

Unit and Dimension

Physics $\rightarrow$ the study of nature and its laws
 $\hookrightarrow$ Mathematics is the language of Physics.

Physical Quantities $\rightarrow$ The quantities which we can measure.

Ex: Height, weight, distance $\rightarrow$ measurable.
happiness, sadness $\rightarrow$ can't measure.

$\rightarrow$ Two types of physical quantities $\Rightarrow$

(a) Fundamental Physical Quantity

(b) Derived Physical Quantity.

They do not depend on any other physical quantity for their measurement.

Ex $\rightarrow$ Mass, length, time.

They ~~are~~ depend on other physical quantities for their measurement. And expressed as some mathematical function of fundamental physical quantities.

Ex: Velocity = ~~dist~~ $\frac{\text{Length}}{\text{time}}$

$$\text{Force} = \text{mass} \times \text{acceleration} \\ = \text{mass} \times \frac{\text{length}}{(\text{time})^2}$$

Units $\rightarrow$ It is an internationally accepted reference standard which is used to measure a physical quantities.

Fundamental Units $\rightarrow$ The units for the fundamental physical quantities.

Derived Units $\rightarrow$ The units of other physical quantities ~~can be~~ are expressed in terms of fundamental units. These are known as derived units.

System of units $\rightarrow$ A complete set of both fundamental & derived units.

(a) C.G.S. System $\rightarrow$ Based on Centimeter, Gram, Second as fundamental units of length, mass and time.

(b) M.K.S. System $\rightarrow$ Based on meter, kilogram and second as fundamental units.

(c) SI Unit $\rightarrow$ Formed by the international Bureau of weights and measures in 1971. It is based on 7 fundamental units and 2 supplementary units.

7 fundamental Units

Mass $\rightarrow$ Kilogram (kg)

Length $\rightarrow$ Meter (m)

Time $\rightarrow$ Second (s)

Temperature $\rightarrow$ Kelvin (K)

Elect. current $\rightarrow$ Ampere (A)

Luminosity $\rightarrow$ Candela (cd)

Amount of substance $\rightarrow$ Mole (mol.)

^{Supplementary}
Units { Angle $\rightarrow$ Radian (Rad)

{ Solid angle $\rightarrow$ Steradian (Sr.)

Dimensions $\rightarrow$ These are the powers to which the fundamental quantities / base quantities are raised to represent a physical quantity.

$\rightarrow$ denoted by putting $[]$ around a quantity.

Ex: $[\text{Length}] = [L] = [L']$

$$[\text{Mass}] = [M] = [M']$$

$$\text{Velocity} = \frac{\text{Length}}{\text{Time}} = \frac{[L]}{[T]} = [M^0 L T^{-1}]$$

$\rightarrow$ Calculation of dimension of physical quantity (Do some examples).

$\rightarrow$ Dimensional equation $\rightarrow$ The eqn obtained by equating a physical quantity with its dimensional formula is called dimensional eqn or check.

Ex:- Velocity $\Rightarrow [V] = [M^0 L^1 T^{-1}]$

Acceleration $\Rightarrow [a] = [M^0 L^1 T^{-2}]$

Density $\Rightarrow [\rho] = [M L^{-3} T^0]$

Dimensional Analysis $\rightarrow$

Principle of homogeneity $\rightarrow$ The dimension of each term of a dimensional eqn on both sides should be same.

- * Conversion from MKS to CGS or vice versa
- * Numericals on dimensional correctness

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Scalar & Vector

Physical Quantities

Scalar

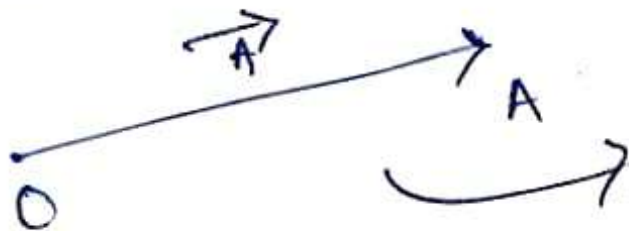
Have ~~both~~ only magnitude)
→ Expressed as a number

Vector

(Have both magnitude and direction)
→ expressed with an arrowhead above.

Ex: Force vector
 $= \vec{F}$

Representation of Vector ⇒



length of OA is $|\vec{A}|$
magnitude of vector $\vec{A}$

Scalar

- Have only magnitude
- a number with an unit.
- Mathematical operations are same as plane Maths.

Temp, pressure,
energy, mass, time

Vector

- has both direction & magnitude.
- displacement vector
- They added by triangle law of vector addition or parallelogram of vector addition.

(Equal vector - same magnitude & same direction)

Representation of vector $\Rightarrow$

- Null vector $\rightarrow$ zero magnitude / arbitrary direction
- $\rightarrow$ a point or dot.
- $\rightarrow$ -Adding with gives same.
- $\rightarrow$ Dot product & cross product $\Rightarrow$ zero.

$\Rightarrow$ Unit vector $\rightarrow$

Magnitude = 1

gives the direction of vector.

written with cap &

$\hat{i}, \hat{j}, \hat{k}$

Collinear vector -

Parallel.

Antiparallel.

Perpendicular vector.

Negative Equal vectors -

Negative vectors $\Rightarrow$

- $\rightarrow$ Same magnitude & opposite direction
- $\rightarrow$ All negative $\vec{v}$ are antiparallel.

Coinitial

Position vector $\vec{r}$.

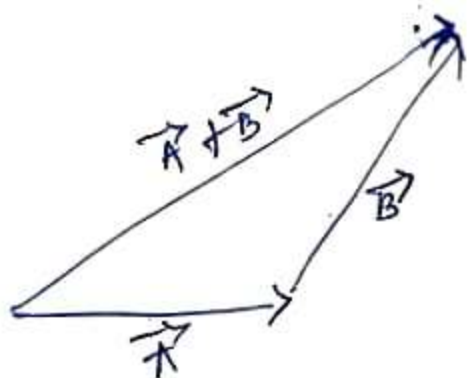
Coplanar

Indicate the position of a point in a coordinate system.

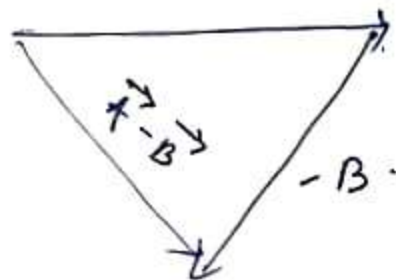
Vector Addition

Parallel shift of vector $\Rightarrow$
If we shift a vector parallelly, it will remain unchanged.

Graphical Method / Triangle law / Head-to-Tail method.

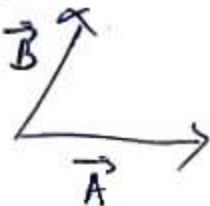


Subtraction

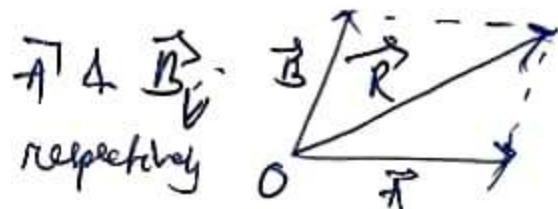


Parallelogram law :-

$\rightarrow$ Join the tails



$\rightarrow$ Draw ~~two~~ ^a lines parallel to from head of B & A



- Parallelogram law is || to triangle law $\Rightarrow$
- The dotted lines (drawn ||) are vectors $\vec{A}$ & $\vec{B}$.

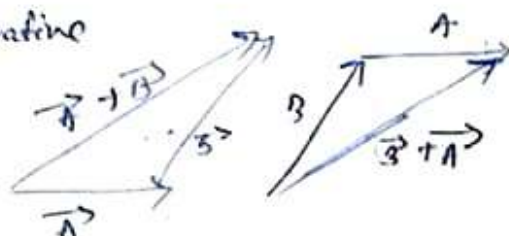
Properties

- 1) Vector addition is commutative

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

- 2) Associative

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

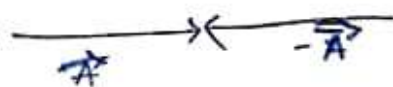


Null Vector / Zero vector

$$\vec{0} = \vec{A} + (-\vec{A})$$

$$\vec{0} = \vec{A} \times \vec{0}$$

$$\vec{A} = \vec{A} + \vec{0}$$



Multiplication with a scalar $\Rightarrow$
 A vector multiplied by a +ve scalar it will give a vector in same direction.

$$\lambda \times \vec{A} = \lambda \vec{A}$$

$\Rightarrow$ Multiplying with -ve scalar, we will get a vector in opposite direction.
 $-\lambda \times \vec{A} = -\lambda \vec{A}$

Equal Vector

Two vectors having same magnitude & same direction are equal vector.

Give examples

Unit vectors

$\Rightarrow$ Unit = 1.

$\Rightarrow$ length/magnitude is 1.

$\Rightarrow$ Points in one direction & gives direction any vector.

$\Rightarrow$ If we multiply a scalar with an unit vector, we will get a vector.

So any vector can be represented by scalar $\times$ unit vector.
 magnitude of $\downarrow$ direction

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|} = \hat{A}$$

→ Unit vectors are dimensionless.

