightarrow$ Consistency limit are the water content at which soil mass passes from 1 state to other. (Atterberg limit)



- $\rightarrow$ In Solid state there will be no change in volume of soil mass accompanying change in water content.
- $\rightarrow$ In the remaining 3 states increase in water content is accompanied by increase in volume of soil mass & vice-versa.
- $\rightarrow$ In Semi-solid state the soil mass cannot be deformed without cracking.
- $\rightarrow$ In plastic state the soil mass can be deformed without cracking.
- $\rightarrow$ In liquid state the soil mass behaves like liquid & can flow.

→ Liquid limit (W_L) :-

It is the boundary between plastic state & liquid state or the minimum water content at which soil flows like liquid.

→ Liquid limit is defined as the water content at which a groove, cut with standard grooving tool, in soil pat taken in the cup of a standard liquid limit device closes for a distance of 13 mm when cup is imparted 25 blows.

→ Plastic limit (W_p) :-

→ It is the boundary b/w semi-solid & plastic states of consistency.

→ It is the minimum water content at which soil mass can be deformed without cracking.

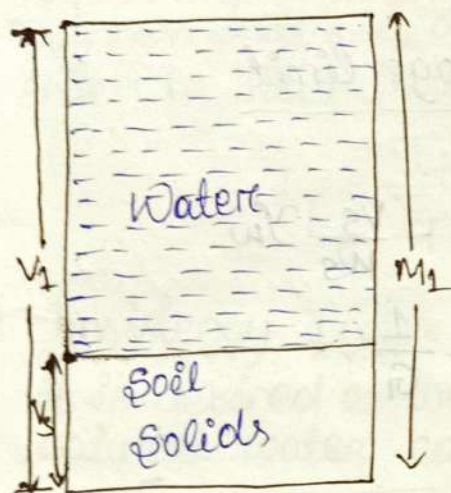
→ Plastic limit is defined as the water content at which the soil mass can be rolled into a thread of 3 mm diameter & the thread shows signs of cracking.

→ Shrinkage limit (W_s) :-

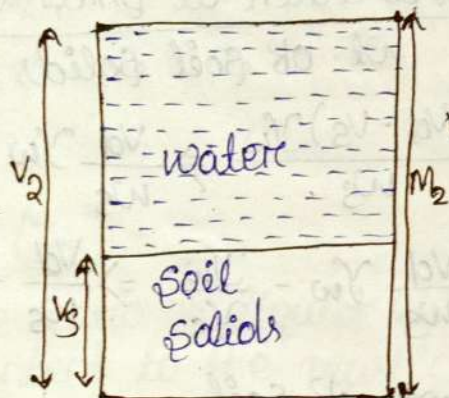
→ It is the boundary between solid & semi-solid states of consistency.

→ It is defined as the maximum water content at which there is no reduction in volume of soil mass accompanying reduction in water content.

→ It test is conducted by Mercury displacement method.



(a) wet soil



(b) soil at Shrinkage limit



(c) dry soil

$$W_s = \frac{(M_1 - M_d) - (V_1 - V_d) \gamma_w}{M_d}$$

Shrinkage Ratio (SR):

It is defined as the ratio of reduction in volume of soil mass expressed as a percentage of its dry volume to the corresponding reduction in water content.

or $S.R = \frac{\gamma_d}{\gamma_w} \rightarrow \text{Formula}$

$$SR = \frac{\frac{V_1 - V_2}{V_d}}{W_1 - W_2} \times 100$$

V_1 = vol. of soil at water content ' W_1 '

V_2 = vol. of soil at water content ' W_2 '

V_d = vol. of dry soil mass

Volumetric Shrinkage (VS):

It is defined as the reduction in volume of soil mass expressed as a percentage of its dry volume when a soil mass is dried from a water content above shrinkage limit to shrinkage limit.

$$V.S = \frac{V_1 - V_d}{V_d} \times 100\%$$

Shrinkage Limit Formula :-

$$w_s = \frac{\text{wt. of water at shrinkage limit}}{\text{wt. of soil solids}}$$
$$= \frac{(V_d - V_s) \gamma_w}{w_s} \Rightarrow \frac{V_d \gamma_w}{w_s} - \frac{V_s \gamma_w}{w_s}$$
$$\Rightarrow \frac{V_d}{w_s} \gamma_w - \frac{\gamma_w}{\gamma_s} \Rightarrow \frac{V_d}{w_s} - \frac{1}{G}$$

V_d = dry volume of soil

V_s = volume of soil solids

w_s = wt. of soil solid = dry wt. of soil = w_d

G = Specific gravity of soil.

Atterberg Indices :-

Following is the

of Atterberg Indices:

1. Plasticity Index
2. Flow Index
3. Toughness Index
4. Consistency Index
5. Liquidity Index

1. Plasticity Index :-

→ It is defined as liquid limit minus plastic limit.

$$I_p = w_L - w_p$$

2. Flow Index :-

→ Flow Index is the slope of Flow curve obtained by plotting water content as ordinate on natural scale against number of blows as abscissa (x-axis) on logarithmic scale.

$$I_f = \frac{w_1 - w_2}{\log_{10} \left(\frac{N_2}{N_1} \right)}$$

w_1 = water content corresponding to no. of blows, N_1

w_2 = water content corresponding to no. of blows, N_2

3. Toughness Index:

Toughness Index is defined as the ratio of plasticity Index to Blow Index.

$$I_T = \frac{I_P}{I_B}$$

4. Consistency Index (I_C):-

It is defined as the ratio of liquid limit minus natural water content to the plasticity Index.

$$I_C = \frac{W_L - W}{I_P}$$

5. Liquidity Index (I_L):-

It is defined as the ratio of natural water content minus plastic limit to plasticity Index.

$$I_L = \frac{W - W_p}{I_P}$$

6. Density Index (I_D):-

It is also called as relative density.

It is the ratio of difference between maximum void ratio & natural void ratio to the difference between maximum void ratio & minimum void ratio.

$$I_D = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

$$\left\{ \begin{array}{l} e_{\max} = \text{void ratio in loosest state.} \\ e_{\min} = \text{void ratio in densest state.} \end{array} \right.$$

$$\begin{aligned} \Delta W &= 29 \\ 7.8 \times 6.0 &= (1)96 \\ \Delta W &= 9 \end{aligned}$$

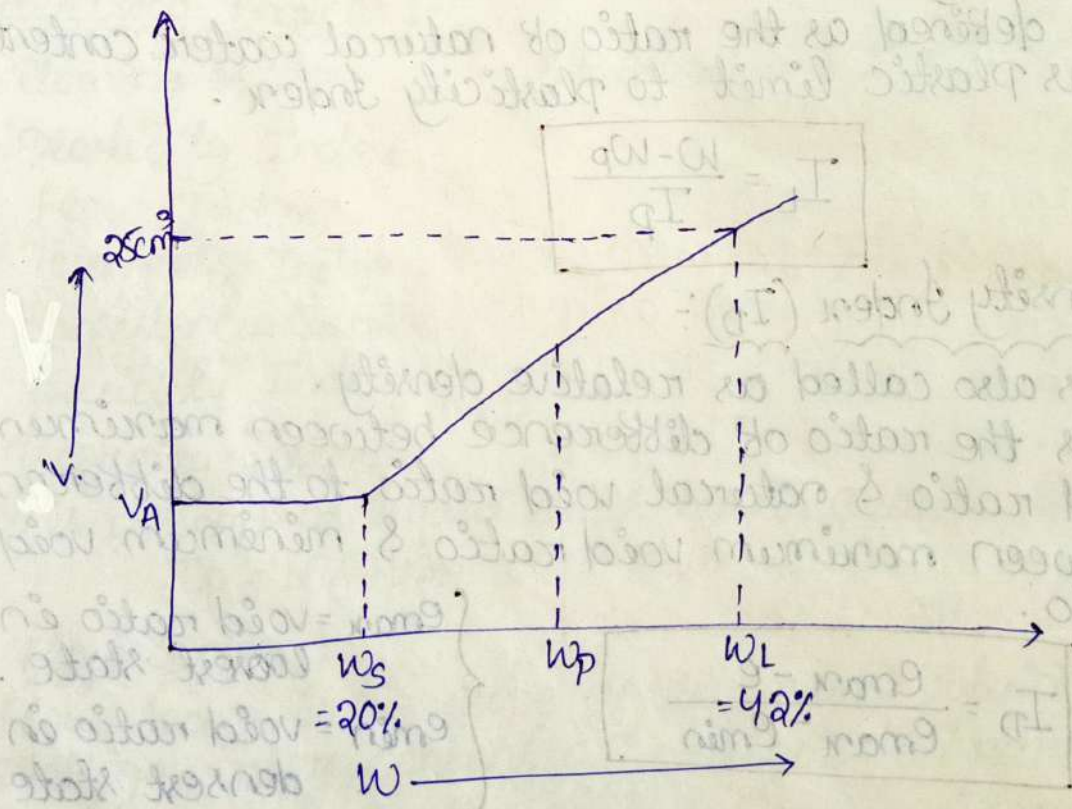
Q. A saturated soil sample has a volume of 25 cm^3 at liquid limit. If the soil mass liquid limit & shrinkage limit of 42% & 20% respectively. Determine the minimum volume which can be attained by the soil specimen. (Take $G = 2.72$)

Ans. The soil specimen will attain minimum volume at shrinkage limit.

$$\text{S.R} = \frac{V_d}{V_w}$$

(Shrinkage ratio)

Let the volume at shrinkage limit $= V_A$



$$e_s = w_s G$$

$$\Rightarrow e(1) = 0.2 \times 2.7$$

$$e = 0.544$$

$$\gamma_d = \frac{G \gamma_w}{1+e}$$

$$\gamma_d = \frac{2.72 \times 1}{1+0.544}$$

$$= 1.76 \text{ g/cc}$$

$$\Rightarrow S.R = \frac{\gamma_d}{\gamma_w} = \frac{1.76}{1} = 1.76$$

$$S.R = \frac{\frac{V_1 - V_2}{V_d}}{w_1 - w_2}$$

$$S.R = \frac{25 - V_A}{V_A}$$

$$0.42 - 0.2$$

$$1.76 = \frac{25 - V_A}{V_A}$$

$$0.22$$

$$\Rightarrow 1.76 \times 0.22 = \frac{25 - V_A}{V_A}$$

$$\Rightarrow 0.3872 = \frac{25 - V_A}{V_A}$$

$$\Rightarrow 0.3872 \times V_A = 25 - V_A$$

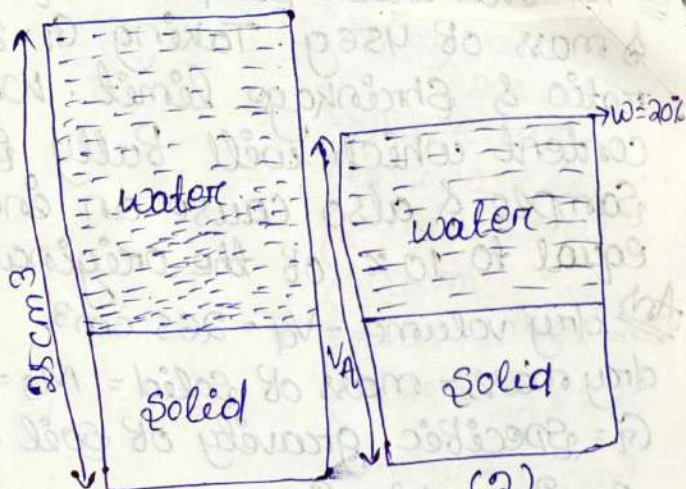
$$\Rightarrow 0.3872 \times V_A + V_A = 25$$

$$\Rightarrow V_A (0.3872 + 1) = 25$$

$$\Rightarrow V_A (1.3872) = 25$$

$$\Rightarrow V_A = \frac{25}{1.3872} = 18.03$$

$$\text{on solving } V_A = V_d = 18.03 \text{ cm}^3$$



(1)

$w = 42\%$

(2)

$w = 20\%$

$V_1 = \text{Vol. @ water content } w_1$
 $V_2 = \text{Vol @ water content } w_2$
 $V_d = \text{dry volume}$
 $= \text{original volume}$

$$V_1 = 25 \text{ cm}^3$$

$$V_A = ?$$

$$V_A = V_d = ?$$

Q. An oven dried sample of soil has a volume of 265 cm^3 & mass of 456 g . Taking $G_s = 2.71$, determine the void ratio & shrinkage limit. What will be the water content which will fully saturate the soil sample & also cause an increase in volume equal to 10% of the original dry volume.

Ans dry volume = $V_d = 265 \text{ cm}^3$

dry mass = mass of solid = $M_s = 456 \text{ g}$.

$G_s = \text{Specific gravity of soil solids} = 2.71$

$e = ?$, $w_s = ?$

$$\gamma_d = \rho_d = \frac{\text{Mass of soil solids}}{\text{Volume of soil}}$$

$$= \frac{456}{265} = 1.72 \text{ g/cc}$$

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

$$\Rightarrow 1.72 = \frac{(2.71)(1)}{1+e}$$

$$\Rightarrow e = 0.575$$

$$e.s.r = w G_s$$

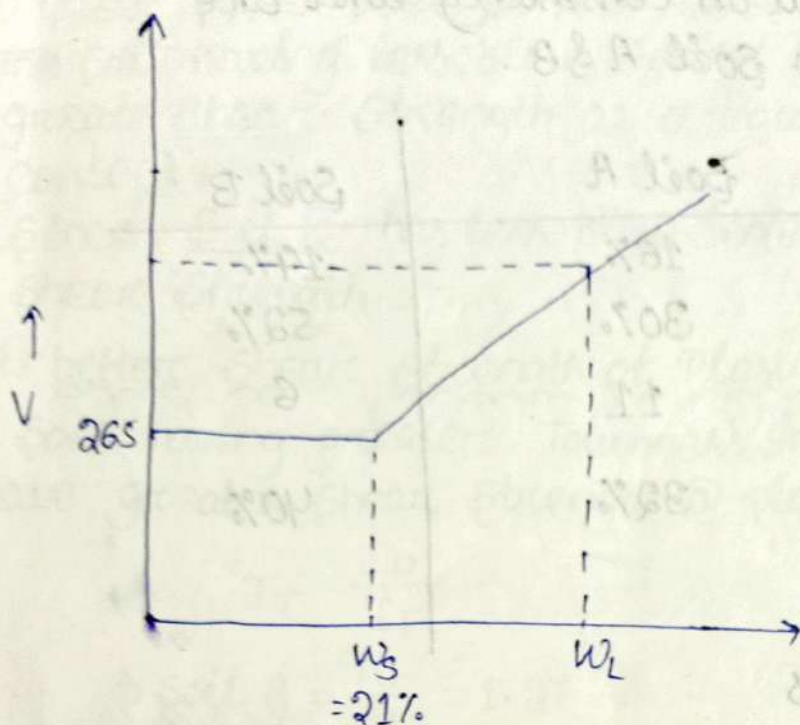
$$\Rightarrow (0.575) \times (1) = w(2.71)$$

$$\Rightarrow w = 21\% = 0.21$$

→ The minimum water content to fully saturate the soil is 21% or 0.21 (21% is called shrinkage limit)

→ After, this 21% of water content, & water content further increases, the vol. of soil also increases but the degree of saturation is 100%.

So, let the water content at which volume increase to 10% of original dry volume = w_1



→ Dry volume
= 265 cm³

→ After 10% increase in original dry volume
= 265 + 10% of 265
= 265 + $\frac{10}{100} \times 265$
= 291.5 cm³

$$S.R = \frac{V_1 - V_2}{V_d} = \frac{w}{\gamma_w} = \frac{1.72}{1}$$

$$\Rightarrow 1.72 = \frac{291.5 - 265}{w_1 - 0.21}$$

$$\Rightarrow 1.72 = \frac{0.1}{w_1 - 0.21}$$

$$\Rightarrow 1.72(w_1 - 0.21) = 0.1$$

$$\Rightarrow w_1 - 0.21 = \frac{0.1}{1.72}$$

$$\Rightarrow w_1 = 0.058 + 0.21$$

$$\Rightarrow w_1 = 0.268 = 26.8\%$$

$$\gamma_d = \frac{G \gamma_w}{1 + e} = \frac{2.71 \times 1}{1 + 0.57} = 1.72 \text{ g/cc}$$

Q. The Following data on consistency limit are available for two Soil A & B.

	Soil A	Soil B
Plastic limit	16%	19%
Liquid limit	30%	52%
Flow Index	11	6
Natural Water Content	32%	40%

Find Which Soil is

- (a) more Plastic
 - (b) better Shear Strength as a function of water content.
 - (c) better Foundation material
 - (d) better Shear Strength at Plastic limit.
- Ans (a) The Soil which has more plasticity Index will be more Plastic.

$$\text{Plasticity Index (Ip) of Soil A} = 30 - 16 = 14\%$$

$$\text{Plasticity Index (Ip) of Soil B} = 52 - 19 = 33\%$$

So, Soil B is more Plastic.

(C) Better Foundation material:

$$\text{consistency Index (Ic)} = \frac{W_L - W}{I_p}$$

The Soil having more (Ic) will be good Foundation material.

$$(I_c) \text{ of Soil A} = \frac{30 - 32}{14} = -0.14$$

$$(I_c) \text{ of Soil B} = \frac{52 - 40}{30} = 0.36$$

So, Soil B is better Foundation material.

(b) better Shear Strength @ water content:
The soil having less blow index (I_F) will have greater shear strength as a function of water content.
Since, Soil B has less blow index. It has better shear strength.

(d) better Shear strength at Plastic limit:
Soil having greater Toughness Index (I_T) will have greater shear strength @ plastic limit.

$$I_T = \frac{I_p}{I_F}$$

$$\Rightarrow \text{Soil A} = \frac{14}{11} = 1.27$$

$$\text{Soil B} = \frac{33}{6} = 5.5$$

CH-3

Classification of Soils

- The purpose of Soil classification is to arrange various types of soils into specific groups based on physical properties & engineering behaviour of soils.
- Several Soil classification system are given as follows:

- 1) Highway Research board classification system (HRB)
- 2) Unified Soil classification system.
- 3) Indian Standard Soil classification system.

Particle Size Classification System:

In these system soil are arranged according to Particle Size range only without considering any other characteristics.

1. U.S Bureau of Soil classification & public Road Administration.
2. M.I.T. classification system.
3. Indian Standard Particle Size classification system.

Highway Research Board Classification System (HRB)

- HRB system is based on both Particle Size range & plasticity characteristics.
- This system is mostly used for Pavement construction.
- Here, soils are divided into 7 Primary groups, designated as A-1, A-2, ..., A-7.
- The higher value of group index the poorer is the quality of material.
- Group index is used to describe the performance of soil when used for Pavement construction.

→ Group Index of soil depends on:-

- (i) Amount of material passing through 75 μ sieve.
- (ii) Liquid limit.
- (iii) Plastic limit.

→ Group Index is given by the following equation.

$$\text{Group Index (GI)} = 0.2a + 0.005ac + 0.01bd$$

Where, a = the portion of % passing 75 μ sieve greater than 35 & less than 75. (as a whole no.) [0-40]

b = the portion of % passing 75 μ sieve greater than 15 & less than 55 (as whole no.) [0-40]

c = the portion of numerical liquid limit greater than 40 as less than 60 expressed as a whole number. [0-20]

d = The portion of numerical plasticity index greater than 10 & less than 30 expressed as whole number. [0-20].

Q. The properties of a subgrade soil are found as:

(i) % Finer than 75 μ = 55%

(ii) liquid limit = 50%

(iii) plastic limit = 40%

Classify the soil according to HRB classification system.

Ans $a = 55 - 35 = 20$

$$b = 55 - 15 = 40$$

$$c = 50 - 40 = 10$$

$$I_p = W_L - W_p = 50 - 40 = 10$$

$$d = 10 - 10 = 0$$

$$G.I = 0.2a + 0.005ac + 0.01bd$$

$$= 0.2 \times 20 + 0.005 \times 20 \times 10 + 0.01 \times 40 \times 0$$

$$= 5$$

Indian Standard classification system, ISCS:

- ISCS, first developed in 1959, was revised in 1970.
- The revised version is essentially based on USCS with some modification, that the fine grained soil have been subdivided into 3 groups (low, medium & high Plasticity).
- Soil are divided into three divisions:

1. Coarse grained soil:

- In these soil more than half the total material by mass is larger than 75 μ m IS sieve.

2. Fine grained soil:

- Here, more than 50% by weight is passes through 75 μ m IS sieve.

3. Highly organic soil:

These soil contains large % of fibrous organic matter such as peat & particles of decomposed vegetation.

1. Coarse grained soil

Coarse grained are further divided into 2-subgroups:

a Gravel (G):

If more than 50% by mass of coarse grained fraction is retained on 4.75 mm IS sieve.

b Sands (S):

In this soil, is more than 50% by mass of coarse grained fraction is passes through 4.75 mm IS sieve.

→ Depending on gradation Gravel (G), Sand (S) are further divided into sub-groups.