→ Unit vectors are called directional vectors too

Resolution of Vectors →

k/e write any vector in terms of their components along X & Y axis.

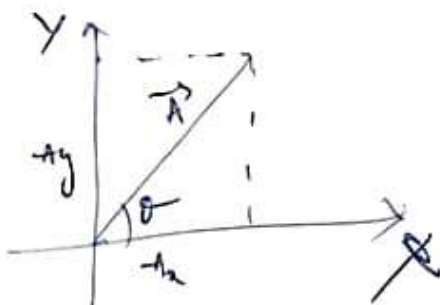
$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

Projection of A
on X-axis

Projection of A
on Y-axis.

$$\sin \theta = \frac{A_y}{A} \Rightarrow A_y = A \sin \theta$$

$$\cos \theta = \frac{A_x}{A} \Rightarrow A_x = A \cos \theta$$



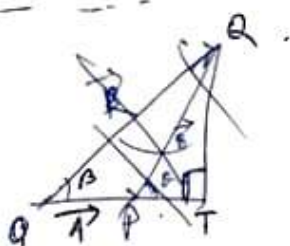
$$A_x^2 + A_y^2 = A^2 \rightarrow \text{Pythagorean thm.}$$

$$\Rightarrow |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\text{Dirn}^n \text{ of } \vec{A} = \tan \theta = \frac{A_y}{A_x}$$

~~ΔOTQ & ΔPTQ.~~

sim.
 ~~triangle~~



Dot product →

$\vec{A} \cdot \vec{B}$ = product of their magnitude and the cosine of the smaller angle betⁿ them.

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta = AB \cos \theta$$

$$\theta = 0, \vec{A} \cdot \vec{B} = AB, \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\theta = 90^\circ, \vec{A} \cdot \vec{B} = 0, \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

Dot product in rectangular coordinates.

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + \dots) \cdot (B_x \hat{i} + \dots)$$

$$= A_x B_x (\hat{i} \cdot \hat{i}) + \dots$$

$$= A_x B_x + A_y B_y + A_z B_z$$

sum of product of rectangular components along the coordinate axes.

$$\begin{aligned} \vec{A} &= 3\hat{i} + 2\hat{j} + 5\hat{k} \\ \vec{B} &= 4\hat{i} + 3\hat{j} + 7\hat{k} \end{aligned} \quad \vec{A} \cdot \vec{B}$$

$$\vec{A} = 2\hat{i} + 5\hat{j} + 6\hat{k}$$

$$3\hat{i} - 6\hat{j} + \hat{k}$$

$$\vec{A} = 5\hat{i} + 2\hat{j} + 3\hat{k}$$

$$2\hat{i} - 3\hat{j}$$

$$\vec{A} = 3\hat{i} + 2\hat{j} \quad \vec{B} = 4\hat{i} + 3\hat{j}$$

Cross product

$$\vec{A} \times \vec{B} = AB \sin \theta$$

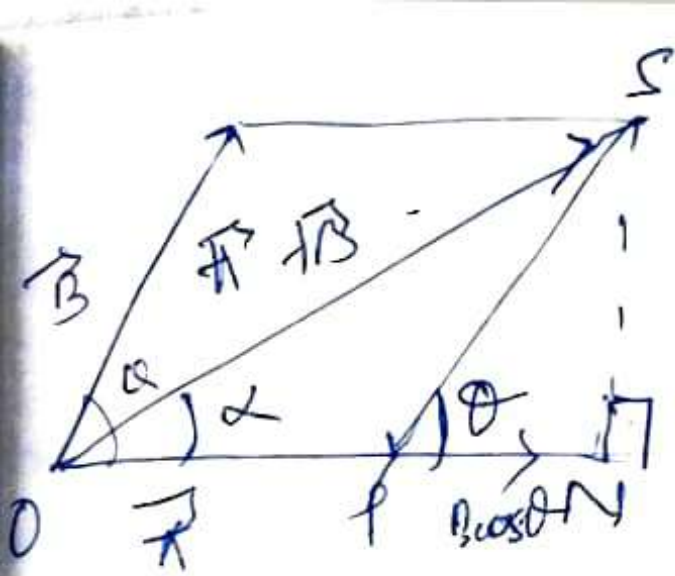
Right hand thumb rule $\rightarrow$
Normal to the plane holding in the right hand.

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

Cross product using rectangular coordinates



$SN \perp ON$ take

ΔOSN ,

4

$$(OS)^2 = (ON)^2 + (SN)^2$$

$$= (OP + PN)^2 + (SN)^2$$

$$= (A + B \cos \theta)^2 + (SN)^2$$

$$= (A + B \cos \theta)^2 + (B \sin \theta)^2$$

$$= A^2 + 2AB \cos \theta + B^2 \cos^2 \theta + B^2 \sin^2 \theta$$

$$\Rightarrow R^2 = A^2 + 2AB \cos \theta + B^2$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

Direction of $\vec{R}$

$$\tan \alpha = \frac{SN}{ON} = \frac{B \sin \theta}{A + B \cos \theta}$$

3. KINEMATICS Curvilinear Motion & Kinematics

Curvilinear motion:- Motion of an object in curved path with variable direction of velocity is called as curvilinear motion.

Projectile:- Any object projected into the space & is moving under the influence of gravity only after projection is called as projectile.

Trajectory:- The path of the projectile is called as trajectory & the motion of the projectile is called as projectile motion.

Angle of Projection:- The angle at which the projectile being projected is called as angle of projection.

Maximum Height (H):- It is the maximum displacement travelled by the projectile in vertical direction.

Horizontal Range (R):- It is the maximum displacement travelled by the projectile along horizontal dirⁿ.

Time of Flight (T):- It is the total time taken by the projectile to come back to the same level from which it is projected.

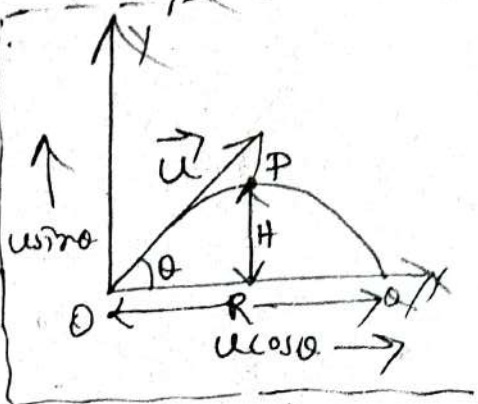
Time of ascent:- It is time taken to reach maximum height in vertical dirⁿ.

Time of descent:- Time taken to come back from max height to the level of projection.

Projectile fired at an angle θ with the horizontal:-

Let a projectile is projected with initial velocity ' u ' at an angle ' θ ' with the horizontal. Suppose the projectile reaches to a height ' h ' & then fall to the pt. 'A' on the ^{slope} level of the projection which it was projected.

As the projectile projected in free space i.e. in 2-d space, hence initial velocity ' u ' will be resolve into two components such as:-



- (i) $u \cos \theta$ along horizontal dirⁿ which is uniform as in this dirⁿ accⁿ due to gravity has no effect.
- (ii) $u \sin \theta$ along vertical dirⁿ which is non-uniform as in this dirⁿ accⁿ due to gravity ' g ' acts exactly opposite to it.

① Equation of Trajectory:-

According to the kinematic eqⁿ, the distance travelled by the projectile after time ' t ' is given by,

$$S = ut + \frac{1}{2}at^2 \quad \text{--- ①}$$

For horizontal direction suppose the distance travelled in ' t ' time is ~~given by~~ ' x ' then it is given by,

$$x = ut + \frac{1}{2}at^2 \quad \text{--- ②}$$

But in horizontal dirⁿ,

3.

$\vec{u} = u \cos \alpha$, accⁿ $a = 0$ as in this dirⁿ velocity is uniform.

Hence from (2) we get, $x = u \cos \alpha \cdot t + \frac{1}{2} \cdot 0 \cdot t^2$

$$\begin{aligned} \Rightarrow x &= u \cos \alpha \cdot t \\ \Rightarrow t &= \frac{x}{u \cos \alpha} \end{aligned} \quad \text{--- (3)}$$

Now for vertical dirⁿ suppose the distance travelled in 't' is 'y' is given by,

$$y = ut + \frac{1}{2} at^2 \quad \text{--- (4)}$$

But in this vertical dirⁿ, $\vec{u} = u \sin \alpha$, accⁿ $a = -g$.

Hence from (4) we get, $\Rightarrow y = u \sin \alpha \cdot t + \frac{1}{2} (-g) t^2$

$$\Rightarrow y = u \sin \alpha \cdot t - \frac{1}{2} g t^2 \quad \text{--- (5)}$$

Now putting value of 't' from (3) in (5), we get,

$$y = u \sin \alpha \cdot \frac{x}{u \cos \alpha} - \frac{1}{2} g \left(\frac{x}{u \cos \alpha} \right)^2$$

$$\Rightarrow \boxed{y = \tan \alpha \cdot x - \frac{g x^2}{2 u^2 \cos^2 \alpha}} \quad \text{--- (6)}$$

This eqⁿ is a form of parabola, hence the path of the projectile is parabolic.

(2) Time of flight:— The total ^{time} taken by the projectile to come back to the level of projection. It can be calculated at pt. 'Q' i.e. at final point.

Now at 'a' the ⁻⁴⁻ vertical displacement covered by the projectile, $y = 0$.

Using this from eqn (5) we can write

$$0 = u \sin \theta \cdot t - \frac{1}{2} g t^2$$

$$\Rightarrow \frac{1}{2} g t^2 = u \sin \theta \cdot t \Rightarrow \frac{1}{2} g t = u \sin \theta$$

$$\Rightarrow \boxed{t = \frac{2u \sin \theta}{g}} \quad \text{--- (7)}$$

③ Time of ascent:- It is the time taken by the projectile at a height H i.e. at point 'P'.

At this point, initial ^(vertical) velocity, $\vec{u} = u \sin \theta$
 final ^(vertical) velocity, $\vec{v} = 0$

distance covered, $s = H$

accelⁿ, $a = -g$

time (time of ascent), $= t$

Now using the kinematic equation,

$$v = u + at$$

we get, $\Rightarrow 0 = u \sin \theta + (-g)t_1$

$$\Rightarrow u \sin \theta - gt_1 = 0 \Rightarrow u \sin \theta = gt_1$$

$$\Rightarrow \boxed{t_1 = \frac{u \sin \theta}{g}} \quad \text{--- (8)}$$

Time of descent:-

Hence, initial ^(vertical) velocity, $\vec{u} = u \sin \theta$
 final ^(vertical) velocity, $\vec{v} = 0$

As total time taken by the projectile is the sum of time of ascent (t_1) & time of descent (t_2),

Hence time of flight, $t = t_1 + t_2$

$$\Rightarrow \frac{2u \sin \theta}{g} = \frac{u \sin \theta}{g} + t_2$$

$$\Rightarrow t_2 = \frac{2u \sin \theta}{g} - \frac{u \sin \theta}{g} = \frac{u \sin \theta}{g}$$

Using eq. (7) & (8)

Hence time of ascent is equal to time of descent.

4) Maximum Height (H) :-

The ~~time~~ ^{distance} ~~time~~ ^{taken} by the projectile after covering the maximum height 'H' and to reach at point 'P' as shown in fig. B. given by, using the kinematic eqn.

$$v^2 = u^2 + 2as$$

Hence at point P,

$$\begin{aligned} v &\rightarrow 0 \\ u &\rightarrow u \sin \theta \\ a &\rightarrow -g \\ s &\rightarrow H \end{aligned}$$

Hence we get,

$$0 = u^2 \sin^2 \theta + 2(-g)H$$

$$\Rightarrow 2gH = u^2 \sin^2 \theta$$

$$\Rightarrow \boxed{H = \frac{u^2 \sin^2 \theta}{2g}} \quad \text{--- (10)}$$

This is the expression for maximum height of a projectile projected with an angle ' θ ' with the horizontal.

(2) Horizontal Range (R) :- The total horizontal distance covered by the projectile to reach at final p. is given by, from eqnⁿ (3) ~~we have~~,

$$X = u \cos \theta \cdot t$$

Now using the value of 't' from eqnⁿ (7) we get,

$$X = u \cos \theta \cdot \frac{2u \sin \theta}{g}$$

$$\Rightarrow X = \frac{u^2 (2 \sin \theta \cdot \cos \theta)}{g}$$

$$\Rightarrow \boxed{X = \frac{u^2 \sin 2\theta}{g}} \text{ or } \boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

~~it is also denoted~~

This is the required expression for the ~~max~~ horizontal range covered by the projectile projected with an angle θ with the horizontal & it is also denoted by R .

NOTE :- Condition for max^m range covered by the projectile :-

max^m range i.e. $R \rightarrow \text{max}^m$

$$\text{But for } R \rightarrow \text{max}^m \Rightarrow \sin 2\theta = 1$$

$$\Rightarrow \sin 2\theta = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ$$

Hence the projectile will cover a max^m range when it is projected by an angle $\theta = 45^\circ$ with the horizontal.

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Friction :-

CHAPTER-4

Defⁿ :- Whenever a body is in contact or in motion with a surface, then an opposing force comes to play tangentially at the point of contact. This force is called as friction.

Ex :- $\rightarrow$ Walking of a person on a floor by friction betⁿ feet & floor.

$\rightarrow$ Friction enable us to drive & stop the vehicle.

Friction are of 4 type, such as

$\rightarrow$ Static friction.

$\rightarrow$ Kinetic friction (dynamic friction)

$\rightarrow$ Sliding friction $\rightarrow$ Fluid friction

① Static Friction :-

② Dynamic Friction

Defⁿ :- It is the force of friction comes to play when a body is forced to move along a surface but movement doesn't start.

Defⁿ :- It is the force of friction comes to play when a body just starts moving along a surface.

$\rightarrow$ The magnitude of static friction remains equal to the applied force & the dirⁿ is always opposite to dirⁿ of motion.

$\rightarrow$ If the magnitude of dynamic friction is ~~less~~ than the external applied force, then only the body will move.

$\rightarrow$ Maximum value of static friction is limiting friction after which only the body starts to move.

$\rightarrow$ Maximum value of dynamic friction is the sliding friction.

Ex - pushing a cart | Ex - pushing a box
across a floor

Reducing Friction

Law of Limiting Friction

- (1) The dirⁿ of force of friction is always opposite to the dirⁿ of motion.
- (2) The force of limiting friction depends upon the nature & state of polish of the surfaces in contact & it acts tangentially to the interface betⁿ the two surfaces.
- (3) The magnitude of limiting friction F_L is directly proportional to magnitude of normal reaction R betⁿ the two surfaces in contact.

$$\text{i.e. } F_L \propto R \Rightarrow F_L = \mu R \Rightarrow \mu = \frac{F_L}{R}$$

where $\mu \rightarrow$ coefficient of limiting friction.

- (4) Magnitude of limiting friction is independent of the area & shape of the surface in contact, so long as normal reaction remains same.

Methods of Reducing Friction:-

- $\rightarrow$ By rubbing & polishing, friction force can be reduced.
- $\rightarrow$ By using lubricant on the surfaces in contact, friction may reduce.
- $\rightarrow$ If we convert sliding friction into rolling friction by streamlining the shape of the body, the fluid friction can be reduced.

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Unit 586 Gravitation Planetary Motion & S.A.M

Gravitation:- Whenever an object is released in free space, it falls towards the earth. It appears as if it appears that earth attracts everything towards it, which is called Gravitation.

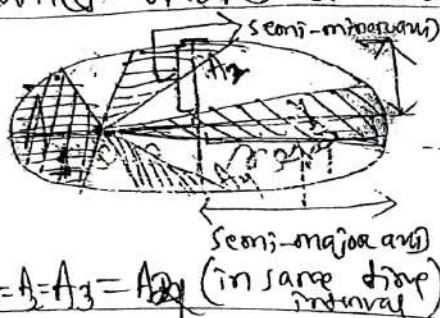
Kepler's Laws of Planetary Motion:-

1st Law:- (Law of elliptical orbits)

It states that "a planet moves around the sun in an elliptical orbit with sun situated at one of its foci."

2nd Law:- (Law of areal velocity)

It states that "a planet moves around the sun in such a way that its areal velocity remains constant."



$$\text{i.e. Areal velocity} = \text{constant}$$

3rd Law:- (The Harmonic law) $\rightarrow$ Law of Time period

It states that "a planet moves around the sun in such a way that the square of the time period is directly proportional to the cube of the semi-major axis of its elliptical orbit."

$$\text{i.e. } T^2 \propto R^3$$

where, $T \rightarrow$ Time period

$\&$ $R \rightarrow$ Semi-major axis

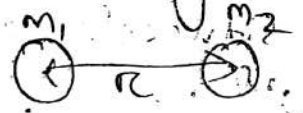
i.e. if the distance between the planet & sun is more, hence more time it will take to complete its rotation.

Newton's law of Gravitation:-

statement:- It states that "every particle of the matter in the universe attracts every other particle with a force which directly proportional to the product of the masses of the two particles & inversely proportional to the square of the distance between them".

→ The force of attraction between any two bodies in the universe is known as ~~grav~~ force of gravitation.

Let m_1 & m_2 → masses of two bodies



F → Force of attraction betⁿ them

r → distance betⁿ the two bodies

Then according to Newton's law, we have

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\Rightarrow \boxed{F = G \frac{m_1 m_2}{r^2}}$$

where G → a constant of proportionality called as Universal gravitational constant. & its value is same

If $m_1 = m_2 = 1$ unit & $r = 1$ unit

Then, $\boxed{F = G}$

Hence, the gravitational constant ' G ' is defined as the magnitude of the force of attraction between two ~~mass~~ bodies each of unit mass & separated by a unit distance from each other.