GW :- Well graded gravel [$C_u > 4$
 $4 C_c \rightarrow (1-3)$]

GP :- Poorly graded gravel [which doesn't meet all requirement of GW]

SW :- Well graded Sand. [$C_u > 6$ & $C_c \rightarrow (1-3)$]

SP :- Poorly graded Sand. [which doesn't meet all requirement of SW]

GM :- Silty gravel, i.e. $I_p < 4 \rightarrow$ For Fine grained Braction.

GC :- Clayey gravel, i.e. $I_p > 7 \rightarrow$ For Fine grained Braction.

SM :- Silty Sand, i.e. $I_p < 4$ For Fine grained Braction.

SC :- Clayey Sand, i.e. $I_p > 7$ For Fine grained Braction.

2. Fine grained Soil :

Fine grained Soil are further divided into 3 sub-division.

(a) Inorganic Silt : (M)

(b) Inorganic clay : (C)

(c) organic Silt & clay & organic matter (O).

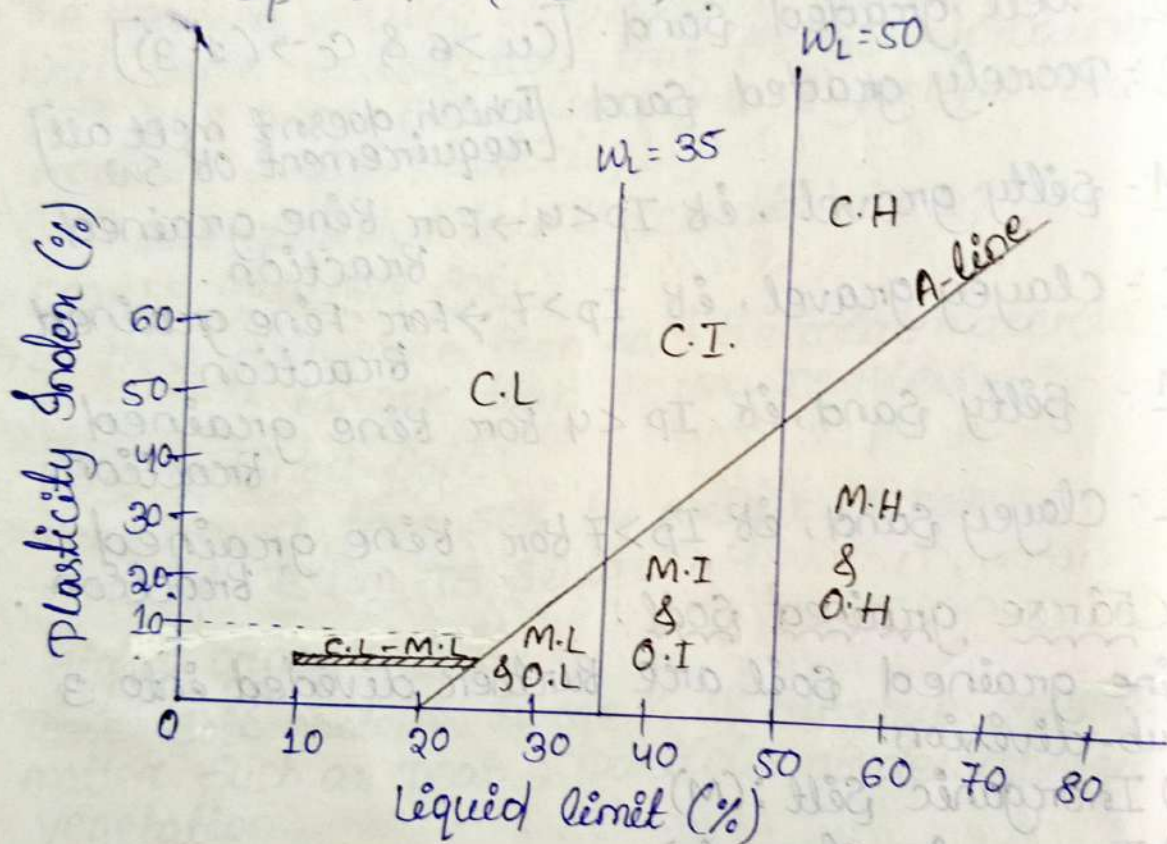
These are further divided into 3 groups depending on liquid limit which is considered a good index of compressibility.

- (i) low compressibility ($w_L < 35\%$) letter $\rightarrow$ L
(ii) medium compressibility ($35\% < w_L < 50\%$) letter $\rightarrow$ I
(iii) Highly compressibility ($w_L > 50\%$) letter $\rightarrow$ H

combination of these symbols indicates type of Fine grained Soil.

- eg - M.L $\rightarrow$ inorganic silt with low compressibility.
- $\rightarrow$ Laboratory classification of fine grained soil is done with the help of plasticity chart.
- $\rightarrow$ The A-line, dividing inorganic clay from silt & organic soil has following eqⁿ.

$$I_p = 0.73 (w_L - 20)$$



There are further divided into 3 groups depending on liquid limit which is considered a good order of compressibility.

Highly compressibility ($w_L > 50\%$)

Medium compressibility ($32\% < w_L < 50\%$)

Low compressibility ($w_L < 32\%$)

Symbol indicates type of fine grained soil.

Permeability

- Permeability is the ability of soil mass to permit flow of water through interconnected void present in soil.
- Flow of water through soil is called seepage.
- Seepage always takes place from higher energy level to lower energy level.
- The total energy at any point can be written from Bernoulli's eqⁿ as:-

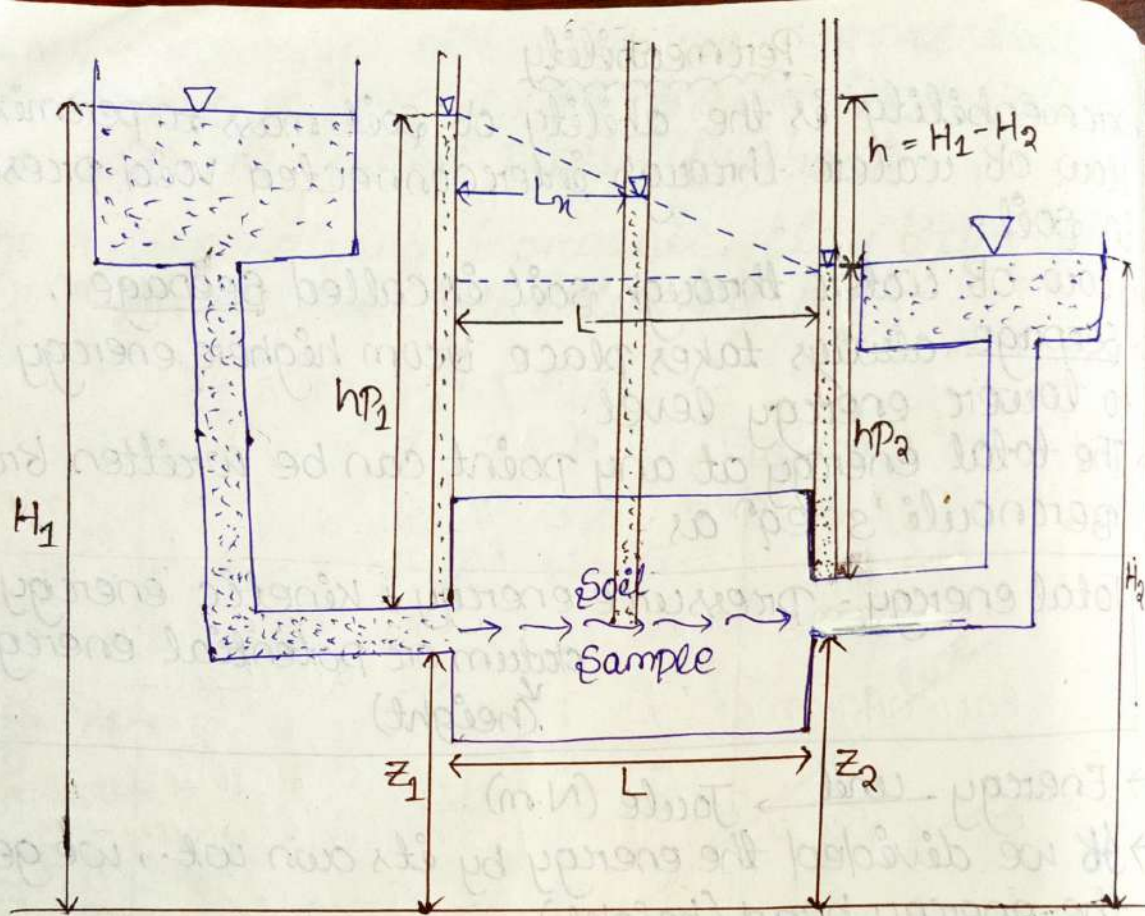
$$\text{Total energy} = \text{pressure energy} + \text{kinetic energy} + \text{datum or potential energy (height)}$$

- Energy unit → Joule (N.m).
- If we divided the energy by its own wt., we get the energy head (height).
- So, total energy in terms of total head is:-

$$T = \frac{P}{\rho g} + \frac{v^2}{2g} + Z$$

T = total head
 $\frac{P}{\rho g}$ = pressure head
 $\frac{v^2}{2g}$ = velocity head
 Z = datum head.

- In soil mass, velocity of flow is very less, so we neglect velocity head.
- In soil, $T = \frac{P}{\rho g} + Z$ is called "piezo metric head".



h = head loss (or) head causing flow (or) seepage head

→ Slope of H.G.L. (Hydraulic gradient line) is "hydraulic gradient".

Hydraulic gradient is denoted by " i "

$$i = \frac{h}{L} = \frac{h_x}{L_x}$$

→ head loss @ any section is given by $h_x = i L_x$

Note :

Ht. of water level rises into piezometer at the point of consideration is given by ' h_p '.

Darcy law & coefficient of permeability

→ According to Darcy's law, for laminar flow condition the velocity of flow (v) is directly proportional to hydraulic gradient (i).

$$V \propto i$$

$$V = ki$$

$\left\{ \begin{array}{l} k = \text{coefficient of permeability} \\ \text{unit of } k = \text{cm/s or m/day} \end{array} \right.$

v = discharge velocity

- discharge velocity (v) is not true velocity, it is superficial velocity as the actual area is ' A_v ' not ' A '.
- True velocity (or) seepage velocity is V_s .

Rate of Flow or discharge (Q):

- It is defined as the quantity of fluid flowing through a section.
- The rate of flow is expressed as the volume of fluid flowing across a section per second.
- Discharge is given as:-

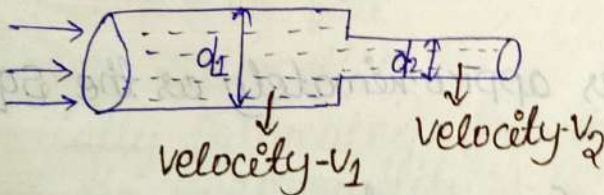
$$Q = A \times V$$

$\left\{ \begin{array}{l} A = \text{cross-sectional area through which water is flowing} \\ V = \text{average velocity of water} \end{array} \right.$

- As continuity equation is valid, the quantity of fluid per second is constant.

$$Q = A_1 V_1 = A_2 V_2$$

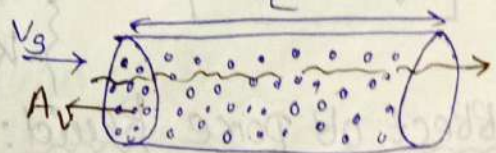
$$= \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} d_2^2 \times V_2$$



In case of soil



theoretical flow



actual flow

$$Q = AV = A_v V_s$$

$$V_s = \frac{A}{A_v} \times v = \frac{A \times L \times v}{A_v \times L} \Rightarrow \left[\frac{\text{volume of soil}}{\text{volume of void}} \times \text{velocity} \right]$$

$$V_s \propto \frac{1}{\eta} \times V$$

Values of k

Soil type	co-efficient of permeability (k) [cm/sec]
Gravel	1 to 100
Sand	10^{-3} to 1
Silt	10^{-6} to 10^{-3}
clay	$< 10^{-6}$

Factors affecting permeability

1. Particle size
2. Properties of pore fluid
3. Void ratio
4. Soil fabric & soil stratification
5. Degree of saturation
6. Presence of foreign matter
7. Adsorbed water.

1. Particle Size :

→ Permeability varies approximately as the square of grain size.

$$K = C D_{10}^2$$

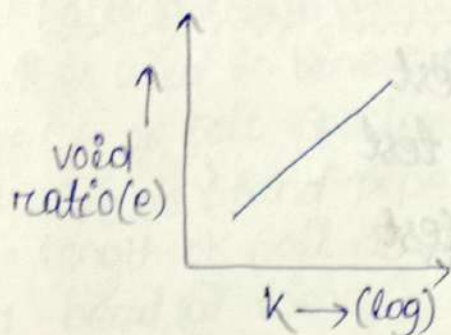
$\left\{ \begin{array}{l} C = \text{constant} \longrightarrow 100 \\ D_{10} = \text{effective size (cm)} \\ K = \text{co-eff. of permeability (cm/sec)} \end{array} \right.$

2. Effect of pore fluid:

→ Permeability is directly proportional to unit wt. of pore fluid & inversely proportional to viscosity.

$$K \propto \frac{\gamma_w}{\mu}$$

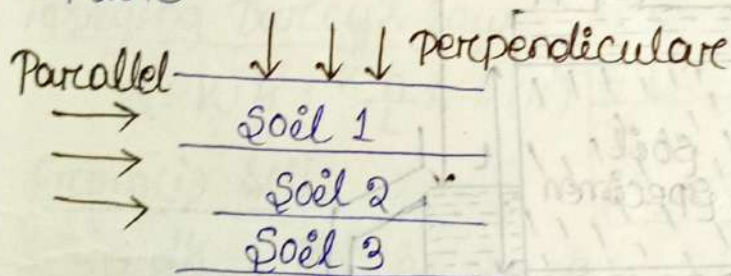
3. Effect of void ratio :



For coarse grained Soil
$$k = \frac{e^3}{1+e}$$

4. Effect of Soil Fabric & Stratification :

→ Stratified (layered) soil masses will have different average permeability in directions parallel & perpendicular to bedding planes. The average permeability parallel to bedding plane will be greater than that of perpendicular to bedding plane.



Stratified Soil

5. Degree of Saturation :

- In partially saturated soil the entrapped air reduces the permeability.
- Permeability test is always conducted on a fully saturated soil sample.

6. Effect of presence of foreign matter :

- Organic foreign matter, if present in soil mass, may be carried by flowing water and choke them up, causing reduction in permeability.

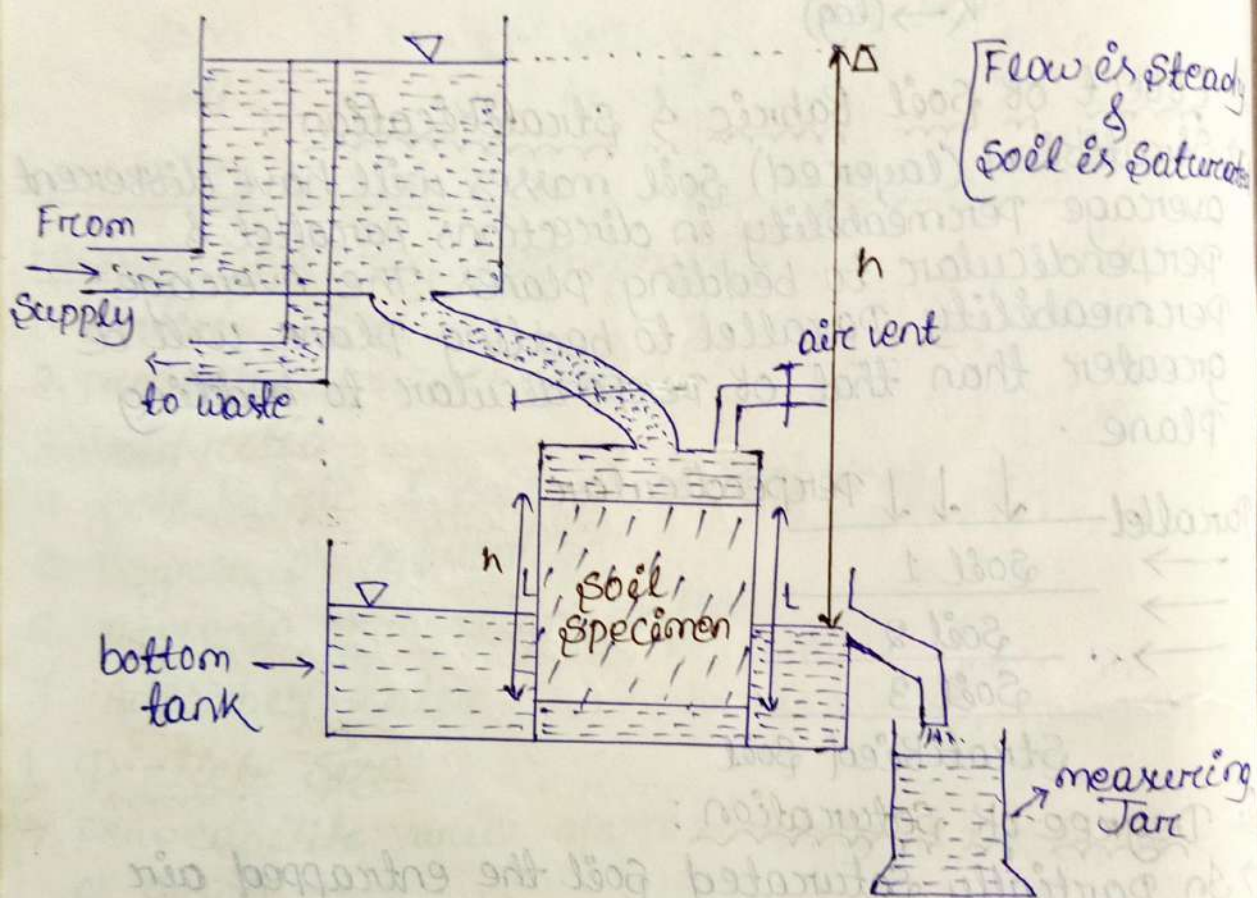
7. Effect of adsorbed water :

The adsorbed water, which is held by soil particles, is not free to move & therefore reduces the effective pore space available for the flow of free water.

Determination of coefficient of permeability

1. Laboratory Methods

- (a) constant head permeability test
 - (b) Falling head permeability test
- (a) constant head permeability test



A = cross sectional area of soil specimen

L = length of soil specimen

h = constant head (difference in water level of overhead & bottom tank)

$$Q = kiA$$

$$\frac{V}{t} = Q = k \frac{h}{L} A$$

$$K = \frac{VL}{Aht}$$

$$Q = A \times v$$

$$\begin{cases} v = ki \\ i = h/L \end{cases}$$

V = volume of water
 t = time

→ It is used for coarse grained soil only.

(b) Falling head (or variable head) permeability test:

→ It is used in fine grained soil

A = C/s of soil specimen

a = C/s of stand pipe

L = length of soil specimen

h_1 = head @ time t_1

h_2 = head @ time t_2

∴ h is the head @ time t & ' dh ' the fall in head at time ' dt '.

$$Q = \frac{-a \cdot dh}{dt} \left[\begin{array}{l} \text{+ve indicates} \\ \text{fall in head} \end{array} \right] \quad (i)$$

Applying Darcy's law

$$Q = k i A = k \frac{h}{L} A \quad (ii)$$

From (i) & (ii)

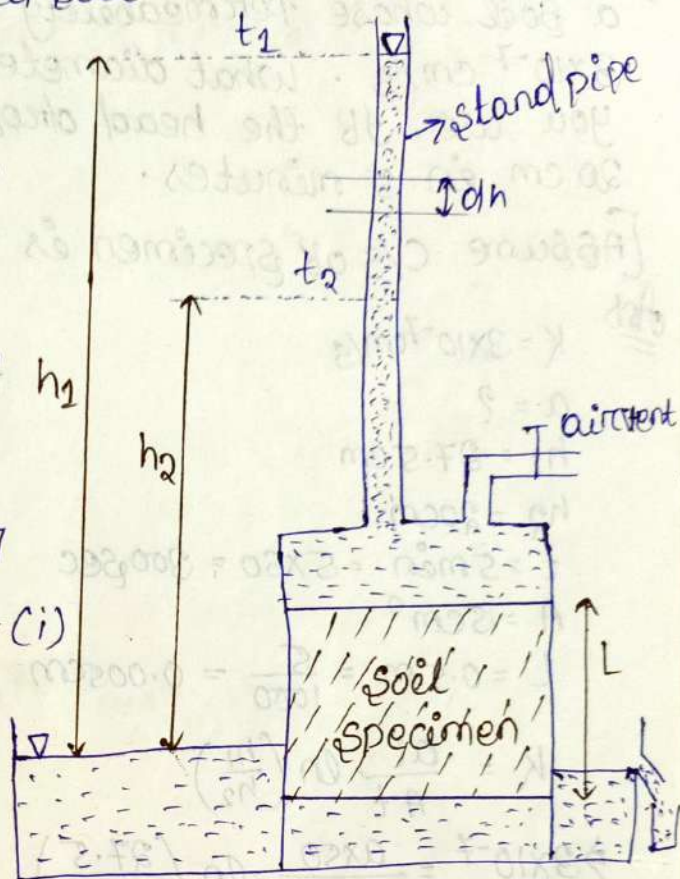
$$-\frac{a \cdot dh}{dt} = k \frac{h}{L} A \Rightarrow k dt = -\frac{aL}{A} \frac{dh}{h}$$

Integration b/w suitable limits

$$\int_{t_1}^{t_2} k dt = -\frac{aL}{A} \int_{h_1}^{h_2} \frac{dh}{h}$$

$$k(t_2 - t_1) = \frac{aL}{A} \ln \frac{h_1}{h_2}$$

$$\Rightarrow \boxed{k = \frac{aL}{At} \ln \frac{h_1}{h_2}}$$



Problem
4th page

Q. A falling head permeability test is to be conducted on a soil whose permeability is estimated to be $3 \times 10^{-7} \text{ cm/s}$. What diameter of standpipe would you use, if the head dropped from 27.5 cm to 20 cm in 5 minutes.