We know, $F = G \frac{m_1 m_2}{r^2}$

$$\Rightarrow G = \frac{FR^2}{m^2}$$

Hence in S.I. $\frac{\text{Newton} \times \text{m}^2}{\text{kg}^2}$

OR $\text{N} \cdot \text{m}^2 \text{kg}^{-2}$

& in C.G.S. $\text{Dyne} \cdot \text{cm}^2 \cdot \text{g}^{-2}$

Dimension

$$G = \frac{FR^2}{m^2}$$

$$\Rightarrow [G] = \frac{[M^1 L^1 T^{-2}][L^2]}{[M^2]}$$

$$= [M^{-1} L^3 T^{-2}]$$

Acceleration due to gravity

The accelⁿ produced by weight of a body is called as accelⁿ due to gravity & is denoted by 'g'.

i.e. $\text{Gravity (weight)} = mg$

If m & $M \rightarrow$ mass of a particle & earth respectively
 $R \rightarrow$ distance betⁿ the particle placed on the earth surface & centre of earth

Acc. to, Newton's law,

$$F = G \frac{Mm}{R^2} \Rightarrow mg = G \frac{Mm}{R^2}$$

$$\Rightarrow \boxed{g = G \frac{M}{R^2}}$$

Unit of g:

In S.I. $\rightarrow g = \text{m/s}^2$

In C.G.S. $\rightarrow g = \text{cm/s}^2$

Dimension of g

$$[g] = [M^0 L^1 T^{-2}]$$

Difference betⁿ G & g

G

- It is called as universal gravitational constant.
- Its value remains constant everywhere, so it is called as universal ^{const of} gravitation.

→ Its units are $\text{Nm}^2\text{kg}^{-2}$ (SI) & $\text{dyne cm}^2\text{g}^{-2}$ (CGS)

→ Its dimension is $[M^{-1}L^3T^{-2}]$

$$G = \frac{Fr^2}{m^2}$$

→ value of $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

Variation of 'g' with Altitude

Consider a body of mass 'm' placed on the surface of the earth.

Let M & $R \rightarrow$ mass & radius of earth respectively.

$g \rightarrow$ accⁿ due to gravity on the surface of earth.

m/m

Then, $g = \frac{GM}{R^2}$, where $G \rightarrow$ Gravitational constant.

If the body is taken to a height 'h' above the surface of earth, the accⁿ due to gravity at this height is 'g'.

$$\text{Then, } g' = \frac{GM}{(R+h)^2}$$

g

→ It is called as accⁿ due to gravity.

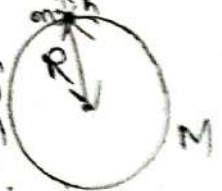
→ Its value changes at diff. place of earth as it depends upon mass & radius of the planet.

→ Its unit is same as accⁿ i.e. m/s^2 or cm/s^2

→ Its dimension is $[M^0L^1T^{-2}]$

$$g = G \frac{M}{R^2}$$

→ value of $g = 9.8 \text{ m/s}^2$



Now, $\frac{g'}{g} = \left[\frac{GM}{(R+h)^2} \right] / \frac{GM}{R^2}$

$$\Rightarrow \frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2 \left(1 + \frac{h}{R}\right)^2} = \frac{1}{\left(1 + \frac{h}{R}\right)^2} = \left(1 + \frac{h}{R}\right)^{-2}$$

$$\Rightarrow \frac{g'}{g} = 1 - \frac{2h}{R} \Rightarrow g' = g \left(1 - \frac{2h}{R}\right) = g - \frac{2gh}{R}$$

$$\Rightarrow g' - g = -\frac{2gh}{R} \Rightarrow \boxed{g - g' = \frac{2gh}{R}}$$

As g & $R \Rightarrow$ are constants at a given place on earth.

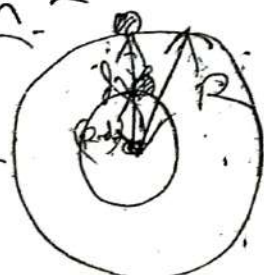
hence, $g - g' \propto h$

It is concluded that if h increases, g' will decrease to increase $(g - g')$ value as g is constant.

Thus the value of accelⁿ due to gravity ' g ' decreases with ~~height~~ in height above the earth surface.

Variation of ' g ' with depth:-

Let's consider the body of mass earth as a homogeneous sphere of radius ' R ', & mass ' M ' & density ' ρ '.



Let's consider a body lying on the surface of earth where the accelⁿ due to gravity is ' g ' & it is given by,

$$g = \frac{GM}{R^2}$$

We know, $\text{Mass} = \text{vol}^m \times \text{density}$

$$= \frac{4}{3} \pi R^3 \times \rho$$

(now) $g = \frac{G \left(\frac{4}{3} \pi R^3 \rho \right)}{R^2} = \frac{4}{3} \pi G R \rho$

Let the body taken to a depth 'd' below the earth's surface, where the accⁿ due to gravity is 'g'' is given by,

$$g' = \frac{4}{3} \pi G (R-d) \rho$$

Now, $g'/g = \frac{(4/3) \pi G (R-d) \rho}{(4/3) \pi G R \rho} = \frac{R-d}{R}$

$$\Rightarrow \frac{g'}{g} = 1 - \frac{d}{R} \Rightarrow g' = g - \frac{dg}{R}$$

$$\Rightarrow \boxed{g - g' = \frac{d}{R} g}$$

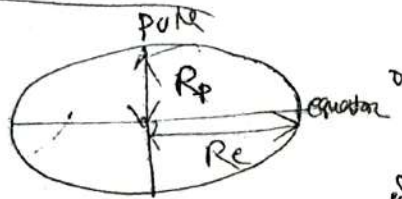
i.e. $(g - g')$ is

If depth 'd' increases, $(g - g')$ value will increase. But 'g' is constant on the surface, hence to increase $(g - g')$ value with increase of h, 'g'' value will decrease.

Thus the value of accⁿ due to gravity 'g' decreases with increase in depth.

Variation of 'g' with latitude:-

The value of "accⁿ" due to gravity changes with altitude due to shape of the earth...



As the shape of the earth is not a perfect sphere. i.e. flattened at poles & bulges out at the equator. Thus $R_e > R_p$.

$$As \quad g = \frac{GM}{R^2}$$

& G, M are constants

$$\text{Hence, } g \propto \frac{1}{R^2}$$

→ As at pole $R \rightarrow \text{min}$, hence g is ^{greater} at pole

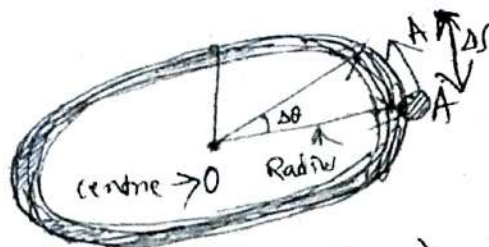
& as at equator $R \rightarrow \text{max}$, hence g is ^{least} at equator.

Circular Motion:-

Defⁿ :- The motion of a body is said to be circular if it moves in such a way, that its distance from a certain fixed ^{pt} always remains the same.

Uniform Circular Motion:-

Defⁿ :- Circular motion is said to be uniform if the speed of the particle along the circular path remains the same.



Angular displacement :- (θ)

Defn :- It is defined as the angle turned by the radius vector of the particle undergoing rotational motion.

If $\Delta s \rightarrow$ linear disp.

$r \rightarrow$ radius of the circular path.

$\Delta \theta \rightarrow$ angular disp.

Then,

$$\Delta s = r \times \Delta \theta$$

$$\text{or } \Delta \vec{s} = \vec{r} \times \Delta \theta$$

Angular velocity :- (ω)

Defn :- It is defined as the rate of change of angular disp. with time.

i.e. Angular velocity, $\omega = \frac{d\theta}{dt}$

$\therefore$ the relation betⁿ angular velocity & linear velocity is,

$$\vec{V} = \vec{r} \times \vec{\omega}$$

$$\text{or } \vec{V} = \vec{r} \times \vec{\omega}$$

where, $\omega \rightarrow$ Angular velocity

Angular acceleration :- (α)

Defn :- It is defined as the rate of change of angular velocity with time.

i.e. angular accelⁿ, $\alpha = \frac{d\omega}{dt}$

$\therefore$ relⁿ betⁿ angular accelⁿ, linear accelⁿ is,

$$\vec{a} = \vec{r} \times \vec{\alpha}$$

$$\text{or } \vec{a} = \vec{r} \times \vec{\alpha}$$

Simple Harmonic Motion (S.H.M):-

Defn:- The motion of a particle is said to be S.H.M if its acceleration is directly proportional to the disp. & is always directed towards the mean position. (Ex: vibration of simple pendulum, etc.)

The eqn for S.H.M is given by,

$$y = r \sin(\omega t + \phi)$$

where,

$y \rightarrow$ displacement.

$r \rightarrow$ amplitude of S.H.M.

$\omega \rightarrow$ angular velocity.

$\phi \rightarrow$ phase angle.

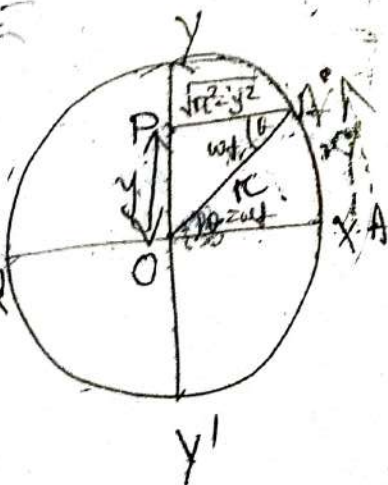
S.H.M parameters:-

(1) Amplitude:- Amplitude of a particle executing S.H.M is defined as its max displacement on either side of its mean position.