[Assume C/S of specimen is 15 cm^2 & length = 0.5 m]

Ans

$$K = 3 \times 10^{-7} \text{ cm/s}$$

$$a = ?$$

$$h_1 = 27.5 \text{ cm}$$

$$h_2 = 20 \text{ cm}$$

$$t = 5 \text{ min} = 5 \times 60 = 300 \text{ sec}$$

$$A = 15 \text{ cm}^2$$

$$L = 0.5 \text{ m} = \frac{5}{1000} = 0.005 \text{ m}$$

$$K = \frac{aL}{At} \ln \left(\frac{h_1}{h_2} \right)$$

$$\Rightarrow 3 \times 10^{-7} = \frac{a \times 50}{15 \times 300} \ln \left(\frac{27.5}{20} \right)$$

$$\Rightarrow 3 \times 10^{-7} = \frac{a \times 50}{4500} \times 0.318$$

$$\Rightarrow a = \frac{3 \times 10^{-7} \times 4500}{50 \times 0.318}$$

$$\Rightarrow a = 8.5 \times 10^{-5} \text{ cm}^2$$

$$A = \pi r^2$$

$$\Rightarrow 8.5 \times 10^{-5} = 3.14 \times \pi r^2$$

$$\Rightarrow \pi r^2 = \frac{8.5 \times 10^{-5}}{3.14}$$

$$\Rightarrow \pi r = \sqrt{\frac{8.5 \times 10^{-5}}{3.14}}$$

$$\Rightarrow \pi r = 5.2029 \times 10^{-3}$$

$$\begin{aligned} d &= 2\pi r \\ &= 2 \times 5.2 \times 10^{-3} \\ &= 0.0104 \text{ cm} \end{aligned}$$

Q. A falling head permeability test was conducted on a soil sample of 4 cm diameter & 18 cm length the head fell from 1m to 0.4m in 20 min. If the c/s of standpipe is 1 cm^2 , determine coefficient of permeability.

Ans $d = 4 \text{ cm}$
given.

Soil sample = 4 cm

$l = 18 \text{ cm}$

$h_1 = 100 \text{ cm}$

$h_2 = 40 \text{ cm}$

$t = 1200 \text{ s}$

$a = 1 \text{ cm}^2$

$$A = \pi r^2$$

$$= \pi \left(\frac{d}{2}\right)^2$$

$$= 3.14 \times \left(\frac{4}{2}\right)^2 = 3.14 \times 2^2 = 3.14 \times 4 = 12.56$$

$$K = \frac{al}{At} \ln\left(\frac{h_1}{h_2}\right)$$

$$= \frac{1 \times 18}{12.56 \times 1200} \ln\left(\frac{100}{40}\right)$$

$$= \frac{18 \times 18}{12.56 \times 1200} \times 0.916$$

$$= 0.001 \text{ cm/s}$$

Q.3 calculate the co-efficient of permeability of a soil sample, 6 cm in height & 50 cm^2 in c/s, if a quantity of water equal to 430 ml passed down in 10 minutes, under a constant head of 40 cm.

On oven-drying, the test specimen has mass of 448 g. Taking $G = 2.65$, calculate seepage velocity of water during the test.

Ans Given:

$$V = 430 \text{ ml} \rightarrow 430 \text{ cm}^3$$

$$t = 10 \text{ min} \Rightarrow 600 \text{ second}$$

$$h = 40 \text{ cm}$$

$$A = 50 \text{ cm}^2$$

$$l = 6 \text{ cm}$$

$$K = \frac{Vl}{Aht} = \frac{430 \times 6}{50 \times 40 \times 600} = 0.0021 \text{ cm/s}$$

$$m = 498 \text{ g}$$

$$G = 2.65$$

$$v_s = ?$$

$$\boxed{v = \frac{v_s}{n}}$$

$$\Rightarrow \gamma_d = \frac{498}{300} = 1.66 \text{ g/cm}^3 \quad (\gamma_d = \frac{M}{V} = \frac{m}{V} = A \times l)$$

$$\Rightarrow \gamma_d = \frac{G \gamma_w}{1+e}$$

$$\Rightarrow 1.66 = \frac{2.65 \times 1}{1+e} \Rightarrow 1+e = \frac{2.65 \times 1}{1.66}$$
$$e = 0.59$$

$$n = \frac{e}{1+e} = \frac{0.59}{1.59}$$

$$\Rightarrow n = 0.37$$

$$\Rightarrow v_s = \frac{1}{n} \times v = \frac{v}{n}$$

$$\Rightarrow \frac{430}{0.37} = 1.16 \text{ cm/s}$$

Ans

Effective Stress & Seepage Analysis

At any plane in a soil mass, the total stress or unit pressure (σ) is the total load per unit area. This pressure may be due to (i) self-weight of soil

(ii) overburden pressure on soil.

→ Total pressure consist of two distinct component :-

(i) Effective pressure or intergranular pressure.

(ii) Pore water pressure or neutral pressure.

→ Effective pressure (σ'): is the pressure transmitted from particle through their point of contact through the soil mass above the plane.

→ Effective pressure is used in decreasing the void ratio of soil mass & in developing shear strength.

→ Pore water pressure (u): is the pressure transmitted through the pore fluid.

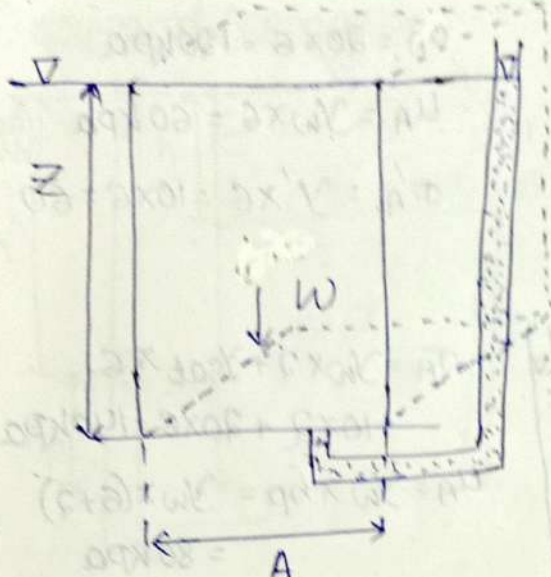
→ It is also called hydrostatic pressure. exerted by water in pores.

$u = \gamma_w h_p \rightarrow$ neutral stress [$h_p =$ piezometric head]

→ This stress does not have any influence on shear strength vertical.

→ The total vertical pressure at any plane is equal to the sum of effective pressure & pore pressure.

$$\sigma = \sigma' + u$$



σ = total stress

$$\sigma = \frac{\text{Weight}}{\text{Area}}$$

Weight = self wt. of soil
+ weight of water (if present)
+ weight of external load (if present)

$$\sigma = \frac{W}{A} = \frac{\gamma_{\text{sat}} (V)}{A} = \frac{\gamma_{\text{sat}} \cdot (A \cdot z)}{A} = \gamma_{\text{sat}} z$$

$$\sigma = \sigma' + u$$

$$\sigma' = \sigma - u$$

→ Effective stress can not be calculated directly.

Conclusion

if W.T present:-

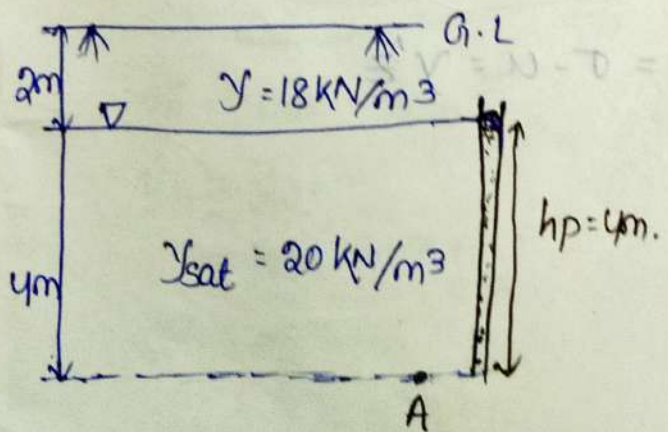
Below W.T, use $\gamma = \gamma_{\text{sat}}$ for σ

use $\gamma = \gamma'$ for σ'

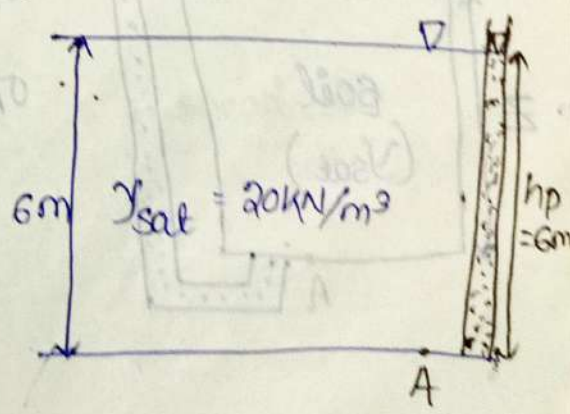
if no W.T. $u = 0$ [$\sigma' = \sigma$]

use $\gamma = \gamma$ for both σ' & σ

(Case-a)



(Case-b)

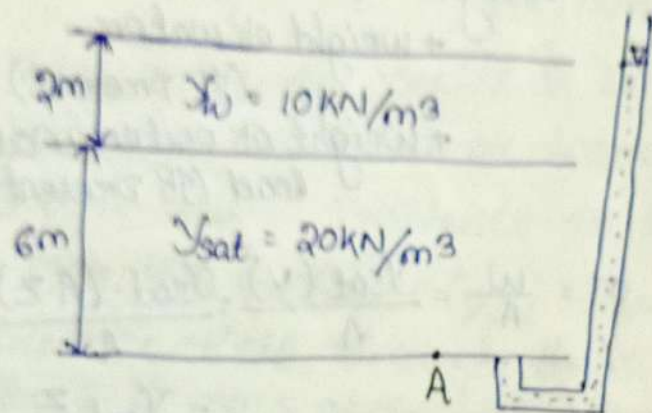


$$\sigma_A = \gamma \times 2 + \gamma_{sat} \times 4 = 18 \times 2 + 20 \times 4 = 116$$

$$u_A = \gamma_w \times h_p = 10 \times 4 = 40 \text{ kPa}$$

$$\sigma'_A = (18 \times 2 + 20 \times 4) - 40 = 76$$

(case - c)



$$\sigma_A = 20 \times 6 = 120 \text{ kPa}$$

$$u_A = \gamma_w \times 6 = 60 \text{ kPa}$$

$$\sigma'_A = \gamma' \times 6 = 10 \times 6 = 60$$

$$\sigma_A = \gamma_w \times 2 + \gamma_{sat} \times 6$$

$$= 10 \times 2 + 20 \times 6 = 140 \text{ kPa}$$

$$u_A = \gamma_w \times h_p = \gamma_w \times (6 + 2)$$

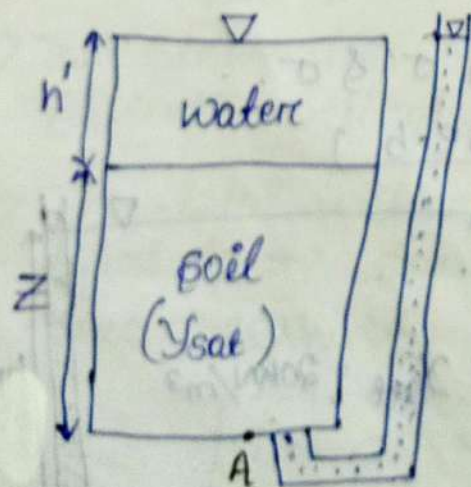
$$= 80 \text{ kPa}$$

$$\sigma'_A = 140 - 80 = 60 \text{ kPa}$$

conclusion

- 1) When W.T. raises to G.L (Ground level), σ , u ($\uparrow$) but σ' ($\downarrow$)
- 2) When W.T. rises beyond G.L, σ , u ($\uparrow$) but no change in σ' .
- 3) Due to W.T. above G.L, σ' below G.L at any depth does not change.

(a) Hydrostatic condition Seepage pressure (NO flow)

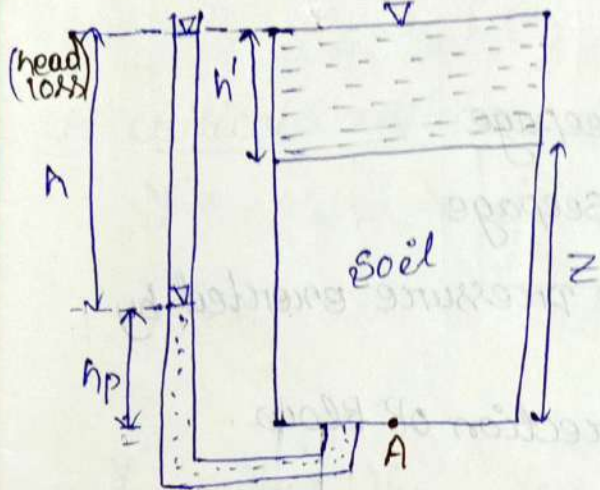


$$\sigma_A = \gamma_w h' + \gamma_{sat} Z$$

$$u_A = \gamma_w h' + \gamma_w Z$$

$$\sigma'_A = \sigma - u = \gamma' Z$$

(b) downward seepage :-



$$\sigma_A = \gamma_w h' + \gamma_{sat} Z$$

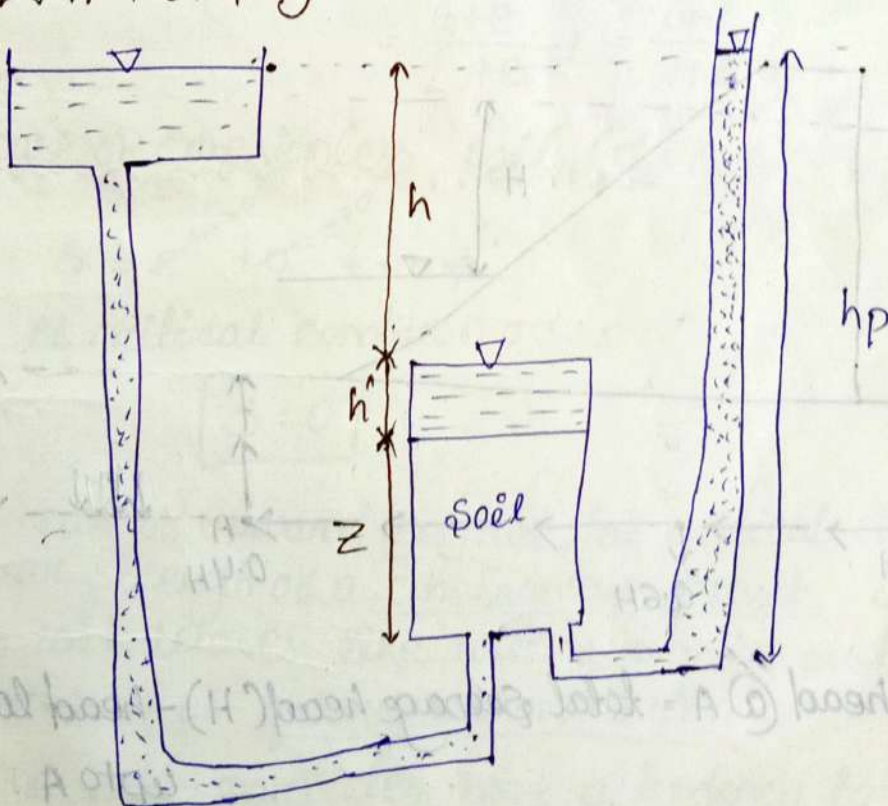
$$u_A = \gamma_w h_p$$

$$u_A = \gamma_w (h' + Z - h) \\ = \gamma_w h' + \gamma_w Z - \gamma_w h$$

$$\sigma_A' = \sigma_A - u_A$$

$$= [\gamma_w h' + \gamma_{sat} Z] - [\gamma_w h' + \gamma_w Z - \gamma_w h] \\ = \gamma' Z + \gamma_w h$$

(c) upward seepage :-



$$\sigma_A = \gamma_w h' + \gamma_{sat} Z$$

$$u_A = \gamma_w h_p$$

$$= \gamma_w (h' + Z + h)$$

$$\sigma_A' = \sigma_A - u_A$$

$$= \gamma' Z - \gamma_w h$$

h = head loss

Summary

$$\sigma' = \gamma' z \Rightarrow \text{No flow}$$

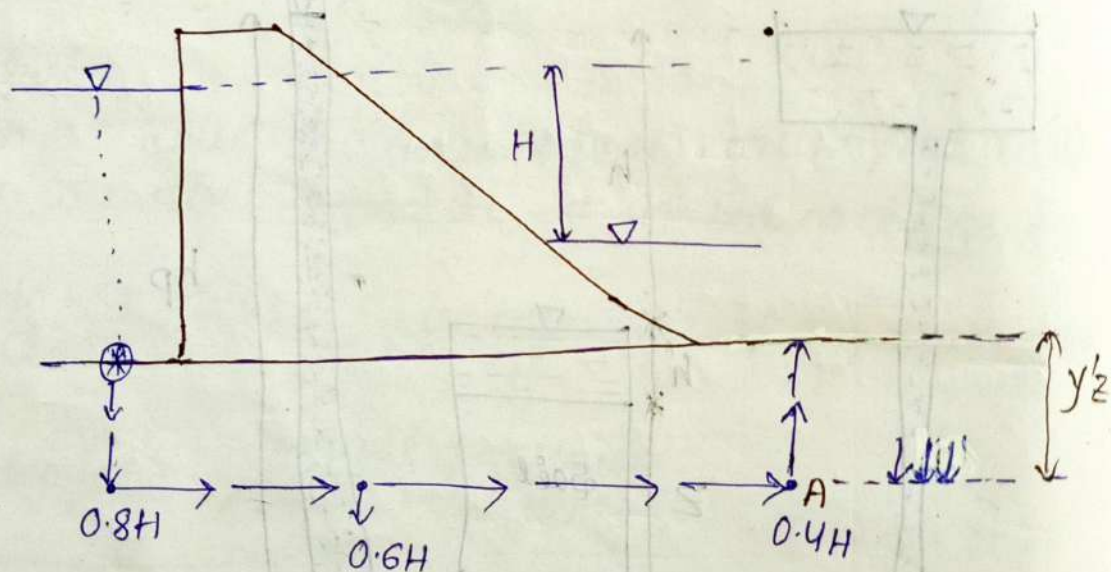
$$\sigma' = \gamma' z + \gamma_w h \Rightarrow \text{Downward seepage}$$

$$\sigma' = \gamma' z + \gamma_w h \Rightarrow \text{upward seepage}$$

Seepage Pressure, It is the pressure exerted by water during seepage.

→ It always acts along the direction of flow.

Practical example



$h = \text{Seepage head @ A} = \text{total seepage head (H)} - \text{head loss upto A.}$

$$F = \frac{\text{Resisting}}{\text{causing}} = \frac{\gamma' z}{\gamma_w h} = \frac{\gamma' / \gamma_w}{h/z} = \frac{i_{\text{check}} + i_{\text{down}}}{i}$$

At critical state $F = 1$

$$\gamma' z = \gamma_w h$$

In upward seepage

$$\sigma' = y'z - \gamma_w h = 0$$

At critical condition, ($\sigma' = 0$)

$$y'z = \gamma_w h \Rightarrow \boxed{\frac{y'}{\gamma_w} = \frac{h}{z}} \quad \boxed{i_c = \frac{G-1}{1+e}}$$

i_c = critical hydraulic gradient

$$F.S. = \frac{i_c}{i}$$

$$\frac{y'}{\gamma_w} = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} = \frac{\gamma_{sat} - 1}{\gamma_w}$$
$$= \frac{G+e}{1+e} - 1 = \frac{G-1}{1+e}$$

In case of cohesionless soil ($c' = 0$)

$$S = c' + \sigma' \tan \phi'$$

At critical condition $\sigma' = 0$

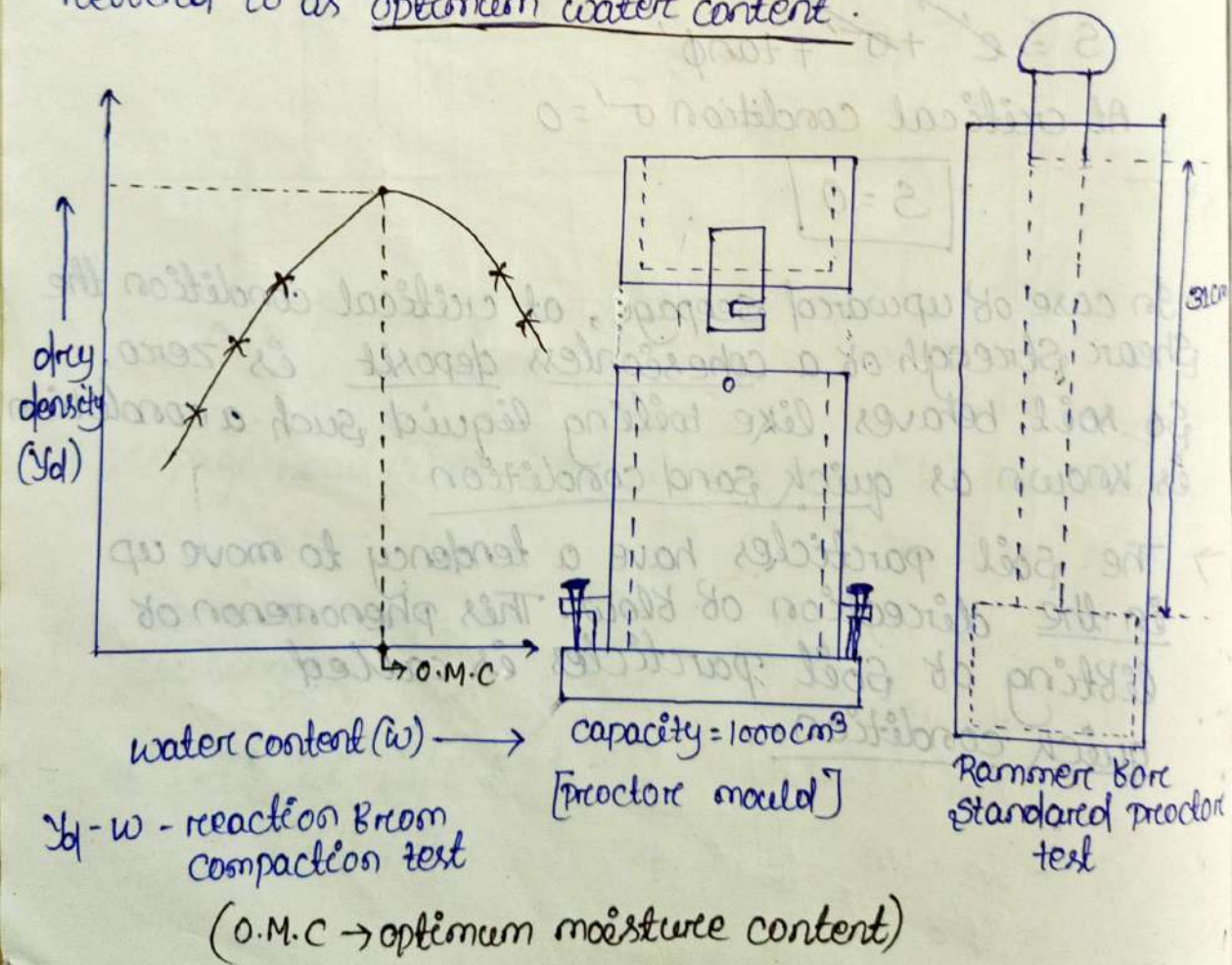
$$\boxed{S = 0}$$

→ In case of upward seepage, at critical condition the shear strength of a cohesionless deposit is zero. So soil behaves like boiling liquid. Such a condition is known as quick sand condition.

→ The soil particles have a tendency to move up in the direction of flow. This phenomenon of lifting of soil particles is called quick condition.

Compaction

- compaction is the process in which rapid reduction of volume takes place due to sudden application of loads as caused by ramming, tamping, rolling and vibration.
- During compaction the reduction in volume is mainly due to expulsion of pore air and rearrangement of particles resulting in closer packing.
- During compaction, the dry density (γ_d or ρ_d) increases.
- The dry density attained depends on water content, amount and type of compaction. [compactive effort]
- For a specific amount of compactive energy applied on soil, the soil mass attains maximum dry density at a particular water content. This water content is referred to as optimum water content.



Standard Proctor test (Laboratory test)

Apparatus required [IS 2720 (Part VII) 1965]

→ Mould with 100mm internal diameter, 127.5mm effective ht. internal volume is 1000ml. Rammer (2.6 kg) & ht. of drop = 310mm

Aim

compacting the soil at different water content & finding corresponding dry densities.

Procedure

- Take about 3kg of dry soil with all lumps pulverised & passing through 4.75mm sieve in a tray.
- Take about (4% of water content for coarse grained soil) & 8% for fine grained soil.
- The computed quantity of water is added to the soil in the tray & mixed thoroughly with hand.
- The mass of mould with base plate is found as (M_1)
- The mould is filled with same quantity of wet soil from the tray & compacted with 25 no. of blows on the surface using standard rammer.
The compacted soil should be abt. $(1/3)^{rd}$ of ht. of mould.
- The collar is fitted on the mould & the soil for the second layer is put inside the mould & compacted similarly, the third layer is compacted.
- Collar is removed & excess soil projected is trimmed off.
- The mass of mould + base plate + compacted soil (M_2) is found.
- The soil is removed & put back in the tray.
Some sample is taken to determine its water content.
- Mass of compacted soil i.e., $(M_2 - M_1)$ is computed & bulk density (γ) is calculated.
- After knowing water content (w), dry density is computed.

- The soil in the tray is again pulverised & the water content (w), is increased for second trial.
- These steps are repeated to get at least 4-5 sets of water content & dry density values.
- The dry density (γ_d) is plotted against water content (w) to obtain the compaction curve.

Percentage air void line

$$\gamma_d = \frac{(1 - o_a) G \gamma_w}{1 + w G}$$

For a particular value of o_a , γ_d can be evaluated for different values of w & plotted to obtain the curve referred to as percentage air void line.

eg- 30% air void line.

Given. $G = 2.7$, $\gamma_w = 9.81 \text{ kN/m}^3$, $o_a = 30\%$

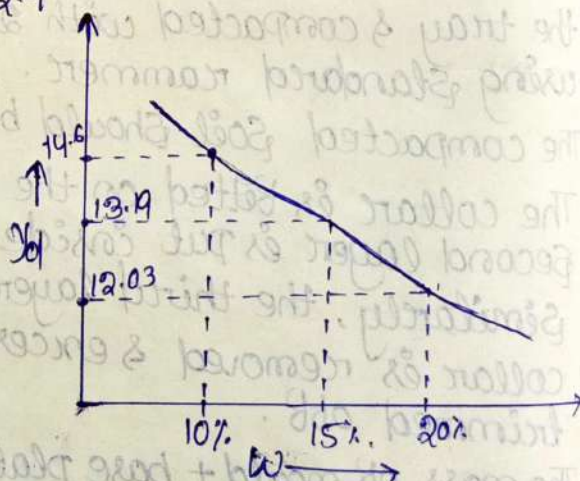
$$\gamma_d = \frac{(1 - 0.3) \times 2.7 \times 9.81}{1 + w(2.7)} = \frac{18.54}{1 + 2.7w}$$

γ_d varies w.r.t. w

$w = 10\% \rightarrow \gamma_d = 14.598 \text{ kN/m}^3$

$w = 15\% \rightarrow \gamma_d = 13.19 \text{ kN/m}^3$

$w = 20\% \rightarrow \gamma_d = 12.03 \text{ kN/m}^3$



(30% air void line)

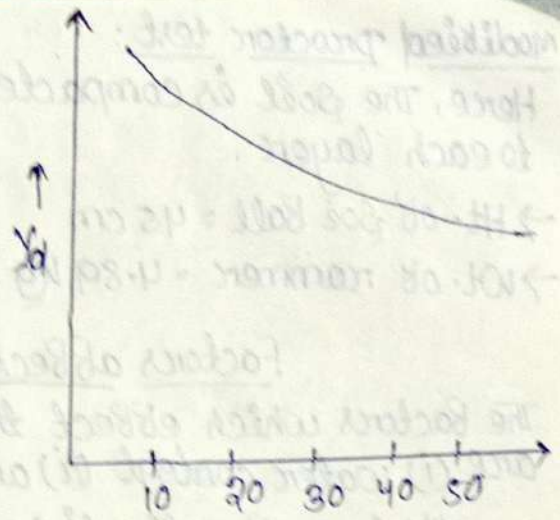
eg- 0 air void line

or 100% saturation line

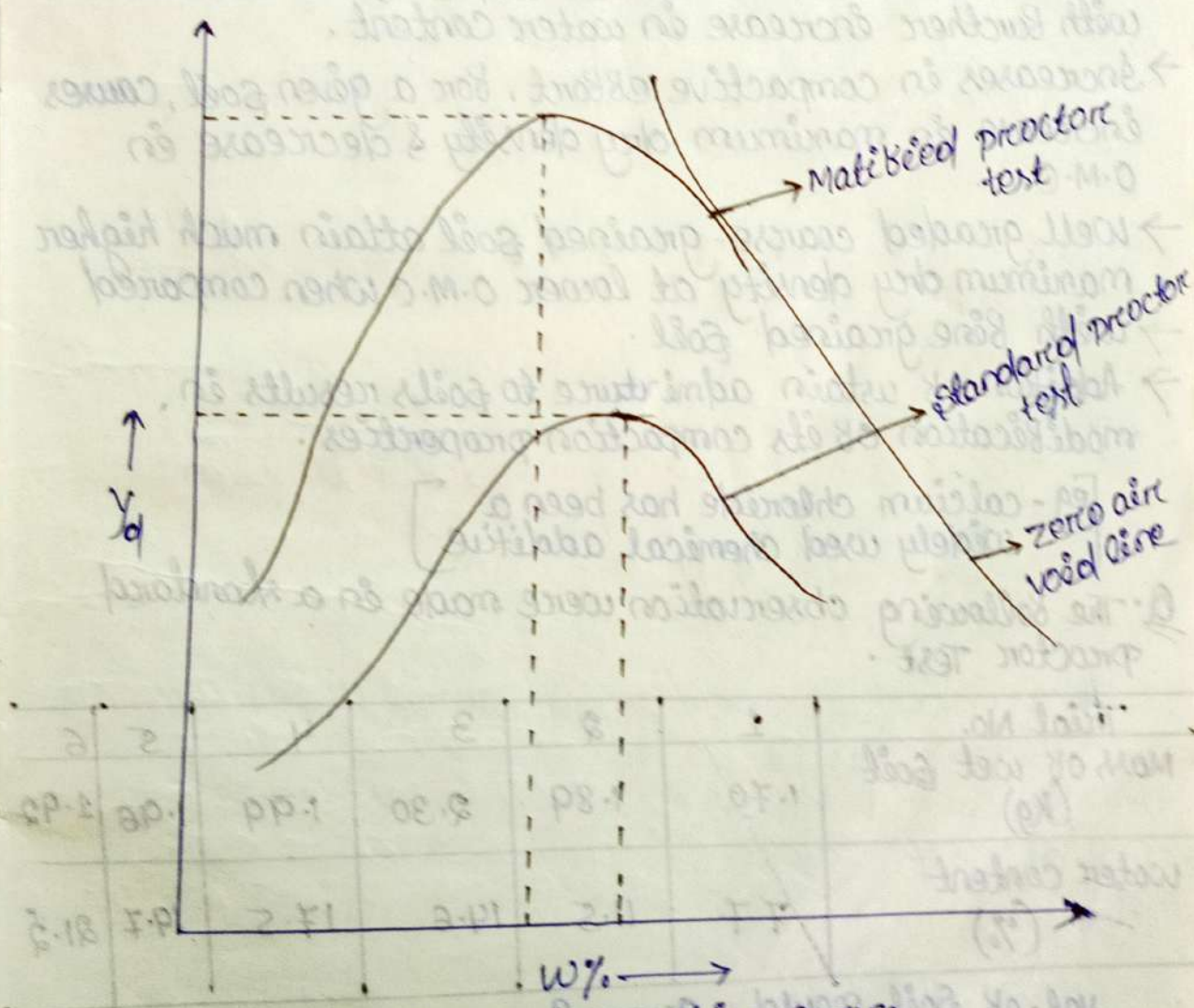
$G = 2.7$, $\gamma_w = 9.81 \text{ kN/m}^3$, $o_a = 0$

$$\gamma_d = \frac{(1 - 0) \times 2.7 \times 9.81}{1 + w(2.7)} = \frac{26.487}{1 + 2.7w}$$

- $w=10\% \rightarrow \gamma_d = 20.86 \text{ kN/m}^3$
 $w=20\% \rightarrow \gamma_d = 17.2 \text{ kN/m}^3$
 $w=40\% \rightarrow \gamma_d = 12.73 \text{ kN/m}^3$
 $w=60\% \rightarrow \gamma_d = 10.11 \text{ kN/m}^3$



→ For any water content w , the theoretical maximum dry density is obtained when $e_a = 0$.



comparison of compaction curves

Modified Proctor test:

Here, The soil is compacted in 5 layers giving 25 blows to each layer.

→ Ht. of Ball = 45 cm

→ Wt. of hammer = 4.89 kg.

Factors affecting compaction

The factors which effect the dry density of compacted soil are (i) water content (ii) amount & type of compaction

(iii) type of soil (iv) addition of admixture.

- From the lab test, the dry density increases with water content, attains a maximum at O.M.C. & then decreases with further increase in water content.
- Increases in compactive effort, for a given soil, causes increase in maximum dry density & decrease in O.M.C.
- Well graded coarse-grained soil attain much higher maximum dry density at lower O.M.C when compared with fine grained soil.
- Addition of certain admixture to soils results in modification of its compaction properties.

[eg - calcium chloride has been a widely used chemical additive]

Q. The following observation were made in a standard Proctor Test.

Trial No.	1	2	3	4	5	6
Mass of wet soil (kg)	1.70	1.89	2.30	1.99	1.96	1.94
Water content (%)	7.7	11.5	14.6	17.5	19.7	21.2

Vol. of Soil mould = 945 cm^3

$G = 2.7$

Calculate O.M.C & M.D.D & draw zero air void line.

CONSOLIDATION

compressibility

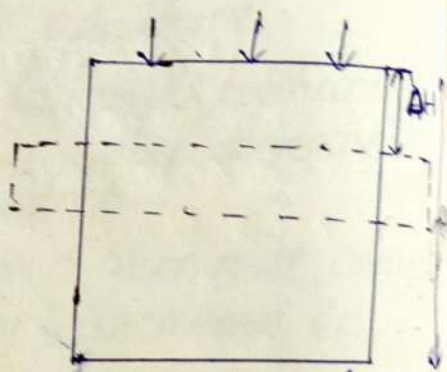
It is the ability of the soil to undergo compression or decrease in volume when subjected to compressive load.

Two process are present in compressibility

- 1) compaction
 - 2) consolidation
- } Both involve decrease in volume.

Assumption in consolidation

- (i) soil is semi-infinite
- (ii) lateral deformation is neglected.



Settlement 's' is relative movement of structure due to compressibility of soil. $\Delta H = S = \text{Settlement}$

Consolidation is the process in which gradual reduction in volume takes place due to sustained loading.

→ In the analysis reason: For reduction in volume:

1. Expulsion of air.
2. Expulsion of pore water.
3. Plastic readjustment of soil particles.

In the analysis both water & soil particles are assumed to be incompressible so the decrease in volume is entirely due to change in relative position of soil particles with particles coming closer to each other.

Compressibility

Compaction

- Reductⁿ in vol. due to expulsion of air.
- It is a short term & dynamic process.
- $S_{rc} < 1$ (85-95%)

Consolidation

(long term & static) process

$$S_{rc} = 100\%$$

Initial compression

- complete expulsion of air.
- Elastic deformation.

Primary consolidation

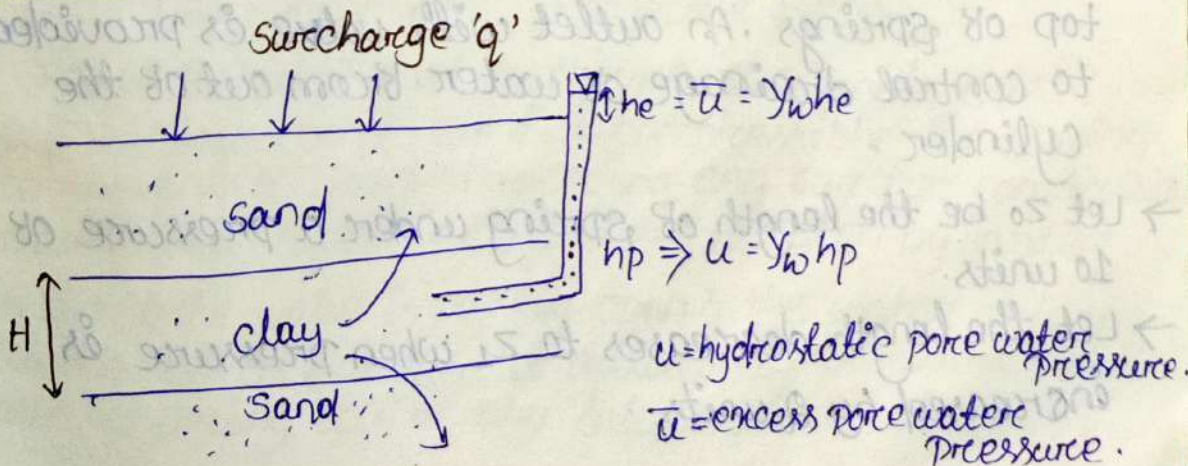
- due to expulsion of pore water

Secondary consolidation

- Plastic readjustment of particles & expulsion of structural water.

- When an external load is applied over a saturated soil sample the entire load is taken by water & pore water pressure gets increased as excess pore pressure.
- During consolidation, as pore water dissipates out, excess pore pressure, reduces & effective stress gets increased.
- At ^{the} end of consolidation dissipation of pore water gets stopped as excess pore pressure becomes zero.

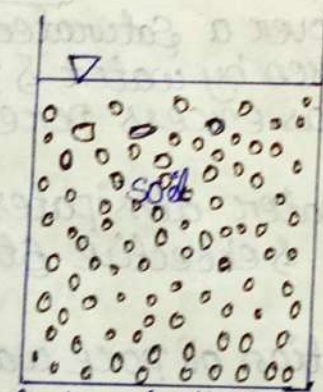
consolidation:- occurs in case of clay.



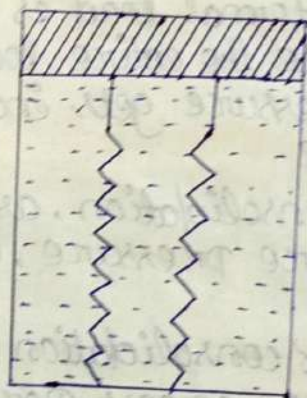
Initial ($t=0$)	During consolidation ($t=t$)	End of consolidation ($t=\infty$)
$\Delta\sigma = q$	$\Delta\sigma = q$	$\Delta\sigma = q$
$\Delta u = \bar{u} = q$	$\bar{u} < q$	$\bar{u} = 0$
$\Delta\sigma' = \Delta\sigma - \Delta u$ $q - q = 0$	$\Delta\sigma' > 0$	$\Delta\sigma' = q$

The Spring Analogy (Piston & Spring Analogy)

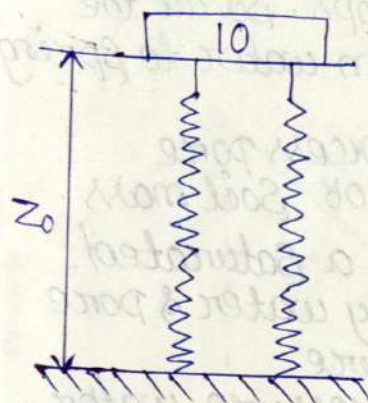
- A saturated soil mass taken in a container.
- In this analogy, soil particles is assumed as spring (replaced by springs) and the water filling voids is the filling the cylinder.



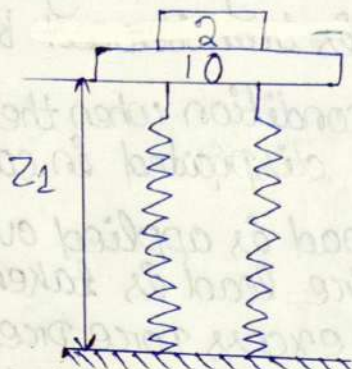
(Saturated soil)



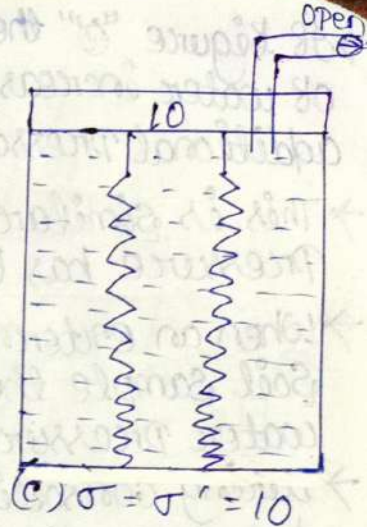
- The compressive load is applied on piston placed on top of springs. An outlet with valve is provided to control drainage of water from out of the cylinder.
- Let z_0 be the length of spring under a pressure of 10 units.
- Let the length decreases to z_1 when pressure is increased by 2 units.