(2) Frequency:- The no. of oscillations made in unit time by the oscillating particle is called its frequency.

It is given by,

$$n = \frac{1}{T} = \frac{1}{\frac{2\pi}{\omega}} = \frac{\omega}{2\pi}$$
$$= \frac{1}{2\pi} \sqrt{\frac{a}{d}}$$



In the above fig, 'P' is the projection of particle 'A' at some instant of time.

$$\text{In } \triangle OAP, \sin \theta = \frac{OP}{OA} = \frac{y}{r}$$

$$\Rightarrow y = r \sin \theta \quad \text{--- (1)}$$

As, angular velocity, $\omega = \frac{\theta \text{ (angular disp)}}{t \text{ (time)}}$

$$\Rightarrow \theta = \omega t \quad \text{--- (2)}$$

Using (2) in (1) we get,

$$y = r \sin \omega t \quad \text{--- (3)}$$

Special case

Disp. y will be max^m when $\sin \omega t$ is max^m & min^m when $\sin \omega t$ is min^m.

Maximum value $y \rightarrow$ max^m if $\sin \omega t = 1$

$$\Rightarrow y = r \quad \left(\begin{array}{l} \text{radius / amplitude of} \\ \text{vibration} \end{array} \right)$$

Min^m value $y \rightarrow$ min^m if $\sin \omega t = -1$

$$\Rightarrow y = -r$$

(b) Velocity :— (V)

From the definition of velocity we know, velocity is the rate of change of disp.

$$\text{i.e. } v = \frac{dy}{dt}$$

But from (3), $y = r \sin \omega t$

Hence, $v = \frac{dy}{dt} = \frac{d}{dt} (r \sin \omega t)$

$$\Rightarrow v = r \cdot \frac{d}{dt} \sin \omega t$$

$$\Rightarrow v = r \cos \omega t \cdot \frac{d}{dt} (\omega t)$$

$$\Rightarrow v = r \cos \omega t \cdot \omega$$

$$\Rightarrow \boxed{v = (r \omega) \cos \omega t} \quad \text{--- (4)}$$

~~But linear velocity = radius $\times$ angular velocity~~

~~i.e. $v = r \omega$~~

~~Thus $v = \omega \cos \omega t$ --- (4)~~

In ΔOAP , $\cos \omega t = \frac{AP}{OA} = \frac{\sqrt{r^2 - y^2}}{r}$

Using this in (4) we get,

$$v = r \omega \frac{\sqrt{r^2 - y^2}}{r} \Rightarrow \boxed{v = \omega \sqrt{r^2 - y^2}}$$

Sp. case

(i) At 0, $y = 0 \Rightarrow v = \omega \sqrt{r^2} = \omega r = v$

(ii) At y/r , $y = r \Rightarrow v = \omega \sqrt{r^2 - r^2} \Rightarrow v = 0$

Thus a particle executing S.H.M, passes with maxⁿ velocity through the mean position and is at rest at the extreme position.

Acceleration :-

As accl^n is the rate of change of velocity,
 $\Rightarrow \text{accl}^n = \frac{dv}{dt}$

$$\Rightarrow a = \frac{d}{dt} (\text{true const}) \quad [\text{using (4)}]$$

$$= r \omega \frac{d}{dt} (\text{const})$$

$$= r \omega (\sin \omega t) \frac{d}{dt} (\omega t)$$

$$= r \omega^2 \sin \omega t \quad \text{--- (5)}$$

In the Δ OAP, $\sin \omega t = \frac{y}{r}$

Using this in (5) we get, T

$$a = -r \omega^2 \cdot \frac{y}{r}$$

$$\Rightarrow \boxed{a = -\omega^2 y}$$

Special case :-

(i) At 0, $y = 0 \Rightarrow a = 0$

(ii) At $y/y', y = \pm r \Rightarrow a = \pm \omega^2 r$

Thus a particle vibrating in S.H.M has zero accl^n while passing through mean position and has max^n acceleration while at extreme position.

(d) Time Period :- (T)

It is the time taken by the particle to complete one ~~rotation~~ oscillation.

It is given by, $T = \frac{2\pi}{\omega}$

where, $\omega \rightarrow$ Angular velocity.

But we know, $a_{\text{c}} = -\omega^2 y$

$\Rightarrow \frac{a_{\text{c}}}{y} = -\omega^2$

$\omega = \sqrt{\frac{a_{\text{c}}}{y}}$

$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{y}{a_{\text{c}}}}$

$\Rightarrow T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

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HEAT & THERMODYNAMICS

Heat → It is a form of energy which produces the sensation of warmth.

Thermodynamics → Thermo + Dynamics
↓ ↓
Means heat Mechanical motion involving work done

Hence the chapter heat & thermodynamics gives the idea about heat & the work done in a system due to motion of heat in it.

Thermal Expansion of solid:-

Thermal Expansion:-

When an object heats, whether it is a solid, liquid or gas, it expands. This expansion in the object due to increase in the temp^{re} is called as thermal expansion.

In case of solid there are 3 types of expansion. Such as:-

① Linear Expansion. (Expansion through length i.e. 1-d expansion)

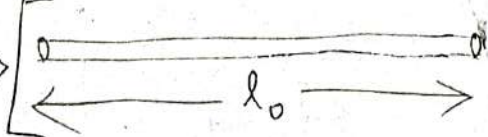
② Superficial / Areal Expansion. (Expansion through area i.e. 2-d expansion)

③ Cubical Expansion. (Expansion through volume i.e. 3-d expansion)

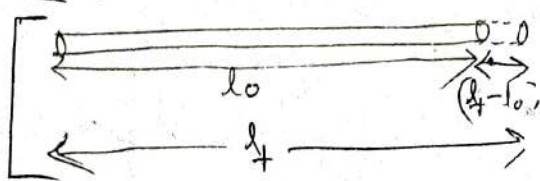
① Linear Expansion:-

Let's consider a long & thin rod, whose length is very large in comparison to its diameter.

Let $l_0 \rightarrow$ Original length at $0^\circ\text{C} \rightarrow$



& $l_t \rightarrow$ Final length at $t^\circ\text{C} \rightarrow$



Then the change in length $= (l_t - l_0)$

two
upon

This increase in length $(l_t - l_0)$ depends upon two factors i.e. upon original length l_0 & upon rise of temp $^{\circ}(t)$ i.e.

$$(l_t - l_0) \propto l_0$$

$$\propto t$$

$$\Rightarrow (l_t - l_0) \propto l_0 t$$

$$\Rightarrow \boxed{l_t - l_0 = \alpha l_0 t} \quad \text{--- (1)}$$

where $\alpha \rightarrow$ constant of proportionality known as co-efficient of linear expansion

Now from (1), $\alpha = \frac{l_t - l_0}{l_0 t}$ --- (2)

$$\& \quad l_t = l_0 + \alpha l_0 t$$

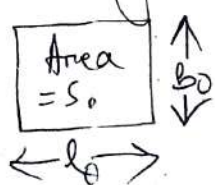
$$\Rightarrow \boxed{l_t = l_0 (1 + \alpha t)} \quad \text{--- (3)}$$

(3) C

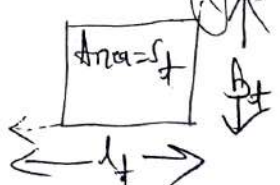
(2) Superficial / Areal Expansion:-

Let's consider a thin sheet, which having some length & breadth but negligible thickness.

At $0^\circ\text{C} \rightarrow$



At $t^\circ\text{C} \rightarrow$



At 0°
Value

Let $s_0 \rightarrow$ original area of the sheet at 0°C

$s_t \rightarrow$ final area of the sheet at $t^\circ\text{C}$

Then change in area $= (s_t - s_0)$

& this increase in area is depending upon two factors i.e. upon original area and upon rise in temp^r.

i.e. $(s_t - s_0) \propto s_0$
 $\propto t$

$$\Rightarrow (s_t - s_0) \propto s_0 t$$

$$\Rightarrow \boxed{s_t - s_0 = \beta s_0 t} \quad \text{--- (4)}$$

where $\beta \Rightarrow$ constant of proportionality known as coefficient of superficial expansion

Now from (4), $\boxed{\beta = \frac{s_t - s_0}{s_0 t}} \quad \text{--- (5)}$

$$\& \quad s_t = s_0 + \beta s_0 t$$

$$\Rightarrow \boxed{s_t = s_0 (1 + \beta t)} \quad \text{--- (6)}$$

(3) Cubical Expansion:-

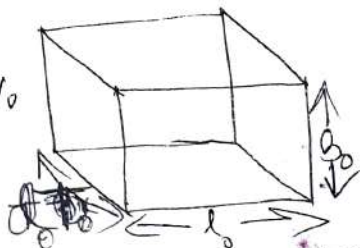
Let's consider a cube which having some specified length, breadth & thickness.