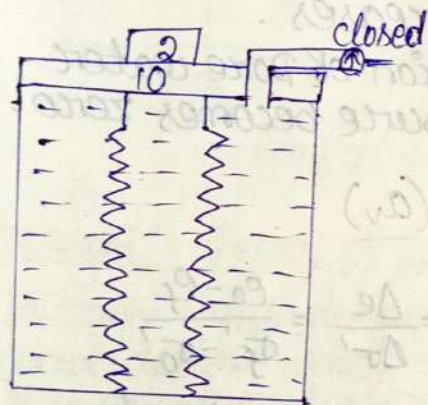
(a) $\sigma = \sigma' = 10$



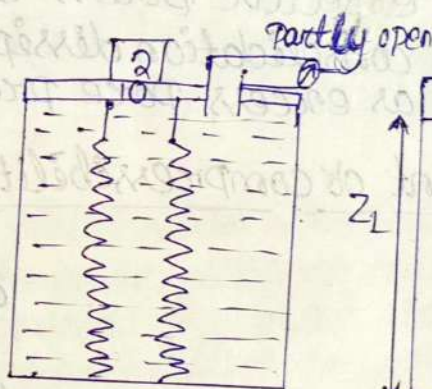
(b) $\sigma = \sigma' = 12$



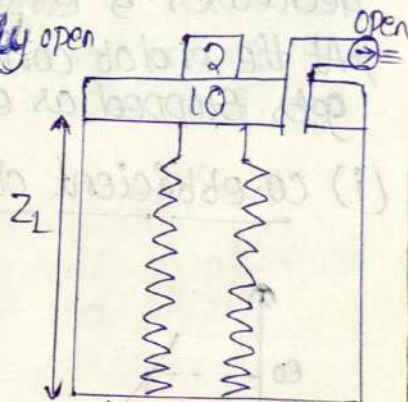
(c) $\sigma = \sigma' = 10$



(d) $\sigma = \sigma' + U$
 $12 = 10 + 2$



(e) $\sigma = \sigma' + U$
 $12 = (10 + \Delta\sigma') + (2 - \Delta\sigma')$



(f) $\sigma = \sigma' + U$
 $12 = 12 + 0$

→ In Figure "c" the valve is open but no drainage takes place as entire 10 unit load is carried by springs & the pressure in water is zero.

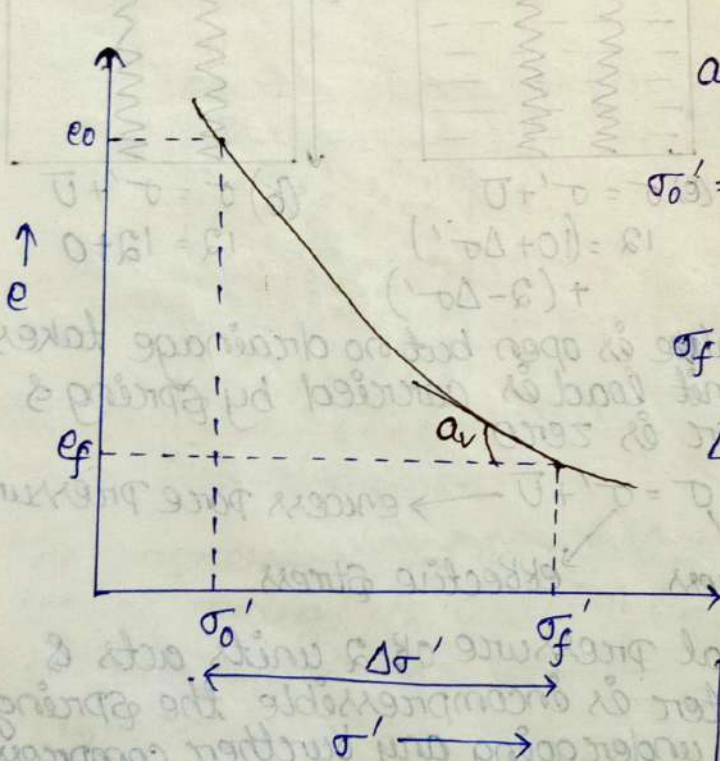
For Soil mass $\sigma = \sigma' + U$ → excess pore pressure
 total stress effective stress

→ In Figure (d) additional pressure of 2 units acts & valve is closed as water is incompressible the springs are prevented from undergoing any further compression so additional pressure will have to be taken by water.

→ In Fig. (e) the valve is partly open & the water starts blowing out, so transfer of additional pressure from water to spring acts at any intermediate stage.

- At figure "b" the valve is fully open the rate of drainage of water increases & finally drainage stops. So all the additional pressure is transferred from water to soil.
- This is similar to condition when the excess pore pressure has fully dissipated in case of soil mass.
- When an external load is applied over a saturated soil sample the entire load is taken by water & pore water pressure as excess pore pressure.
- During consolidation $\Rightarrow$ excess pore pressure water decreases dissipates out excess pore pressure decreases & effective stress increases.
- At the end of consolidation dissipation of pore water gets stopped as excess pore pressure becomes zero.

(i) co-efficient of compressibility (a_v)



$$a_v = \frac{\Delta e}{\Delta \sigma'} = \frac{e_0 - e_f}{\sigma_f' - \sigma_0'}$$

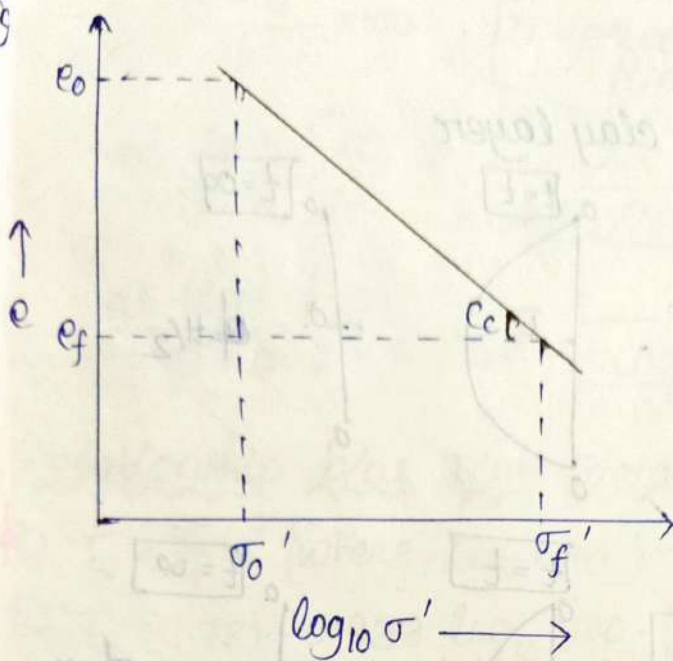
σ_0' = Initial overburden pressure
(σ' due to self wt.)

σ_f' = final effective stress

$\Delta \sigma'$ = Increase in vertical stress due to external load.

$$\sigma_f' = \sigma_0' + \Delta \sigma'$$

ii) compression index (C_c) :



$$C_c = \frac{e_0 - e_f}{\log \left[\frac{\sigma'_f}{\sigma'_0} \right]}$$

$$C_c = \frac{\Delta e}{\log_{10} \left(\frac{\sigma'_f}{\sigma'_0} \right)}$$

→ remoulded soil

$$C_c = 0.007 (w_L - 10)$$

$$C_c = 0.009 (w_L - 10)$$

→ undisturbed clay

Terzaghi's theory of one-dimensional consolidation:-

Terzaghi (1923) derived the basic differential equation of consolidation which represent the first step in the theoretical analysis of consolidation process.

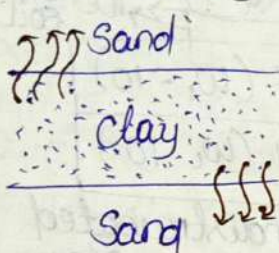
- 1) The soil mass is homogenous & fully saturated.
- 2) The soil particles & water are incompressible.
- 3) Darcy's law for flow of water through soil mass is applicable during consolidation.
- 4) The coefficient of permeability (k) is constant during consolidation.
- 5) Load is applied in one direction only & deformation occurs only in the direction of load application.
- 6) The deformation is entirely due to change in volume.
- 7) The drainage of pore water occurs in 1-direction.
- 8) Boundary drainage face offers no resistance to flow of water.
- 9) During consolidation the change in thickness is continuous but final value of compression is related to initial thickness.

10) Time lag in consolidation is entirely due to permeability of soil.

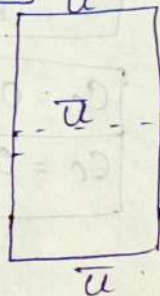
$$\frac{\partial U}{\partial t} = C_v \frac{\partial^2 u}{\partial x^2}$$

$H = Ht$ of compressible clay layer

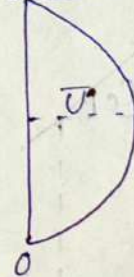
Double drainage



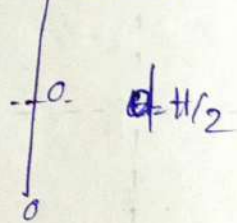
$t=0$



$t=t$



$t=\infty$

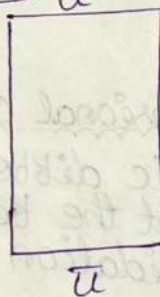


$d=H/2$

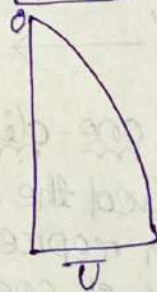
Single drainage



$t=0$



$t=t$



$t=\infty$



$d=H$

Solution of above one dimensional equation:

(1) Time factor (T_v) (2) Degree of consolidation (U)

Time factor (T_v):

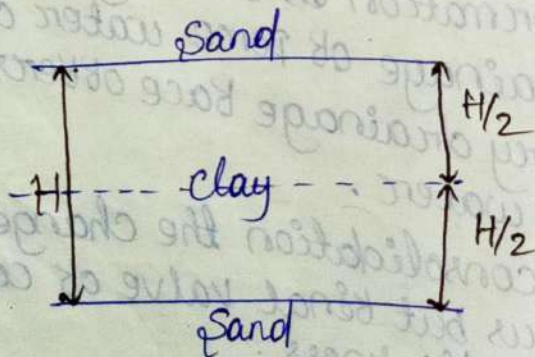
$$T_v = \frac{C_v t}{d^2}$$

T_v is dimensionless parameter.
 C_v = co-efficient of consolidation
 [It is constant for a soil]
 t = time reqd. for consolidation.

d = drainage path.

For single drainage, $d=H$

For double drainage, $d=H/2$



Degree of consolidation:

$$U = \frac{U_i - U}{U_i} \times 100$$

U_i - Initial excess pore pressure
 U - excess pore pressure at any time (t)

at time $t = 0$

$$U = U_i$$

so $U = 0$

at time $t = \infty$

$$U = 0$$

so, $U = 100\%$

another formula of

$$U = \frac{S_t}{S_f} \times 100$$

degree of consolidation

S_t - settlement at time t

S_f - Final settlement

Relationship b/w Time Factor (T_v) & degree of consolidation

(i) $T_v = \frac{\pi}{4} U^2$ [where, $U \leq 60\%$] $\rightarrow$ (decimal)

(ii) $T_v = 1.781 - 0.933 \log_{10} (100 - U)$ [where, $U > 60\%$]
(percentage)

eg $T_v = ?$ [$U = 30\% \text{ \& } 70\%$]

(a) $U = 30\%$

$$T_v = \frac{\pi}{4} \times 0.3^2 = 0.07$$

(b) $U = 70\%$

$$T_v = 1.781 - 0.933 \log_{10} (100 - 70) = 0.4$$

I.V.T.

Depending on the state of consolidation soil deposit is divided into 3 types:

- (i) pre consolidated soil / over consolidated / precompressed soil
- (ii) Normally consolidated soil
- (iii) under consolidated soil

1) Pre-consolidated soil:

A soil is said to be preconsolidated (precompressed or over consolidated) if it has been subjected to maximum stress in its history than present.

(or)

It has in the past been fully consolidated under a pressure greater than present overburden pressure acting on soil.

max. stress in past > present stress

Normally consolidated soil:

A soil is said to be normally consolidated soil if it has never been subjected to a pressure greater than the present overburden pressure & has been fully consolidated under the present acting stress.

Under-consolidated soil: maximum stress = present stress

In this type of soil, it is still not fully consolidated under the present overburden pressure.

maximum stress < present stress.

Over consolidated Ratio:

$$O.C.R. = \frac{\text{Max. Stress}}{\text{Present Stress}}$$

OCR

P.C.S $\Rightarrow > 1$ (greater than 1)

N.C.S $\Rightarrow = 1$ (equal to 1)

U.C.S $\Rightarrow < 1$ (less than 1)

Q. A soil sample 20mm thick takes 20 minutes to reach 20% consolidation. Find the time taken for a clay layer 6m thick to reach 40% consolidation. Assume double drainage in both cases.

Ans

20mm

20%

25 min

6m

40%

?

} double drainage

$$T_v = \frac{\pi}{4} \times U^2$$

$$= \frac{\pi}{4} \times (0.2)^2 = 0.031$$

$$T_v = \frac{C_v t}{d^2}$$

$$\Rightarrow 0.031 = \frac{C_v (25 \times 60)}{(20)^2}$$

$$\Rightarrow C_v = 8.267 \times 10^{-3} \text{ mm}^2/\text{s}$$

$$(i) T_v = \frac{\pi}{4} \times 0.4^2 = 0.125$$

$$T_v = \frac{C_v t}{d^2}$$

$$\Rightarrow 0.125 = \frac{8.267 \times 10^{-3} \times t}{(6000)^2}$$

$$\Rightarrow t = 544332889.8 \text{ sec}$$

20mm	→ 20%	→ 20min	} double drainage
6m	→ 40%	→ ?	

For 20mm sample

$$d = \frac{20}{2} = 10 \text{ mm}$$

$$T_v = \frac{\pi}{4} \times U^2$$

$$= \frac{\pi}{4} \times (0.2)^2 = 0.031$$

$$T_v = \frac{C_v t}{d^2}$$

$$\Rightarrow 0.031 = \frac{C_v \times (20 \times 60)}{(10)^2}$$

$$\Rightarrow C_v = 2.6 \times 10^{-3} \text{ mm}^2/\text{s}$$

For 6m sample

$$6\text{m} = 6000 \text{ mm}$$

$$d = \frac{6000}{2} = 3000 \text{ mm}$$

$$T_v = \frac{\pi}{4} \times U^2$$

$$= \frac{\pi}{4} \times (0.4)^2 = 0.125$$

$$T_v = \frac{C_v t}{d^2}$$

$$\Rightarrow 0.125 = \frac{2.6 \times 10^{-3} \times t}{(3000)^2}$$

$$\Rightarrow t = 432692307.7 \text{ sec}$$

Shear Strength of Soils

Shear Strength:

- When a soil is loaded shearing stress are developed in it.
- When shearing stress reach a limiting value shear deformation takes place causing failure of soil mass.
- It may be in the form of sinking or boiling or movement of soil behind a retaining wall.
- The shear strength of soil is the resistance towards deformation or it is the ability of soil mass to with stand shear stress.
- The failure condition for a soil can be expressed in terms of limiting shear stress or shear strength.
- The shearing resistance of soil consists of the following:

- 1/ The structural resistance
- 2/ The frictional resistance
- 3/ cohesion

1. Structural resistance:

It is the resistance against the displacement of soil because of interlocking of particles.

2. Frictional resistance:

It is the resistance against translocation between the individual soil particles at their point of contact.

3. cohesion:

- It is the property by which particles are attracted towards each other.
- It is a surface phenomenon.

The shear strength in cohesionless soil results from intergranular friction alone while in all other soil it results from both internal friction & cohesion. In case of pure clay the strength is due to ~~an~~ cohesion only.

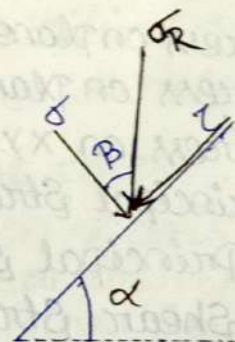
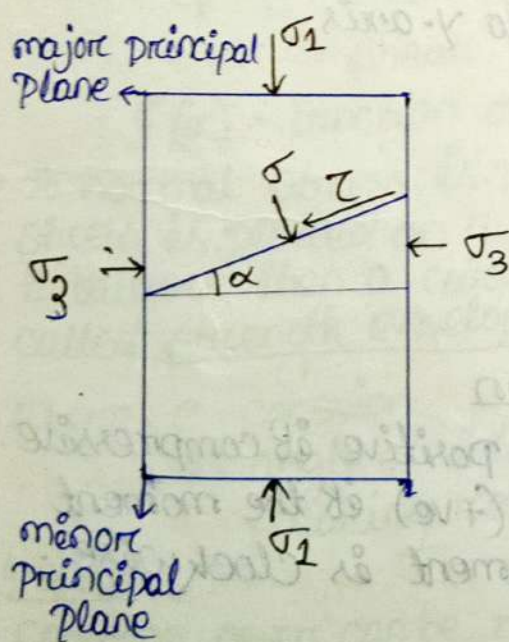
Mohr stress circle

- Through a point in a loaded soil mass infinite plane passes through it & the stress component on each plane depends on the direction of plane.
- There exist three typical planes mutually perpendicular to each other, on which only normal stress acts & no shear stress is present, these planes are called Principal planes & the normal stress acting on these planes are called Principal Stress.
- In decreasing order of normal stress these planes are major, intermediate & minor principal planes & corresponding normal stresses on them are called

σ_1 = major principal stress

σ_2 = Intermediate principal stress

σ_3 = minor principal stress



α = angle made by any plane w.r.t. major principal plane.

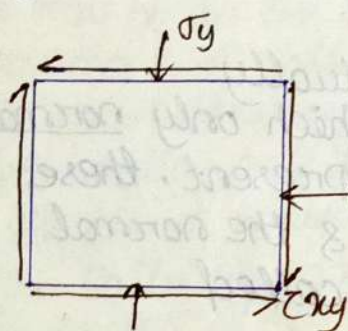
$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha$$

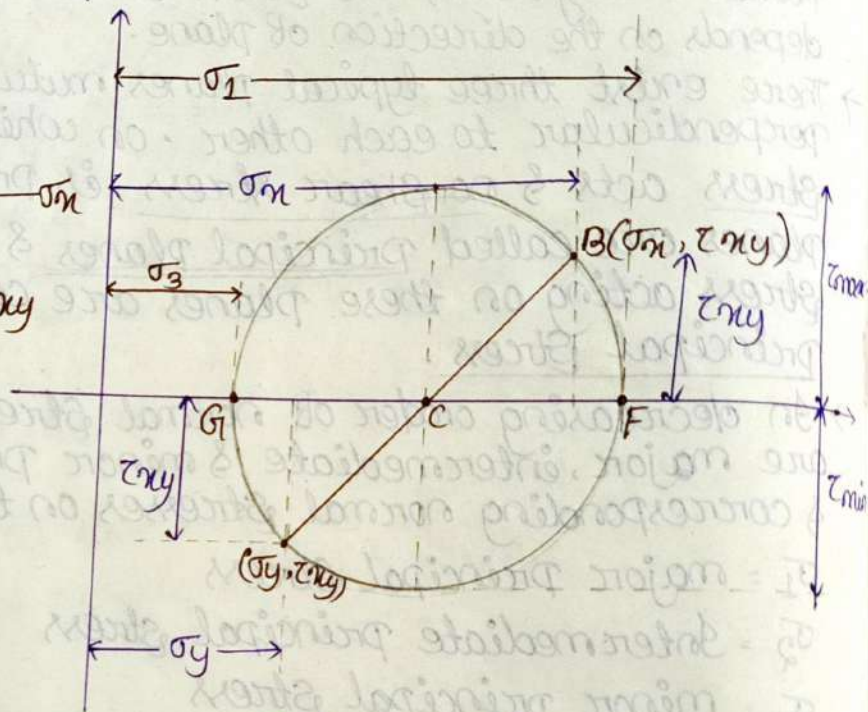
$$\tau_R = \sqrt{\sigma^2 + \tau^2}$$

θ = angle of obliquity

→ A two dimensional stress system is one in which all stresses act in a plane.



(a)



1 σ_n = normal stress on plane $\perp^r$ to x -axis.

σ_y = normal stress on plane $\perp^r$ to y -axis.

τ_{xy} = shear stress on xy plane.

2 σ_1 = major principal stress

σ_3 = minor principal stress

τ_{max} = Max. shear stress

τ_{min} = Min. shear stress

Sign. Convention

→ Normal stress is considered positive if compressive.

→ A shear stress is considered (+ve) if the moment about any point inside the element is clockwise.

Theory of Failure of Soil / Mohr - Coulomb Failure theory

Mohr (1900) developed useful theory in case of soils :-

- (a) Materials fails essentially by shear. The critical shear stress causing failure depends on properties of material as well as normal stress on failure plane.
- (b) The ultimate strength of material is determined by stresses on potential failure plane or plane of shear. When the material is subjected to 3D principal stresses i.e. σ_1, σ_2 & σ_3 . The intermediate principal stress does not have any influence or effect on strength of material or failure critical is independent of intermediate principal stress.

Mohr - Coulomb Theory

- This theory was first developed by Coulomb (1776) & later simplified by Mohr in (1900).
- This theory can be expressed by the equation

$$\boxed{\tau_f = C + \sigma \tan \phi}$$

$$y = C + m x$$

$$[y = f(x)]$$

$$[\tau_f = f(\sigma)]$$

Where, $\tau_f = S$ = shear stress on failure plane.
= shear resistance of material.

$f(\sigma)$ = Function of normal stress.

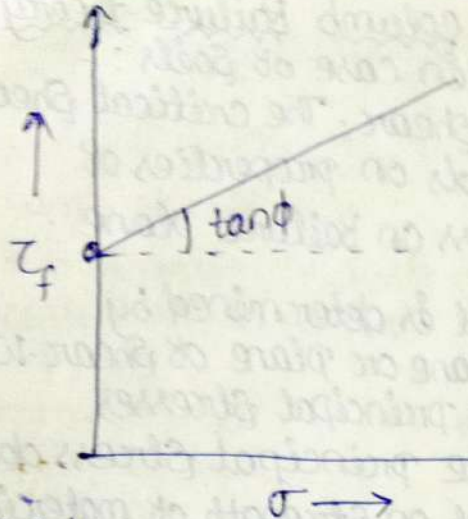
- If normal stress is plotted on x-axis & shear stress is plotted on y-axis (ordinate). Corresponding to failure then a curve is obtained this curve is called strength envelope given by $\tau_f = C + \sigma \tan \phi$

Where, C = cohesion

ϕ = angle of internal friction or angle of shearing resistance.

Imp Note

Cohesion never can be zero.



- c & ϕ are shear strength parameters & variable for any soil depending on condition of testing such as drainage condition & rate of strain.
- Mohr generalised or simplified the strength envelope called failure envelope.

$$\tau_f = c + \sigma \tan \phi$$

where, τ_f = shear stress on failure plane
 = shear resistance of material.

$f(\sigma)$ = function of normal stress.

As normal stress is plotted on x-axis & shear stress is plotted on y-axis (ordinate). Corresponding to failure then a curve is obtained this curve is called strength envelope given by $\tau_f = c + \sigma \tan \phi$

where, c = cohesion
 ϕ = angle of internal friction or angle of shearing resistance.

Cohesion never can be zero.

Determination of Shear Strength Parameters:

- Shear Strength Parameters are determined in laboratory by conducting Shear test & using of these tests to obtain failure envelope.
- The tests are conducted on undisturbed soil samples obtained from the field taking care to stimulate field drainage condition.
- Based on methods of application of loads the shear test used in laboratories are as follows.

(i) Direct Shear Test

(ii) Triaxial test

(iii) Vane-Shear test

(iv) Unconfined test

Direct Shear Test:

- Soil specimen used in the test is a square in plan (60 mm x 60 mm) & thickness of 20-25 mm.

- Equipment of this test include

(i) Shear box

(ii) Loading yoke for applying normal force

(iii) geared yoke for applying shear force & facilities for measuring shear force, shear displacement & vertical displacement or volume change.

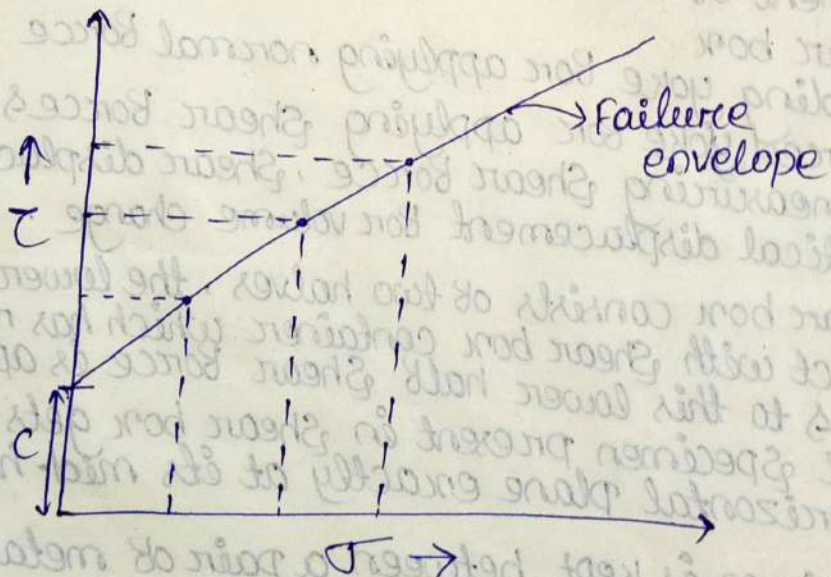
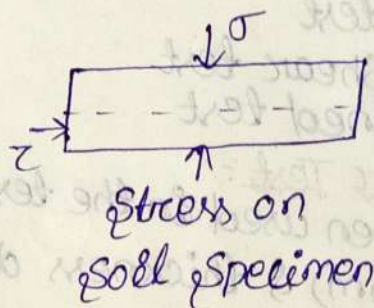
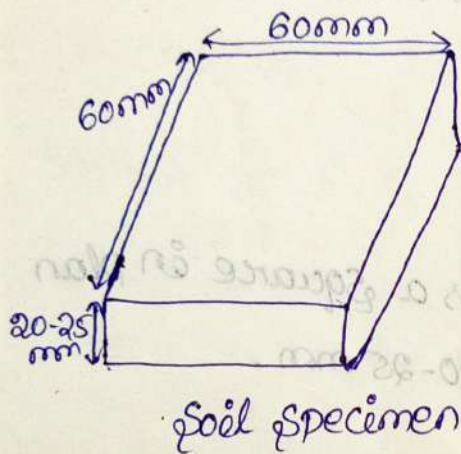
- The shear box consists of two halves, the lower half is in contact with shear box container which has rollers at base & to this lower half shear force is applied.
- The soil specimen present in shear box gets sheared on a horizontal plane exactly at its mid-height.

The specimen is kept between a pair of metal grids & porous plate (or non-porous plate).

Principle of test:

- A normal stress is applied on the specimen & is kept constant throughout the test.
- The shear stress (τ) is caused by applying shear force through geared jack & transmitted to top half of shear box.

- The shear force is gradually increased until the specimen fails (or if the strain is beyond 20%)
- The test is conducted on minimum of three specimen subjected to three different values of σ .
- By plotting τ_f against σ the failure envelope is obtained, c & ϕ are obtained by measurement from the plot.



Adv. of direct shear test

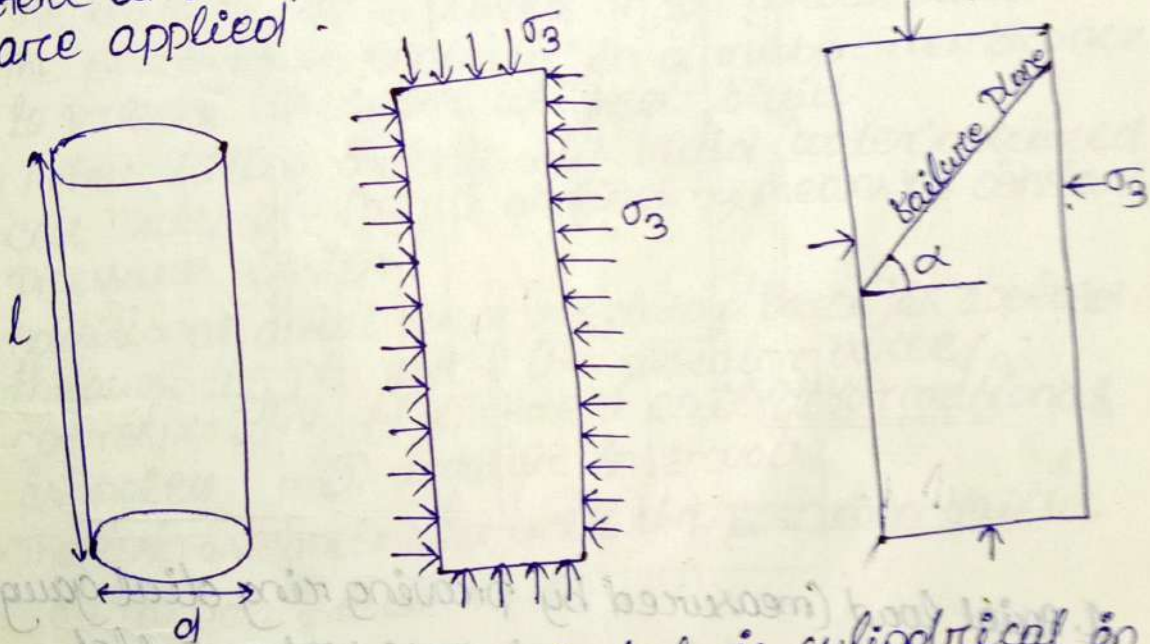
- This test is a simple test
- Thickness of sample is small so quick drainage & rapid dissipation of pore pressure is possible.

- Disadvantages of direct Shear test:
- The shear stress is not uniformly distributed being more @ the edge than at the centre
 - The failure plane is predetermined, so the specimen is not allowed to fail along its weakest plane.
 - Measurement of pore pressure is not possible.
 - Shear displacement causes reduction in area under shear. corrected area should be used in computing normal & shear stresses.

Triaxial compression Test

This test was introduced by Casagrande & Terzaghi

→ Here are 3 principal stresses (σ_1, σ_2 & σ_3) are applied.

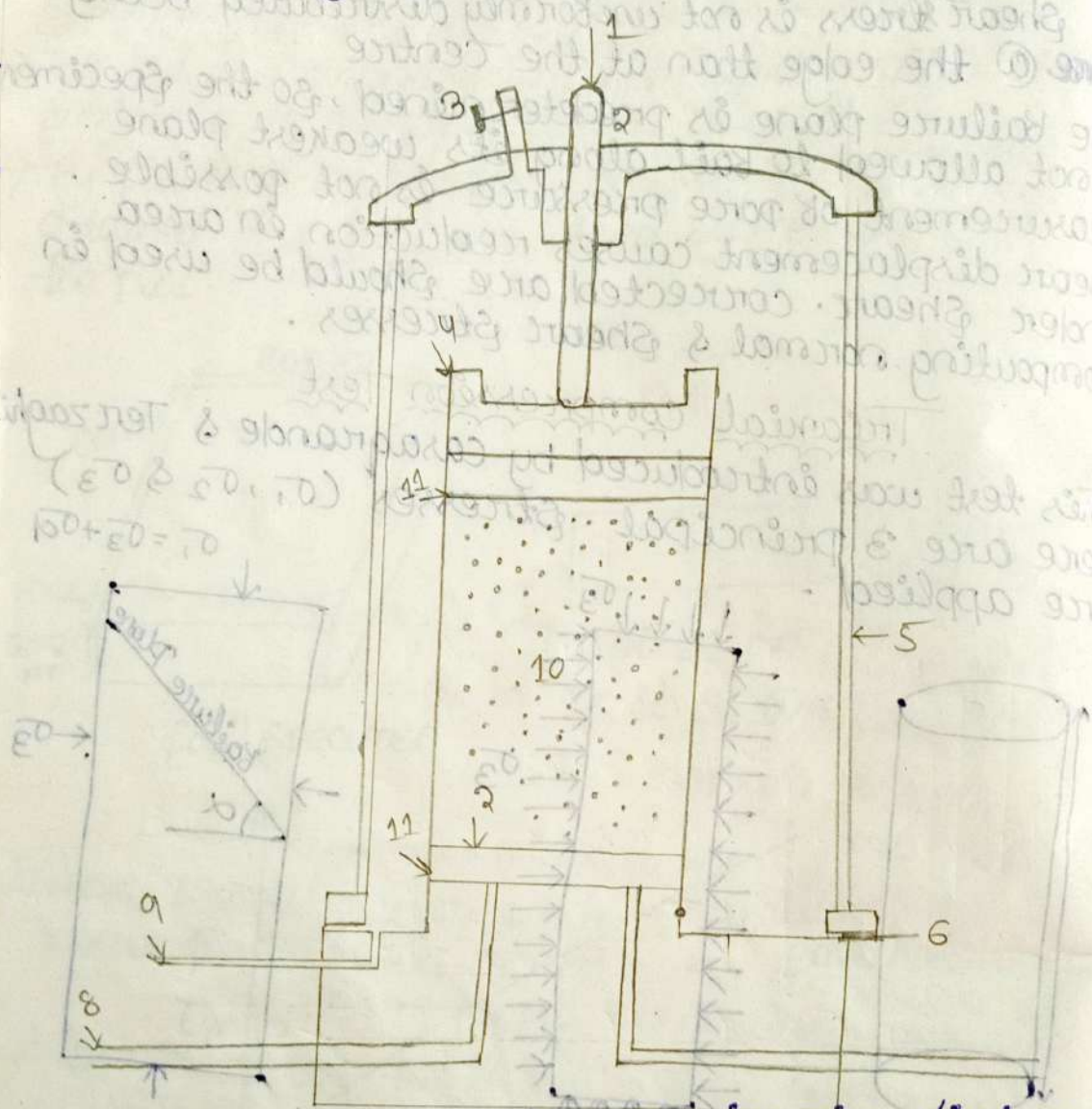


The soil specimen used in test is cylindrical in Shape

Equipment of the test

- Triaxial cell
- Loading frame with accessories for applying gradually increasing axial load on specimen at constant rate of strain
- Provision for measuring axial force & axial displacement.
- Constant pressure system to apply & maintain constant cell pressure.
- Pore pressure measuring apparatus

(vi) volume change gauge



1. Axial load (measured by proving ring dial gauge)
2. Loading ram
3. Air release valve
4. Top cap
5. Perspex cylinder
6. Sealing ring
7. Pore water outlet
8. Additional pore water outlet
9. Cell fluid inlet
10. Soil specimen (enclosed in rubber membrane with 'g' rings at the ends)
11. Porous disc

- The triaxial cell consist of high pressure cylindrical cell, fitted between top & base cap.
- At the base inlet for cell fluid, outlet for draining of pore water from specimen & measurement of pore pressure.
- At the top an air release valve is present to expel air.
- A steel plunger for applying axial load on specimen is provided.
- The soil specimen is kept inside the triaxial cell with porous plate (or non-porous plate) at top & bottom.
- The loading cap is placed on top porous plate.
- The specimen is enclosed in a rubber membrane to prevent its contact with cell fluid.
- After filling the cell with fluid (water) required cell pressure (σ_3) is applied by means of constant pressure system.
- Additional axial force [deviator force] is applied through the plunger & the deviator force corresponding to different axial deformations is noted @ regular intervals.
- The test is continued until the specimen fails. (or @ 20% strain level)

$$\sigma_d = \frac{F}{A_c}$$

$\left\{ \begin{array}{l} F = \text{deviator force} \\ A_c = \text{corrected area of specimen} \end{array} \right.$

- After finding deviator stress (σ_d) at failure we find principal stress @ failure.

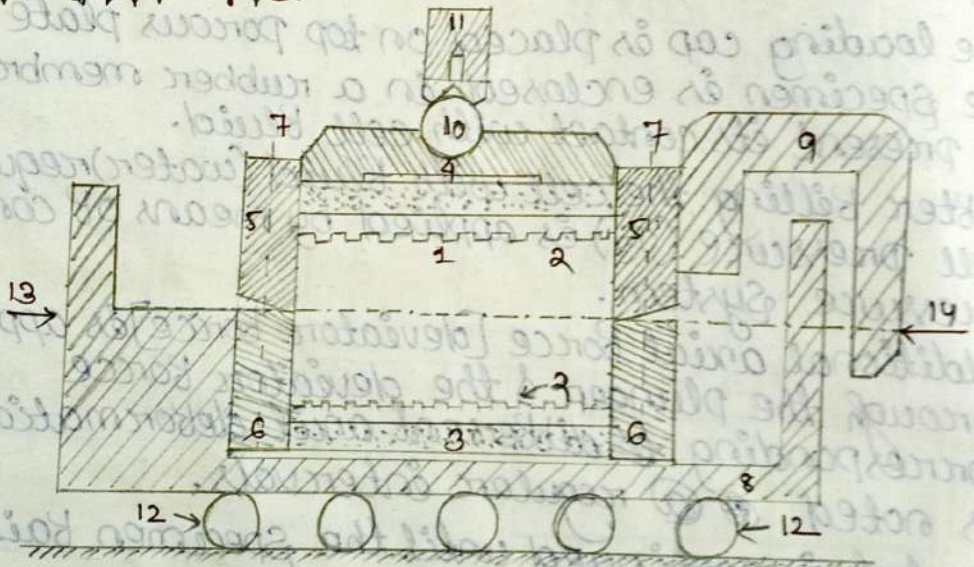
$$\sigma_1 = \sigma_3 + \sigma_d$$

with this set of (σ_1, σ_3) values mohr circle @ failure is drawn.

- The test is conducted on 3 soil sample.
- The Mohr circle at failure is drawn for each specimen & common tangent touching all circle is failure envelope.
- From failure envelope we get c & ϕ values.

Shear
Diagram
Incomplete

Direct shear test:-



$\sigma = \frac{F}{A}$
 $\tau = \frac{F}{A}$
 $\sigma = \frac{F}{A}$
 $\tau = \frac{F}{A}$

$$\sigma = \frac{F}{A}$$

After finding deviator stress (σ) at failure we find principal stress (σ_1) at failure.

$$\sigma_1 = \sigma_3 + \sigma$$

With this set of (σ_1, σ_3) values Mohr circle is drawn.

Bearing capacity

Footing:-

It is a portion of the foundation of a structure that transmits (transfers) loads directly to the soil.

Foundation:-

It is that part of the structure which is below ground level and transmits loads to the ground.

Foundation Soil:-

It is the upper part of the earth mass carrying the load of the structure.

Bearing capacity:-

It is the supporting capacity or the load carrying capacity of the soil.

* Gross Pressure Intensity (q):

It is the total pressure at the base of the footing due to the weight of the superstructure, self-weight of the footing and weight of earth fill, if any.

* Net pressure Intensity (q_n):

It is defined as the excess pressure, or the difference in intensities of the gross pressure after the construction of the structure and the original overburden pressure, if D is the depth of footing.

$$q_n = q - \bar{\sigma} = q - \gamma D$$

(γD = overburden pressure)

(γ = unit wt. of soil above foundation base)

* Ultimate bearing capacity (q_u):

It is defined as the minimum gross pressure intensity at the base of the foundation at which the soil fails in shear.

or

Max. capacity of the soil (max. pressure that a soil can take without shear failure).

* Net ultimate bearing capacity (q_{nu}):
 Max. net pressure that a soil can take without shear failure.

$$(q_{nu} = q_u - \gamma D)$$

* Net safe bearing capacity (q_{ns}):

Net safe pressure that a soil can take safely without shear failure.

[F.S = Factor of safety]

$$q_{ns} = \frac{q_{nu}}{F.S}$$

$$[F.S = (2.5 \text{ to } 3)]$$

$$F.S = \frac{\text{Resisting load}}{\text{Applied load}}$$

* Safe bearing capacity (q_s):

Gross pressure that a soil can take safely without shear failure.

$$q_s = q_{ns} + \gamma D \quad [F.S = (2.5 \text{ to } 3)]$$

* Net safe settlement pressure (q_{np}):

Net pressure that a soil can take without any excessive settlement.

Allowable bearing capacity (q_a):

It is the net loading intensity at which neither the soil fails in shear nor there is excessive settlement of the structure.

Types of bearing capacity failures

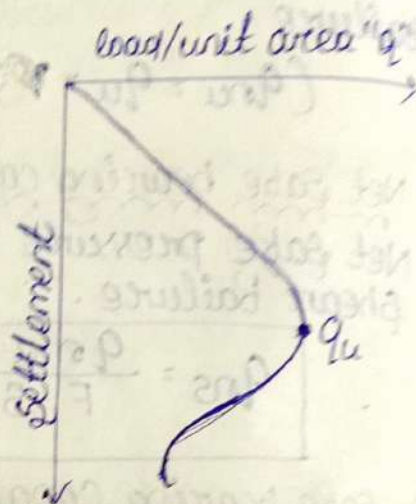
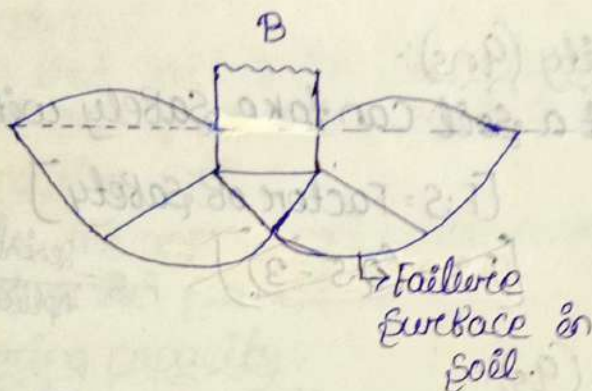
→ When a footing fails due to insufficient bearing capacity, distinct failure patterns are developed, depending upon type of failure mechanism.