Let $V_0 \rightarrow$ ^{original} Volume of the cube at 0°C

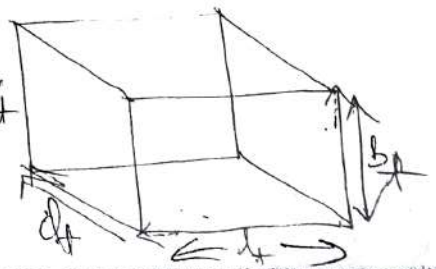
$V_t \rightarrow$ Final volume of the cube at $t^\circ\text{C}$

which neglect

At 0°C
Volume $= V_0$



At $t^\circ\text{C}$
Volume $= V_t$



Then the change in volume = $V_t - V_0$
 The increase in volume depends upon two factors i.e. upon original volume and upon the change in temp.
 i.e. $(V_t - V_0) \propto V_0$
 $\propto t$

$$\Rightarrow (V_t - V_0) \propto V_0 t$$

$$\Rightarrow \boxed{V_t - V_0 = \gamma V_0 t} \quad \text{--- (7)}$$

Hence $\gamma \rightarrow$ constant of proportionality known as co-efficient of cubical expansion

From (7), $\boxed{\gamma = \frac{V_t - V_0}{V_0 t}} \quad \text{--- (8)}$

$$\& V_t = V_0 + \gamma V_0 t$$

$$\Rightarrow \boxed{V_t = V_0 (1 + \gamma t)} \quad \text{--- (9)}$$

Relation between α , β & γ :-

Relation betⁿ α & β :-

We know, $\beta = \frac{l_t - l_0}{l_0 t} \quad \text{--- (10)}$

Again area = length² $\rightarrow$
 i.e. at $0^\circ\text{C} \rightarrow \text{area}, S_0 = l_0^2$
 & at $t^\circ\text{C} \rightarrow \text{area}, S_t = l_t^2$

Putting this in equⁿ (10) we get,

$$\beta = \frac{l_t^2 - l_0^2}{l_0^2 t}$$

As from eqn (2) we have, $l_t = l_0(1 + \alpha t)$, so putting this in above we get,

$$\begin{aligned} \beta &= \frac{[l_0(1 + \alpha t)]^2 - l_0^2}{l_0^2 t} = \frac{l_0^2(1 + \alpha t)^2 - l_0^2}{l_0^2 t} \\ &= \frac{\cancel{l_0^2} [(1 + \alpha t)^2 - 1]}{\cancel{l_0^2} t} = \frac{(1 + \alpha t)^2 - 1}{t} \\ &= \frac{1 + 2\alpha t + \alpha^2 t^2 - 1}{t} = \frac{2\alpha t + \alpha^2 t^2}{t} \\ &= \frac{t(\alpha^2 t + 2\alpha)}{t} = \alpha^2 t + 2\alpha \end{aligned}$$

Neglecting the higher ^{ordered} term of α we get,

$$\boxed{\beta = 2\alpha} \quad \text{--- (11)}$$

Relation betⁿ α & γ :-

$$\text{We know, } \gamma = \frac{V_t - V_0}{V_0 t} \quad \text{--- (12)}$$

Again, volume of a cube = (length)³
 i.e. " " " " at 0°C, $V_0 = l_0^3$
 & " " " " " t°C, $V_t = l_t^3$.

Putting the value of V_0 & V_t in (12) we get,

$$\gamma = \frac{l_t^3 - l_0^3}{l_0^3 t}$$

As from eqn (2) we have, $l_t = l_0(1 + \alpha t)$, so putting this in above eqn we get,

$$r = \frac{[l_0(1+\alpha t)]^3 - l_0^3}{l_0^3 t} = \frac{l_0^3(1+\alpha t)^3 - l_0^3}{l_0^3 t}$$

$$\Rightarrow r = \frac{l_0^3 [(1+\alpha t)^3 - 1]}{l_0^3 t} = \frac{(1+\alpha t)^3 - 1}{t}$$

$$\Rightarrow r = \frac{1 + \alpha^3 t^3 + 3\alpha t + 3\alpha^2 t^2 - 1}{t}$$

$$\Rightarrow r = \frac{\alpha^3 t^3 + 3\alpha t + 3\alpha^2 t^2}{t} = \frac{\alpha^3 t^2 + 3\alpha + 3\alpha^2 t}{1}$$

$$\Rightarrow r = \alpha^3 t^2 + 3\alpha + 3\alpha^2 t$$

As α is very small, hence neglecting the higher ordered term of α we get,

$$\boxed{r = 3\alpha} \quad \text{--- (13)}$$

From eqn (11) & (13) we get,

$$\beta = 2\alpha \quad \& \quad r = 3\alpha$$

$$\text{i.e.} \quad \boxed{\alpha = \frac{\beta}{2} = \frac{r}{3}} \quad \text{--- (14)}$$

First Law of Thermodynamics :-

Statement:- It states that "if the quantity of heat supplied to a system is capable of doing some work, then the quantity of heat ^{absorbed} supplied by the system is equal to the sum of the increase in the internal energy of the system & external work done by it".

i.e. $\boxed{dQ = dU + dW}$

Where $dQ \rightarrow$ Amount of heat absorbed
 $dU \rightarrow$ change in internal energy.
 $dW \rightarrow$ External work done.

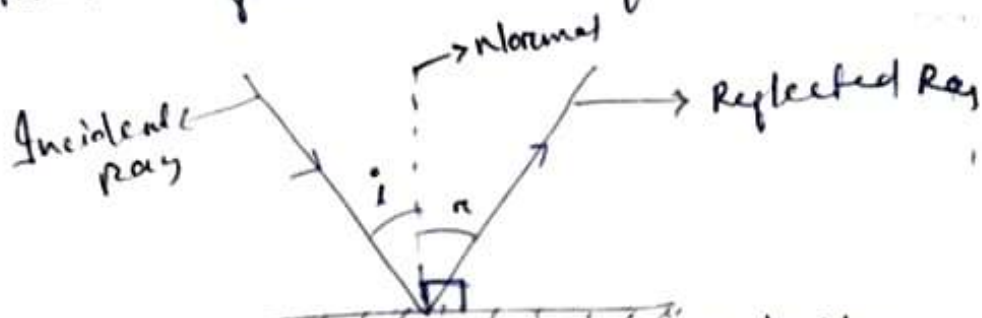
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Optics

Optics $\rightarrow$ Geometrical Optics / Ray optics (Particle nature)
 $\rightarrow$ Wave optics / Physical optics (Wave nature)
Study of properties & behaviour of light.

Reflection $\rightarrow$

The phenomenon in which light travelling from one medium to another returns back to the same medium from the interface.



$\angle i$ = angle of incidence
 $\angle r$ = angle of reflection
O $\rightarrow$ point of incidence

Laws $\rightarrow$

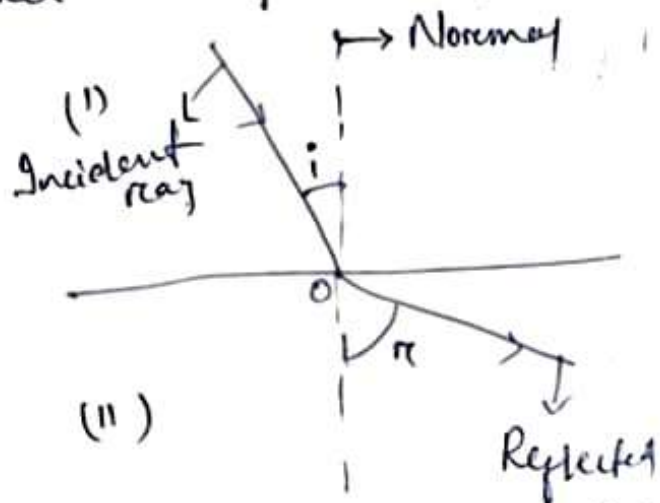
- (i) Incident ray, Reflected ray, normal, all lie in one plane and that plane is $\perp$ to the interface.
- (ii) $\angle i = \angle r$

Refraction $\rightarrow$

The phenomenon in which light travelling from one medium to another changes its path or bends.

$\angle i$ = angle of incidence

$\angle r$ = angle of refraction



Laws →

(i) ~~Reflected~~ Incident ray, Refracted ray, normal, all lie in one plane and that plane is $\perp$ to the interface.

(ii) $\frac{\sin i}{\sin r} = \text{constant} \rightarrow \text{Snell's law.}$

$\Rightarrow \frac{\sin i}{\sin r} = {}^1\mu_2 \rightarrow \text{Refractive index of}$
 $2^{\text{nd}} \text{ medium w.r.t } 1^{\text{st}} \text{ medium}$

Refractive Index

→ Property of medium, that decides how to what extent the direction of light will change in a medium. i.e. the speed of light in the medium.

→ R.I. measures the optical density of the medium.

→ R.I. can be related to relative speed of propagation of light in a medium.

$$\text{Relative speed} = \frac{v}{c}$$

→ $\frac{\sin i}{\sin r} = \mu_2 = \mu_{21}$ → R.I. of 2nd medium w.r.t 1st medium

$$\mu_{21} > 1, \quad \sin i > \sin r$$

$$\Rightarrow i > r$$

i.e. the ray bends towards the normal.
if medium 2 is optically denser.

$$\mu_{21} < 1, \quad \sin i < \sin r \Rightarrow i < r$$

i.e. Ray moves away from normal.

→ Optical density $\neq$ mass density.