There are 3 types of bearing capacity failures.

(1) General Shear Failure: (G.S.F)

This type of failure occurs in dense sand or stiff cohesive soil, as a load is gradually applied to the foundation, settlement load per unit area equals q_u . A sudden failure in the soil supporting the foundation

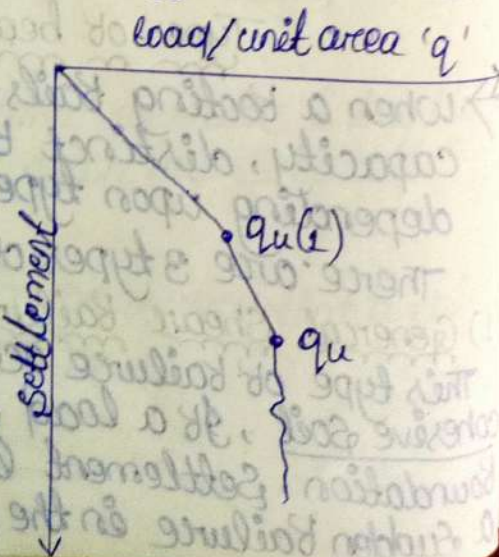
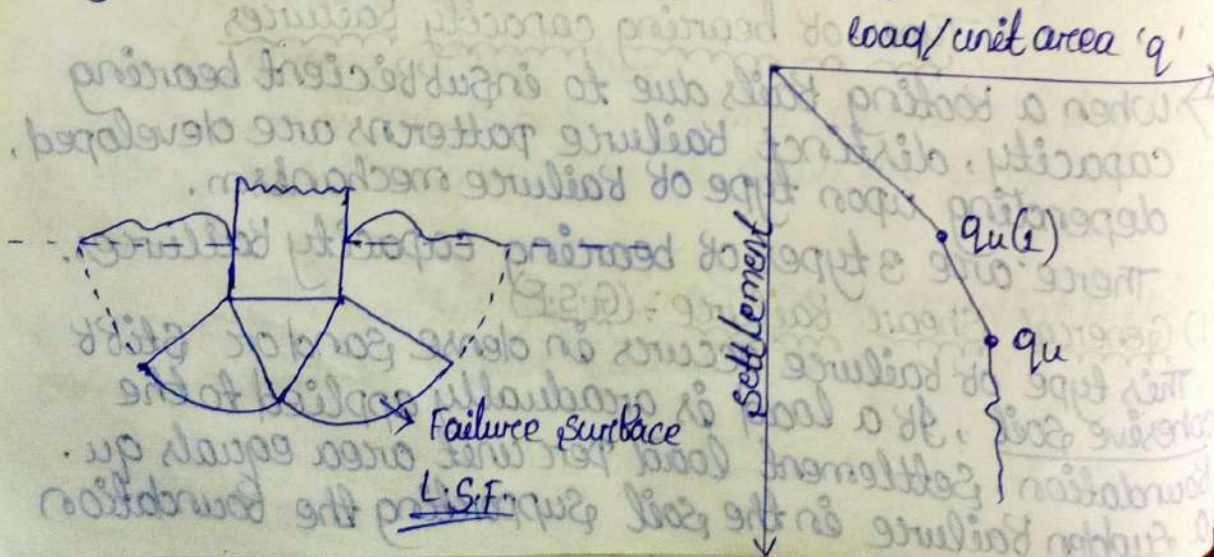
will take place, and the failure surface in the soil will extend to the ground surface.



G.S.F

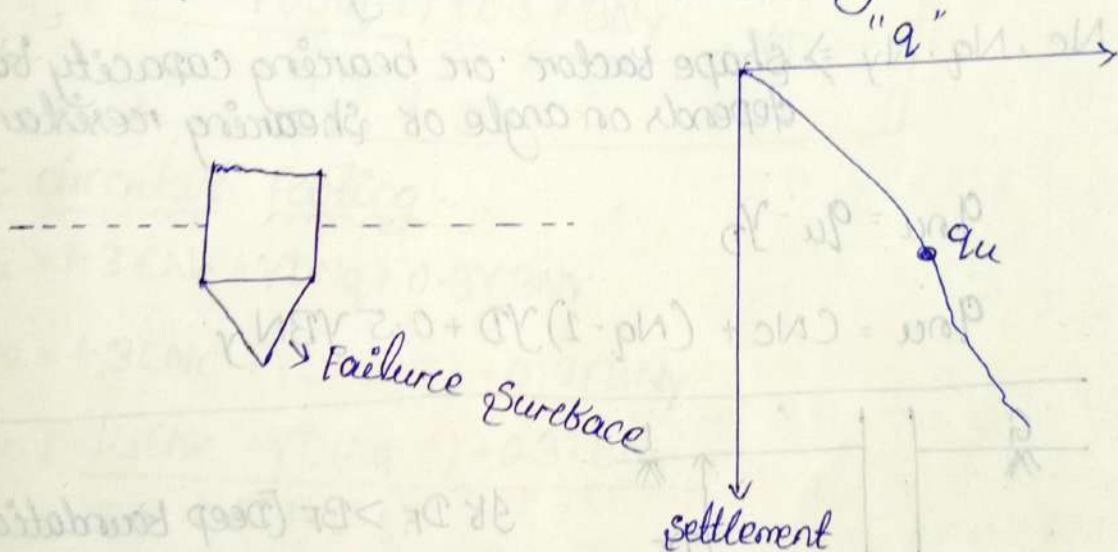
Local Shear Failure :- (L.S.F.)

- It occurs if the foundation rests on sand or clayey soil of medium compaction.
- When the load on foundation increases, settlement also increases.
- Here failure surface in the soil will extend outward from the foundation.
- When the bearing capacity equals to q_{u1} , movement of foundation causes sudden jerks.
- Further when the load increases to " q_u ", there is large increase in foundation settlement.



Punching Shear Failure (PSF):

- It occurs in deep foundation & when the foundation soil is fairly loose.
- Here, the failure surface in the soil will not extend to the ground surface.
- After the ultimate failure load " q_u ", the load-settlement plot will be steep & practically linear.



P.S.F.

Terzaghi's bearing capacity theory:

- Terzaghi was the first scientist to present a theory for calculation of ultimate bearing capacity for shallow foundation only.
- Shallow foundation is a footing if the depth of the footing (D_f) is less than or equal to the breadth of footing (B_f).
- Terzaghi suggested for a continuous strip footing, and assumed failure to be general shear failure. The base of the footing is rough & side resistance is neglected.

For strip Footing:

$$q_u = C N_c + \gamma D N_q + 0.5 \gamma B N_\gamma$$

C = cohesion of soil

γ = Unit wt. of soil

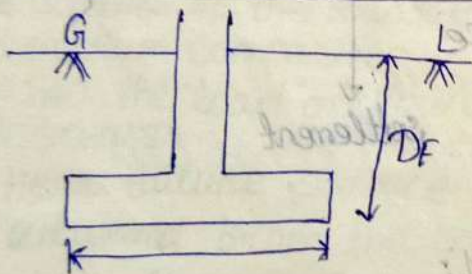
D = Depth of the footing.

B = Breadth of the footing.

$N_c, N_q, N_\gamma \Rightarrow$ shape factor or bearing capacity factor depends on angle of shearing resistance.

$$q_{nu} = q_u - \gamma D$$

$$q_{nu} = C N_c + (N_q - 1) \gamma D + 0.5 \gamma B N_\gamma$$



If $D_f > B_f$ [Deep Foundation]

If $D_f < B_f$ [Shallow Foundation]

$$\Rightarrow q_{nu} = q_u - \gamma D$$

$$q_{nu} = [C N_c + \gamma D N_q + 0.5 \gamma B N_\gamma] - \gamma D$$

$$\Rightarrow q_{ns} = \frac{q_{nu}}{F.O.S.}$$

$$q_{ns} = \frac{C N_c + \gamma D (N_q - 1) + 0.5 \gamma B N_\gamma}{F.O.S.}$$

$$\Rightarrow q_s = q_{ns} + \gamma D$$

$$\Rightarrow \frac{C N_c + \gamma D (N_q - 1) + 0.5 \gamma B N_\gamma}{F.O.S.} + \gamma D$$

only Forc Strip Footing:

$$q_u = C N_c + \gamma D N_q + 0.5 \gamma B N_\gamma$$

$$q_{ou} = C N_c + \gamma D (N_q - 1) + 0.5 \gamma B N_\gamma$$

$$q_{os} = \frac{C N_c + \gamma D (N_q - 1) + 0.5 \gamma B N_\gamma}{F.O.S.}$$

$$q_s = \frac{C N_c + \gamma D (N_q - 1) + 0.5 \gamma B N_\gamma}{F.O.S.} + \gamma D$$

Forc circular Footing:

$$q_u = 1.3 C N_c + \gamma D N_q + 0.3 \gamma B N_\gamma$$

$$q_{ou} = 1.3 C N_c + \gamma D (N_q - 1) + 0.3 \gamma B N_\gamma$$

$$q_{os} = \frac{1.3 C N_c + \gamma D (N_q - 1) + 0.3 \gamma B N_\gamma}{F.O.S.}$$

$$q_s = \frac{1.3 C N_c + \gamma D (N_q - 1) + 0.3 \gamma B N_\gamma}{F.O.S.} + \gamma D$$

Forc Square Footing:

$$q_u = 1.3 C N_c + \gamma D N_q + 0.4 \gamma B N_\gamma$$

$$q_{ou} = 1.3 C N_c + \gamma D (N_q - 1) + 0.4 \gamma B N_\gamma$$

$$q_{os} = \frac{1.3 C N_c + \gamma D (N_q - 1) + 0.4 \gamma B N_\gamma}{F.S.}$$

$$q_s = \frac{1.3 C N_c + \gamma D (N_q - 1) + 0.4 \gamma B N_\gamma}{F.S.} + \gamma D$$

Forc rectangular Footing:

$$q_u = \left(1 + 0.3 \frac{B}{L}\right) C N_c + \gamma D N_q + 0.5 \left(1 - 0.2 \frac{B}{L}\right) \gamma B N_\gamma$$

G.S.F				L.S.F			
ϕ	N_c	N_q	N_γ	ϕ	N_c	N_q	N_γ
0	5.7	1	0	0	5.7	1	0
5	7.3	1.6	0.5	5	6.8	1.4	0.2
10	12.9	4.4	2.5	10	8	1.9	0.5
20	17.7	7.4	5	20	11.8	3.9	1.7
30	37.2	22.5	19.7	30	19	8.3	5.7
35	51.8	41.4	42.4	35	23.2	12.6	10.1

Q. A square footing (2.5×2.5) is built in homogenous sand of unit wt. 20 kN/m^3 & angle of shearing resistance 35° , the depth of the footing is 1.5 m below ground level. calculate q_u , q_{au} , q_s , q_{as}
 $F.O.S = 3$.

Ans $L = 2.5 \text{ m}$, $b = 2.5 \text{ m}$

$\gamma = 20 \text{ kN/m}^3$

$\theta = 35^\circ$

$D = 1.5 \text{ m}$

$F.O.S = 3$.

$N_c = 51.8$

$N_q = 41.4$

$N_\gamma = 42.4$

$C = 0$

$$q_u = 1.3 C N_c + \gamma D N_q + 0.4 \gamma B N_\gamma$$

$$= 1.3 \times 0 \times 51.8 + 20 \times 1.5 \times 41.4 + 0.4 \times 20 \times 2.5 \times 42.4$$

$$= 2090 \text{ kN/m}^2$$

$$q_{au} = 1.3 C N_c + \gamma D (N_q - 1) + 0.4 \gamma B N_\gamma$$

$$= 1.3 \times 0 \times 51.8 + 20 \times 1.5 (41.4 - 1) + 0.4 \times 20 \times 2.5 \times 42.4$$

$$= 2060 \text{ kN/m}^2$$

$$q_{ns} = \frac{1.3 C N_c + \gamma D (N_q - 1) + 0.4 \gamma B N_\gamma}{F.O.S.}$$

$$= \frac{1.3 \times 0 \times 51.8 + 20 \times 1.5 \times \frac{(11.4 - 1)}{41.4} + 0.4 \times 20 \times 2.5 \times 42.4}{3}$$

$$= 686.67 \text{ KN/m}^2$$

$$q_s = \frac{1.3 C N_c + \gamma D (N_q - 1) + 0.4 \gamma B N_\gamma}{F.O.S.} + \gamma D$$

$$= 686.67 + 20 \times 1.5$$

$$= 716.67 \text{ KN/m}^2$$

Q. A square footing located at a depth of 1.3 m below G.L., it has to carry a safe load of 800 kN. Find the size of the footing. If F.O.S is 3, the soil has the following properties.

$$e = 0.55$$

$$S_r = 50\%$$

$$G_1 = 2.67$$

$$C = 8 \text{ kN/m}^2$$

$$\phi = 30^\circ$$

$$N_c = 37.2$$

$$N_q = 22.5$$

$$N_y = 19.7$$

Ans
 $D = 1.3 \text{ m}$

$$L = 800 \text{ kN}$$

$$\text{F.O.S} = 3$$

$$\gamma = \frac{(G_1 + e \cdot S_r) \gamma_w}{1 + e}$$

$$= \frac{(2.67 + 0.55 \times 0.5) \times 9.810}{1 + 0.5}$$

$$= 18.64 \text{ kN/m}^3$$

$$q_s = \frac{1.3 C N_c + \gamma D (N_q - 1) + 0.4 \gamma B N_y + \gamma D}{\text{F.O.S.}}$$

$$\Rightarrow (q_s \times A) = \left[\frac{(1.3 \times 8 \times 37.2 + 18.64 \times 1.3 (22.5 - 1) + 0.4 \times 18.64 \times B \times 19.7)}{3} \right] \times B^2$$

$$\Rightarrow 800 = \left[\frac{386.88 + 520.98 + 146.8 B}{3} + 24.232 \right] B^2$$

$$\Rightarrow 800 = \left[\frac{907.8 + 146.8 B}{3} + 24.232 \right] B^2$$

$$\Rightarrow 800 = \left[\frac{907.8}{3} + \frac{146.8B}{3} + 24.232 \right] B^2$$

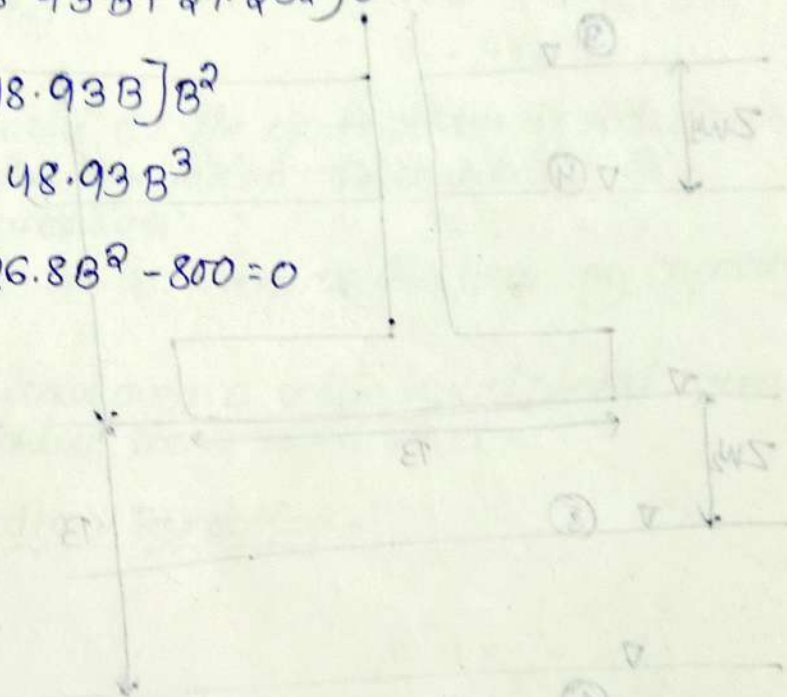
$$\Rightarrow 800 = [302.6 + 48.93B + 24.232] B^2$$

$$\Rightarrow 800 = [326.8 + 48.93B] B^2$$

$$\Rightarrow 800 = 326.8 B^2 + 48.93 B^3$$

$$\Rightarrow 48.93 B^3 + 326.8 B^2 - 800 = 0$$

$$\Rightarrow B = 1.42$$



$$x_1 = -6.26$$

$$x_2 = 1.42$$

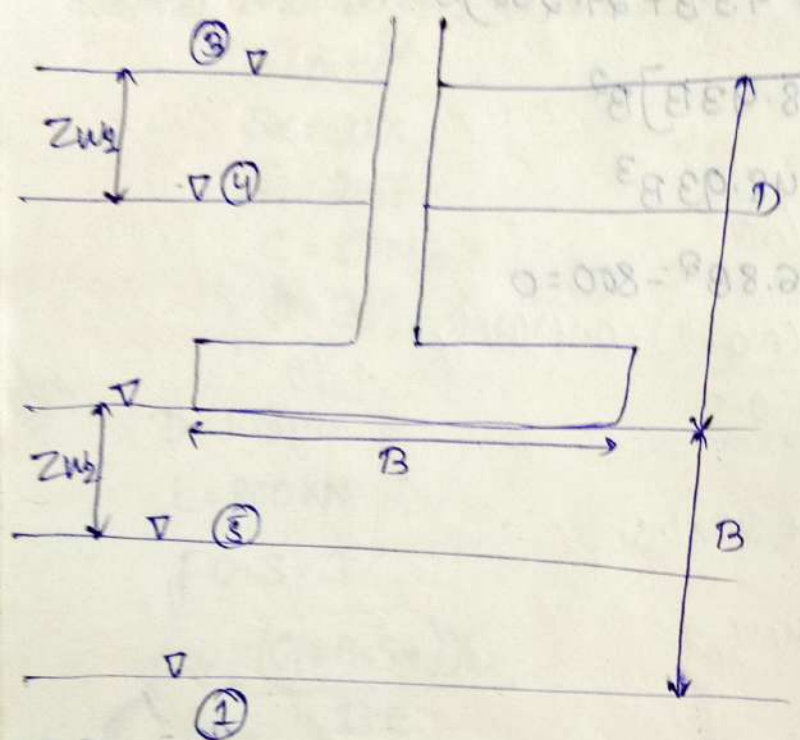
$$x_3 = -1.83$$

Factor of safety & R_{w2} are water table correction factor.

$$R_{w1} = 0.2 \left[1 + \frac{Z_{w1}}{D} \right] ; 0 \leq Z_{w1} \leq D$$

$$R_{w2} = 0.2 \left[1 + \frac{Z_{w2}}{D} \right] ; 0 \leq Z_{w2} \leq B$$

Effect of water table on bearing capacity:



Z_{w1} = depth of water table above base of footing.

Z_{w2} = depth of water table below, footing.

Formula:

$$q_u = C N_c + \gamma D N_q R_{w1} + 0.5 \gamma B N_\gamma R_{w2}$$

R_{w1} & R_{w2} are water table correction factors.

$$R_{w1} = 0.5 \left[1 + \frac{Z_{w1}}{D} \right], 0 \leq Z_{w1} \leq D$$

$$R_{w2} = 0.5 \left[1 + \frac{Z_{w2}}{D} \right], 0 \leq Z_{w2} \leq B$$

Types of Foundation:-

There are 2 types of Foundation

(a) Shallow Foundation

(b) Deep Foundation

Shallow Foundation:-

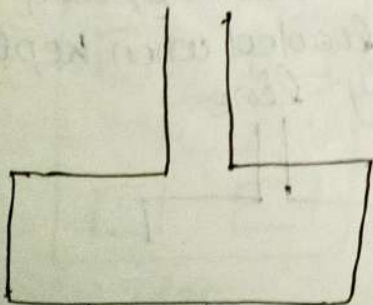
- According to Terzaghi if the depth of the foundation is less than the width of the foundation. The foundation is called shallow foundation.
- In general shallow is used where the bearing capacity of soil is high.
- It distributes the load over a wide horizontal area at a shallow depth below the ground level.

Various types of Shallow Foundation:-

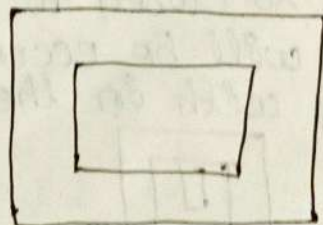
1. Isolated Footing
2. Strip Footing
3. Combined Footing
4. Strap Footing
5. Raft or mat Footing

Isolated Footing:-

- Isolated footing can be called as individual or separate, or pad footing.
- It is provided to support an individual column.
- A spread footing can be rectangular, circular, square in shape.
- Some time it is stepped to spread load gradually over a large area.