→ It is the measure of absorbance.
↓
More optically denser, light will travel slower.

→ R.I. indicates the no. of times light will be slower in a medium than its in vacuum.

$$\mu_{21} = \frac{1}{\mu_{12}}$$

→ R.I. is related to relative speed of light in diff. media.

$$\mu = \frac{c}{v} \rightarrow \text{Absolute R.I.}$$

$$\mu_2 = \frac{v_1}{v_2}$$

$$\frac{\sin i}{\sin r} = \mu_2 \Rightarrow \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\frac{c}{\mu_1}}{\frac{c}{\mu_2}} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$$

→ higher R.I. → Higher optical density.
 slower speed of light.
 smaller R.I. → optically rarer.

→ R.I. is unitless and dimensionless.

→ Q. R.I. of water w.r.t. air = ${}^a\mu_{aw} = \frac{4}{3}$
 R.I. of glass = ${}^a\mu_{ag} = \frac{3}{2}$
 R.I. of glass w.r.t. water.

$$= {}^w\mu_{ag} = \frac{{}^a\mu_{ag}}{{}^a\mu_{aw}} = \frac{{}^a\mu_{ag}}{{}^a\mu_{aw}} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

$\angle i = 30^\circ$
 $\mu_g = \frac{3}{2}$, $\mu_w = \frac{4}{3}$
 $\angle r = ?$

$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_w} = \frac{\frac{3}{2}}{\frac{4}{3}}$$

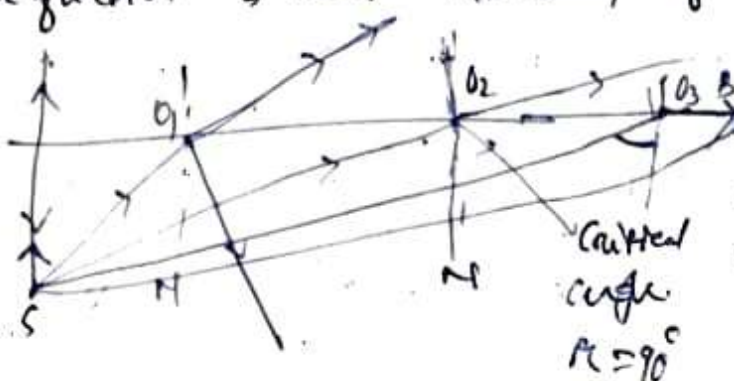
$\sin 30^\circ = \frac{4}{8} \sin r$

$\sin r = \frac{8}{4} \times \frac{1}{2} = \frac{4}{4}$

$\Rightarrow r = \sin^{-1}(1)$

Total Internal Reflection →

When light travels from optically denser medium to optically rarer medium, at the interface, it is partly reflected back to the same medium and partly gets refracted. This reflection is called internal reflection.



Critical angle
 $\theta_c = 90^\circ$

→ At a point when the angle of incidence is further increased, no refraction possible and the incident ray will be totally reflected. It is called total internal reflection.

→ In total internal reflection no transmission of light takes place.

→ The angle of incidence corresponding to the angle of refraction $= 90^\circ$, is called critical angle.

→ If, $n_2 < n_1$, then $\frac{\sin i}{\sin 90^\circ} = \sin i_c$

~~$\Rightarrow \frac{\sin i}{\sin r} < 1$~~

$\Rightarrow \sin i_c = n_2$

$\Rightarrow \sin i < \sin r$

→ As angle of incidence increases, angle of refraction increases.

$$i_c = \sin^{-1} \frac{n_2}{n_1}$$

And max^m value of $\sin \theta = 1$, so, n_2 can not exceed n_1 . So, total internal reflection can not occur when light travels from rarer to denser medium.

2 conditions for T.I.R.

- ① Must travel from denser to rarer medium.
- ② Angle of incidence must be greater than critical angle.

Relate betⁿ i_c and R.I. → .

Snell's law $\frac{\sin i}{\sin r} = n_2 \Rightarrow \frac{\sin i_c}{\sin 90^\circ} = n_2 \Rightarrow \sin i_c = n_2$

For angle of incidence = C , angle of refraction = 90°
 By Snell's Law, R.I. of 1st medium w.r.t. 2nd
 medium i.e. ${}^2\mu_1 =$

$${}^1\mu_2 = \frac{\sin i}{\sin r} = \frac{\sin C}{\sin 90^\circ} = \frac{\sin C}{1} = \sin C$$

$$\Rightarrow \frac{1}{\mu_2} = \frac{1}{\sin C}$$

$$\Rightarrow {}^2\mu_1 = \frac{1}{\sin C}$$

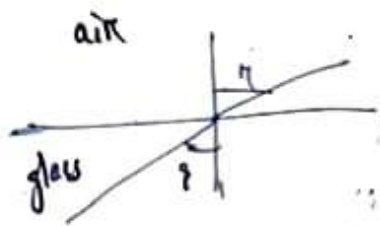
$\therefore$ 2nd medium is air,

$${}^2\mu_1 = \mu_1$$

So,

$$\boxed{\mu_1 = \frac{1}{\sin C}}$$

$\rightarrow$ i.e. Absolute R.I. of a medium is equal to reciprocal of the sine of the critical angle for that medium.



The critical angle of incidence of a glass slab immersed in air is 30° . What will be the critical angle when it is immersed in a medium of R.I. $\sqrt{2}$

Soln

$$\sin C = \frac{1}{{}^a\mu_g} \Rightarrow \sin 30^\circ = \frac{1}{{}^a\mu_g}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{{}^a\mu_g}$$

$$\Rightarrow {}^a\mu_g = 2$$

$$\sin C_m = \frac{1}{{}^m\mu_g} = \frac{1}{{}^m\mu_a \times {}^a\mu_g} = \frac{{}^a\mu_m}{{}^a\mu_g}$$

$$= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow C_m = \sin^{-1} \frac{1}{\sqrt{2}} = 45^\circ$$

What is the critical angle for a ray going from glass to water? The R.I. of glass and water are 1.62 & 1.32.

Soln :-

$$\sin C = \frac{1}{\mu_g} = \frac{\mu_w}{\mu_g} = \frac{1.32}{1.62}$$

R.I. of material of prism $\rightarrow$

$$n = \frac{\sin\left(\frac{A+S_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Optical Fiber :-

- $\rightarrow$ They work on the principle of TIR.
- $\rightarrow$ Cylindrical wave guide consisting of two layers. core & outside cladding. R.I. of core is higher.
- $\rightarrow$ It's made of high quality ^{transparent} glass or plastic. Successive internal reflection.
- $\rightarrow$ Study of optical fibers is called fiber optics.
- $\rightarrow$ No appreciable loss of ~~energy~~ intensity of light signal.

Properties

- $\rightarrow$ It has higher ^{Can contain larger variety of frequency} band width. So, optical fibers have higher information carrying capacity. and permit transmission over long distances than electrical cables.
- $\rightarrow$ Small in size, light weight, have high tensile strength so flexible. They can be bent or twisted easily.
- $\rightarrow$ It has high degree of signal security. As
- $\rightarrow$ It's free from electrical interference.
- $\rightarrow$ They do not carry high voltage or current. So, they are safer than electrical cable.

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889 Current Electricity & Electromagnetism

Kirchoff's Laws:-

① Kirchoff's Current Law (KCL):-

Statement:- It states that "the algebraic sum of all the currents meeting at an electrical junction/node is equal to zero."
 or Sum of currents entering is equal to leaving at a junction.

i.e

$$\sum_n I_n = 0$$

Sign Convention:- To determine the algebraic sum of currents we have to follow the sign convention given below. Such as:-

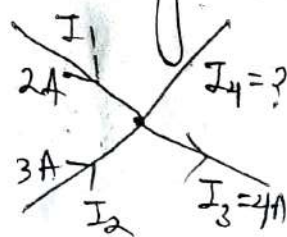
- The current entering at a given junctⁿ is taken as +ve.
- The current leaving " " " " " " taken as -ve.

Explanation:- Let I_1, I_2, I_3, I_4 are the four currents meeting at a junction where I_1, I_2 are the entering currents & I_3 leaving the junction. We have no idea about I_4 i.e. it is entering or leaving.

According to KCL, $\sum I = 0$

$$\Rightarrow I_1 + I_2 + I_3 + I_4 = 0 \Rightarrow I_4 = -I_1 - I_2 - I_3$$

Taking sign convention in account & putting the values of I_1, I_2, I_3 we get



$$\Rightarrow I_4 = -(2A) - (3A) - (-4A)$$

$$\Rightarrow I_4 = -5A + 4A = -1A$$

i.e. $I_4 = -1A$ & this -ve sign indicating that I_4 current is leaving the junction.