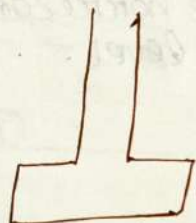
(Elevation)



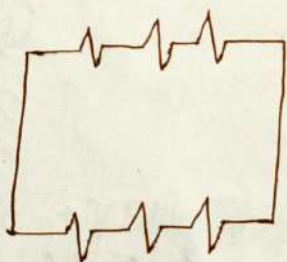
Plan

Strip Footing:-

- A strip footing is provided for a load bearing wall.
- It is also provided for a row of columns which are so closely spaced that their individual footing overlap or nearly touch each other.
- In this case it is more economical to provide strip footing than to provide a no. of isolated footing.
- A strip footing can also be called as continuous footing.



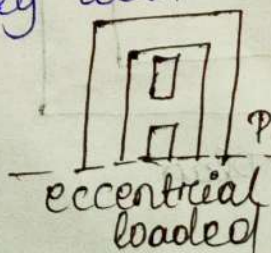
Elevation



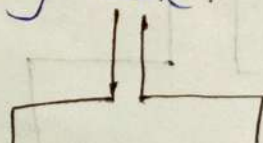
Plan

3. Combined Footing:-

- A combined footing supports two columns.
- It is used when the two columns are so close that their individual footing would overlap.
- A combined footing is also provided when the properly line is so close to one column that a spread footing will be eccentrically loaded when kept entirely within the properly line.



Properly line



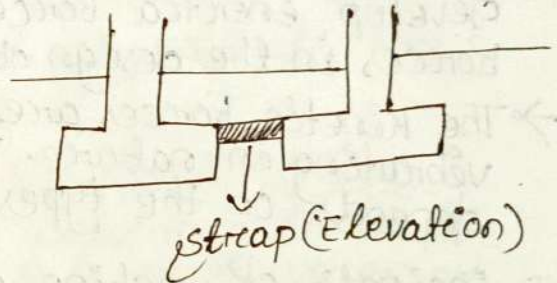
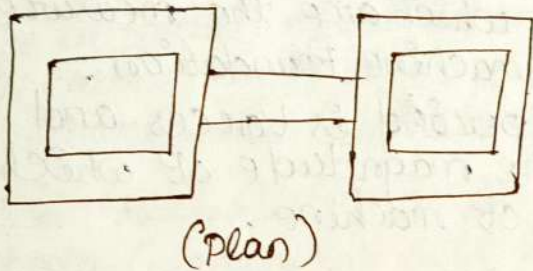
So by combining with the interior column the load is evenly distributed.

4. Strap footing or cantilever footing:-

→ It consists of two isolated footings connected by a strap.

→ The strap consists of a beam which connects two footings which behave as a single unit.

→ A strap is more economical than a combined footing. It is preferred when the soil pressure is higher and the distance between two columns is high.



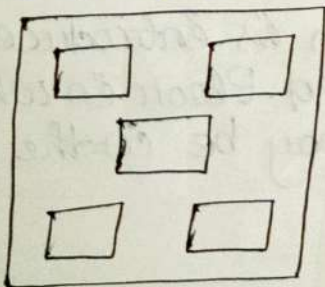
5. Raft or mat footing:-

→ A mat foundation is a large slab supporting a no. of columns and walls under the entire structure or large part of structure.

→ A mat is required when the allowable soil pressure (bearing capacity) is low or less or when the columns & walls are so close that the individual footing would overlap.

→ Mat footing are useful in reducing the differential settlement on non-homogenous soil or where the load variation is high on columns.

→ If their total area covered by each footing is more than 50% of entire area.



Machine Foundation:

- Foundation provided below the Super Structure of vibrating and rotating machine for installation is known as machine foundation.
- It consist of a mass of concrete.
- Design of machine foundation involves consideration of static loads & kinetic forces.
- The load of machine is the static load which is of minor importance in the design of machine foundation the moving part of machine develop inertia force which are the measure forces in the design of machine foundation.
- The kinetic forces are periodic forces and vibrating in nature. The magnitude of which depends on the types of machine.

Principals of machine design:

- Machine foundation should be isolated from the adjoining part by giving a gap around it to avoid transmission of vibration. The gap is filled with suitable insulator.
- The foundation should be stiff and rigid to avoid possibilities of tilt unit.
- In static state the resultant of forces acting on the machine foundation should pass through the CG of the contact area.
- The weight of a foundation block should be adequate so, that it can absorb vibration.
- A vibration absorbing medium is introduced between the bottom of foundation block and floor on which it is resting, the medium may be in the form of rubber or leather.

Earth Pressure & Retaining wall

- In the design of retaining wall, sheet piles or other earth retaining structures. It is necessary to compute the lateral pressure, exerted by the retaining mass of soil.
- The plastic state of stress when failure is imminent (about to happen) was found by Rankine in 1860.
- The retaining wall or retaining structure is used for maintaining the ground surface at different elevation on either side of it.
- The material retained or supported by the structure is called back fill, which may have its top ... horizontal or inclined.

Plastic equilibrium in soil: Active & Passive

A body (element) of soil is said to be in plastic equilibrium if every point on it is on the verge of failure.

Plastic equilibrium equation

$$\sigma_1 = \sigma_3 \tan^2 \alpha_f + 2c \tan \alpha_f$$

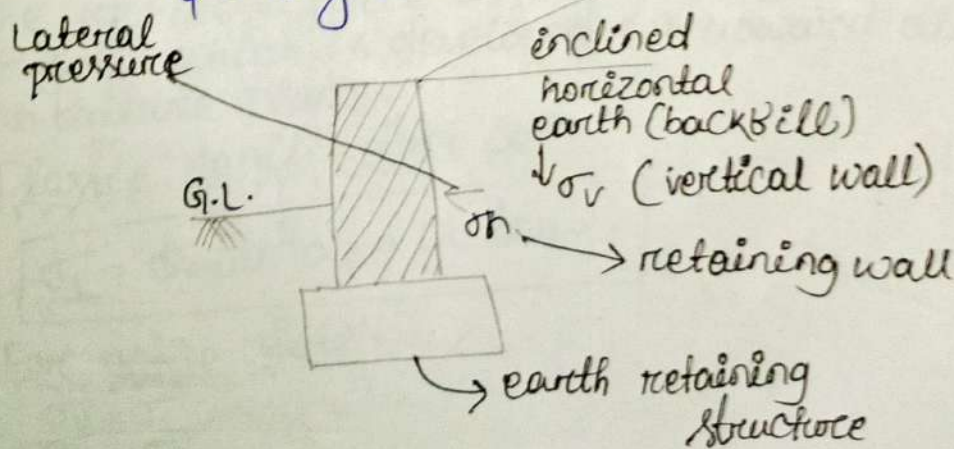
Where, σ_1 = Major Principal stress

σ_3 = Minor Principal stress

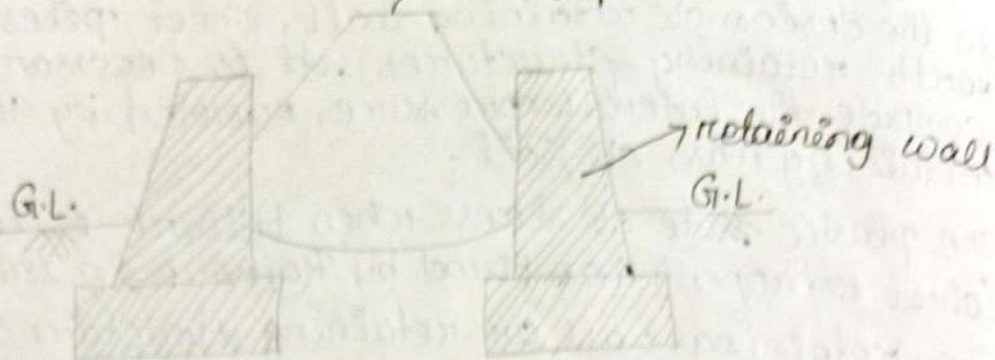
$$\alpha_f = 45 + \frac{\phi}{2}$$

c = cohesion

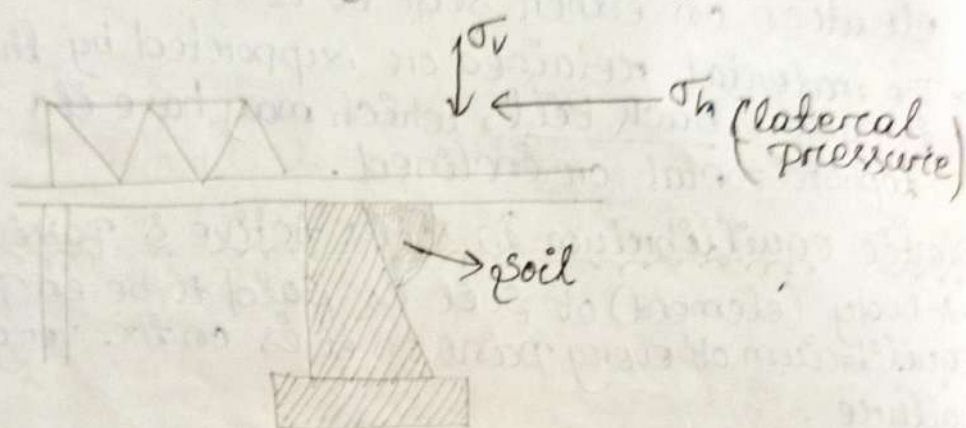
ϕ = Angle of internal friction.



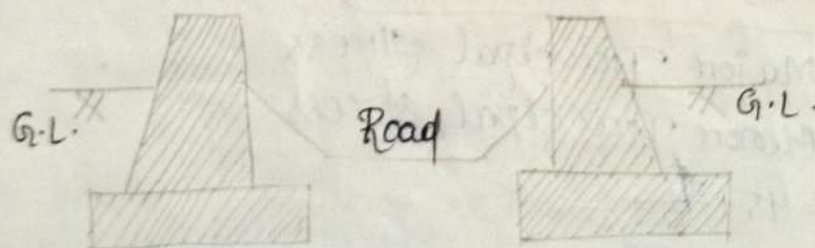
Examples of Retaining structure:-



(embankment elevation)



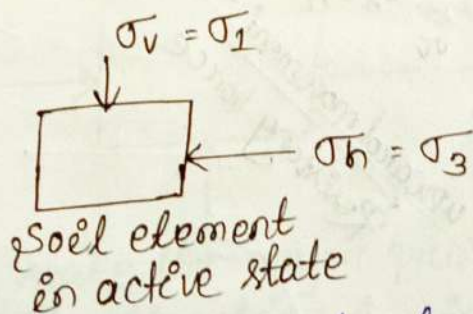
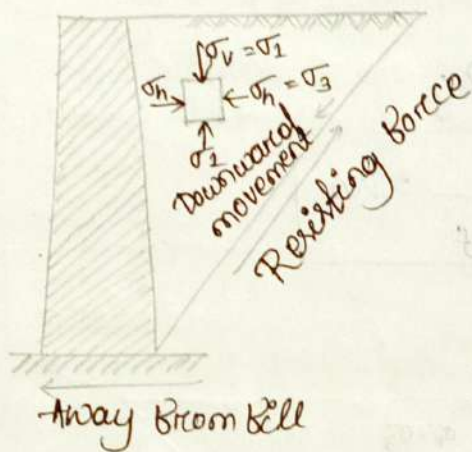
(Bridge Abutment)



cut (elevation)

Active Earth Pressure:-

- During the active pressure the wall moves away from the backfill.
- A certain portion (some) of backfill located immediately behind the wall breaks away from the rest of soil mass.
- This portion of backfill is triangular in shape or wedge shape.
- This wedge shape portion of backfill moves downward. This action of backfill is called failure wedge.



- The resisting force is due to the shear strength of soil, which is developed in upward direction on failure plane.

Plastic equilibrium eqⁿ

$$\sigma_1 = \sigma_3 \tan^2 \alpha_f + 2c \tan \alpha_f$$

For active state:-

$$\frac{\sigma_v}{\sigma_h} = \frac{\sigma_1}{\sigma_3}$$

$$\sigma_v = \sigma_h \tan^2 \alpha_f + 2c \tan \alpha_f$$

The cohesion less soil

$$c = 0$$

$$\sigma_v = \sigma_h \tan^2 \alpha_f$$

Note

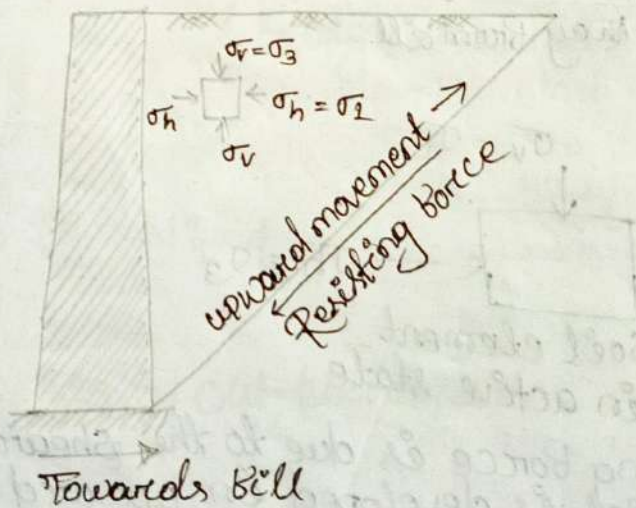
The ratio of horizontal stress (σ_h) to the vertical (σ_v) is called co-efficient of earth-pressure.

$$\frac{\sigma_h}{\sigma_v} = K$$

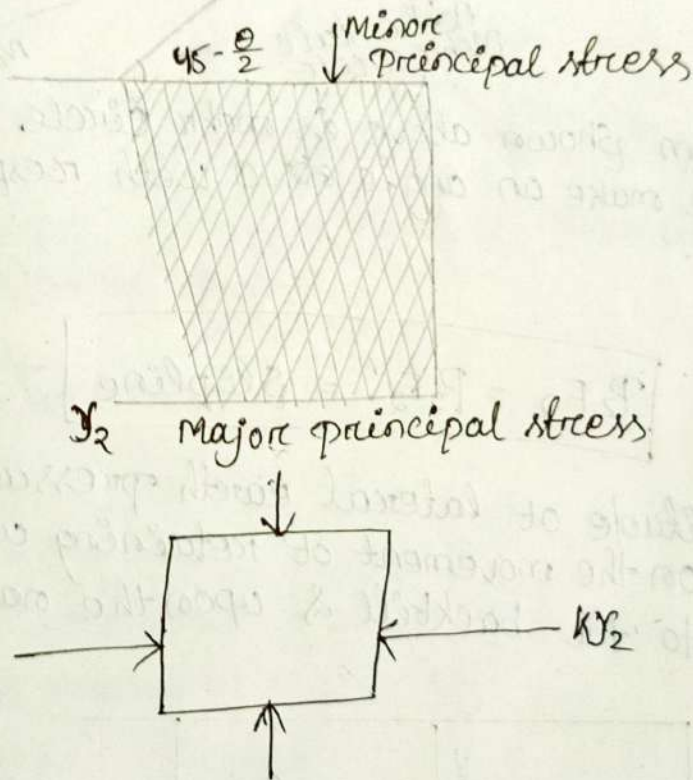
$$\sigma_v = \sigma_h \tan^2 \alpha_f$$

$$\frac{\sigma_h}{\sigma_v} = \frac{1}{\tan^2 \alpha_f} = K_a$$

Passive state :



- During the passive state the retaining wall moves forwards the backfill & a certain portion of backfill located immediately behind the wall breaks away from the rest of soil mass.
- This wedge shape portion of backfill tending to move with the wall is called Railure wedge.
- The resisting force due to the shear strength of soil which is developed in down ward direction along the Railure plane.



- In passive state the major principal stress (σ_1) is in horizontal direction & minor principal stress (σ_3) is in vertical direction.

The soil is cohesion less & ground surface as horizontal.

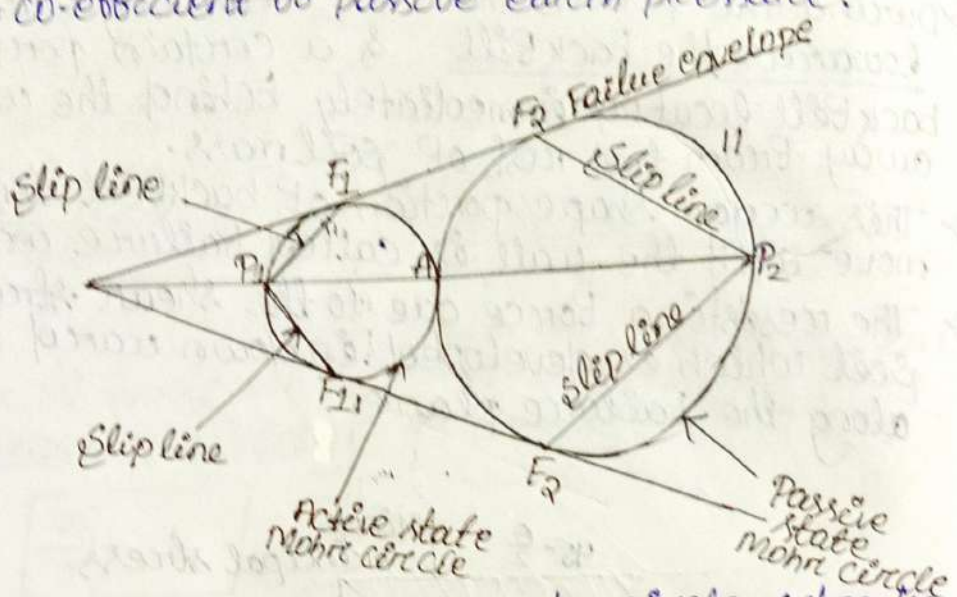
$$\sigma_1 = \sigma_3 \tan^2 \alpha_f + 2c \tan \alpha_f$$

$$\Rightarrow \boxed{\sigma_1 = \sigma_3 \tan^2 \alpha_f}$$

$$\sigma_h = \sigma_v \tan^2 \alpha_f$$

$$\boxed{\frac{\sigma_h}{\sigma_v} = K_p = \tan^2 \alpha_f = \frac{1 + \sin \phi}{1 - \sin \phi}}$$

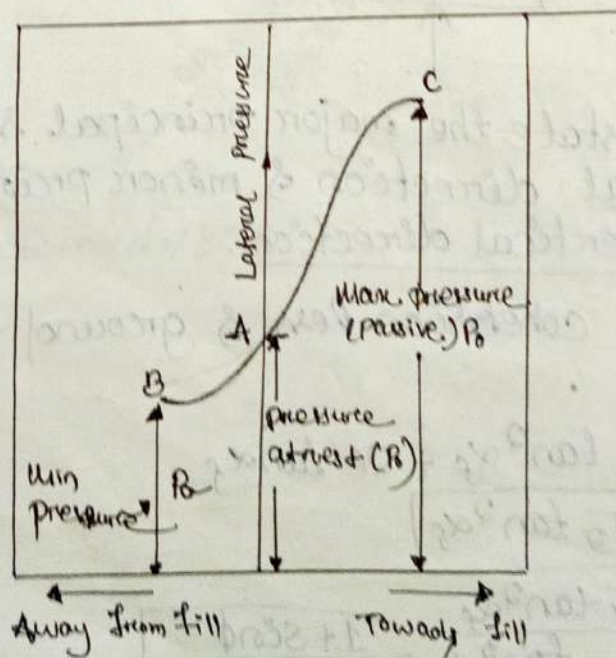
where, k_p = coefficient of passive earth pressure.

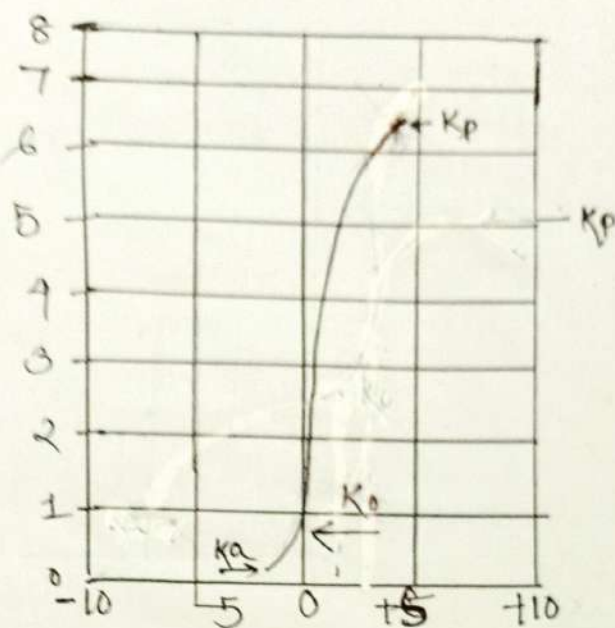


→ The diagram shown above is Mohr circle. when two sets of line make an angle ϕ with respect to horizontal.

$$P_2 F_2 = P_2 F_2' = \text{slip line}$$

→ The magnitude of lateral earth pressure depends on the movement of retaining wall relative to the backfill & upon the nature of soil.





vertical ob crest ob earth pressure with horizontal strain.

$$\frac{\sigma_h}{\sigma_v} = k$$

$$\sigma_h = k \sigma_v$$

$$\text{active pressure} = \sigma_h = k_a \sigma_v$$

$$\text{passive pressure} = \sigma_h = k_p \sigma_v$$

$$k_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$k_p = \frac{1 + \sin \phi}{1 - \sin \phi}$$