Hence, $I_1 + I_2 = I_3 + I_4$ [$\because I_1 + I_2 = 5A$
 $I_3 + I_4 = 5A$]

i.e. sum of entering currents = sum of leaving currents

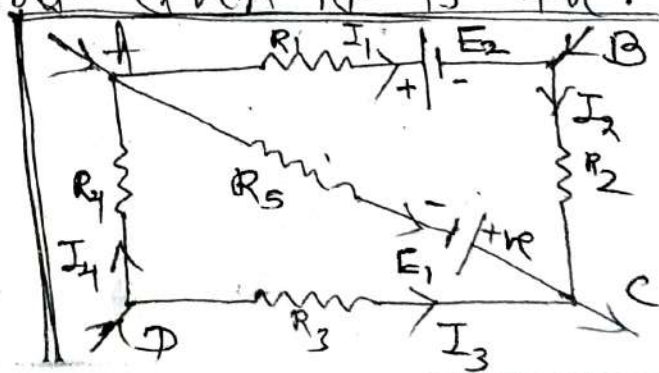
Kirchoff's Voltage Law (KVL):-

Statement:- It states that "in a closed loop the algebraic sum of e.m.f. ~~voltage~~ ~~equal to~~ & product of resistance & current is equal to zero

i.e. $\boxed{\sum_k V_k + I_k R = 0}$

Sign convention:-

- If the current is flowing from -ve to +ve then the e.m.f. is taken as +ve.
- If the current is flowing from +ve to -ve then the e.m.f. is taken as -ve.
- If the voltage is along the dirⁿ of current then it is -ve & if the voltage is in the opposite dirⁿ to current then it is +ve.
- Resistance has no sign convention.



Explanation:- In the given closed circuit ABCD, R_1, R_2, R_3, R_4, R_5 are the resistances & I_1, I_2, I_3, I_4, I_5 are the currents in the five arms of the circuit, with two e.m.f. sources E_1 & E_2 .

Applying KVL to mesh/loop ABC we get,

$$I_1 R_1 + I_2 R_2 - I_5 R_5 = E_1 - E_2 \quad \text{--- (1)}$$

& in mesh/loop ACD, ~~we~~ we get. F: $I_1 \rightarrow +ve$
 $I_5 \rightarrow -ve$

$$I_5 R_5 - I_3 R_3 - I_4 R_4 = E_1 \quad \text{--- (2)}$$

In general from (1) & (2) we get,

$$\boxed{\sum IR + \sum E = 0}$$

Application of Kirchhoff's law to Wheatstone bridge:-

→ Wheatstone bridge is an electrical arrangement used to determine the value of an unknown resistance in an electrical circuit.

→ Let's consider a wheatstone bridge ABCD consisting of 4 resistors P, Q, R, S ^{in four arms} and an e.m.f. source 'E' connected betⁿ A & C.

~~through~~ → Again one galvanometer 'G' is connected between the terminals B & D, as shown in the given fig.

→ After closing the circuit the resistances P, Q, R, S are so adjusted that the galvanometer shows no deflection, hence the wheatstone bridge is in balanced condition.

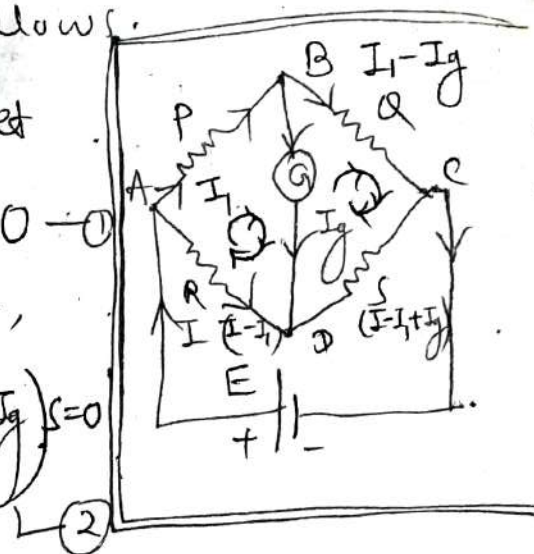
→ Now Using KCL, the current distribution in the circuit is as follows.

In the mesh ABD, we get

$$I_1 P + I_g R - (I - I_1) R = 0 \quad \text{--- (1)}$$

8. In the mesh BCD, we get,

$$(I_1 - I_g) Q - I_g R - (I - I_1 + I_g) S = 0$$



→ The R.H.S of both eqn's are zero as there is no e.m.f in the closed ckt. ABCD

→ Since the bridge is balanced, so there is no fluctuation current through galvanometer i.e. $I_g = 0$ --- (3)

Using (3) in (1) & (2) we get,

$$I_1 P - (I - I_1) R = 0 \Rightarrow I_1 P = (I - I_1) R \quad \text{--- (4)}$$

$$I_1 Q - (I - I_1) S = 0 \Rightarrow I_1 Q = (I - I_1) S \quad \text{--- (5)}$$

Dividing (4) by (5) we get,

$$\frac{I_1 P}{I_1 Q} = \frac{(I - I_1) R}{(I - I_1) S}$$

$$\Rightarrow \boxed{\frac{P}{Q} = \frac{R}{S}} \quad \text{--- (4)}$$

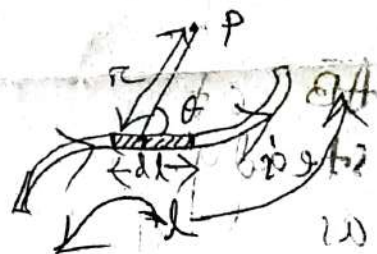
This is the required ~~balance~~ condition for which wheatstone bridge is said to be balanced.

Biot-Savart's Law:-

~~As we all know a current carrying conductor~~

Statement:- It states that "the differential contribution (dB) of a magnetic field around a wire of length ' l ' carrying current ' i ' at any point is directly proportional to amount of current flow, sine of the angle betⁿ posⁿ vector and current, length of the differential portion but inversely proportional to square of the distance betⁿ observation point & wire.

$$\begin{aligned} \text{i.e. } dB &\propto i \\ &\propto dl \\ &\propto \sin\theta \\ &\propto \frac{1}{r^2} \end{aligned}$$



$$\Rightarrow dB \propto \frac{idl \sin\theta}{r^2} \Rightarrow dB = \mu \frac{idl \sin\theta}{r^2} \quad \text{--- (1)}$$

Where $\mu \rightarrow$ constant of proportionality and
in S.I system $\mu = \frac{\mu_0}{4\pi}$ & in cgs $\mu = 1$.

Now, Using $\mu = \frac{\mu_0}{4\pi}$ in (1) we get,

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{idl \sin\theta}{r^2}}$$

Special case:-

Case-1:- If $\theta = 0^\circ/180^\circ \Rightarrow \sin\theta = 0$

Hence, $|dB| = 0 \rightarrow$ which is the min^m value
i.e there is zero magnetic field intensity at any pt.

on the axial line.

Case-2:- If $\theta = 90^\circ \Rightarrow \sin\theta = 1$

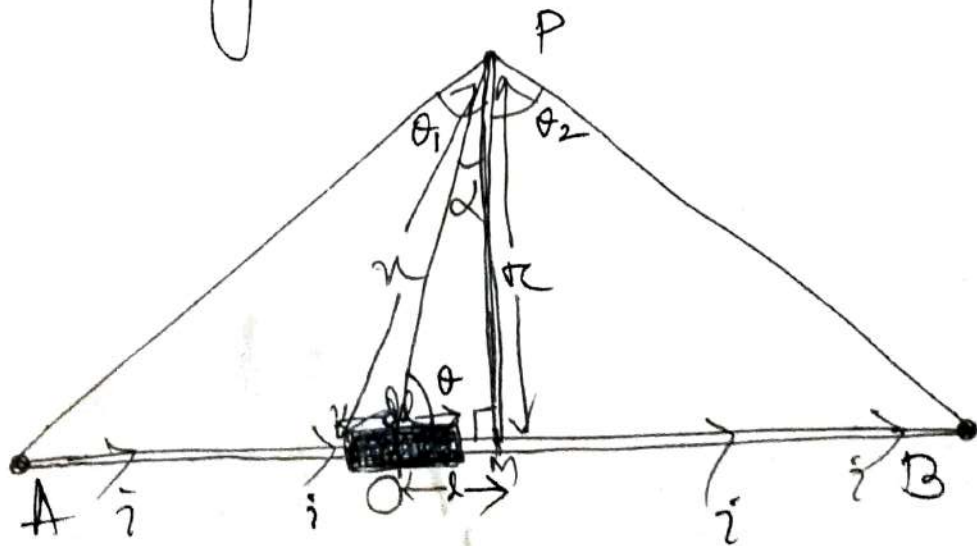
Hence, $dB = \frac{\mu_0}{4\pi} \frac{idl}{r^2} \rightarrow$ maximum value.

i.e. the magnetic field intensity at any point lying on a line $\perp$ to the length of the wire is maximum.

Applications of Biot-Savart's law:-

(1) Magnetic field intensity due to a straight conductor carrying current:-

Let's consider a long straight conductor ~~AB~~ of finite length which is carrying a steady current ' i ' in dirⁿ A to B as shown in the fig. below. Now taking the small element dl of ~~AB~~ the wire ~~AB~~ & let's find out the magnetic field intensity using Biot-Savart's law.



→ Let ~~the~~ have to find out the magnetic field intensity at point 'p' which is at a distance 'u' from the small element 'dl' such that 'p' is at a distance 'r' on the line $\perp$ to ~~the~~ ^{which} centre of $\overline{AB}$.

→ According to Biot-Savart's law for the small element 'dl' we can write

$$dB = \frac{\mu_0}{4\pi} \frac{i dl \sin\theta}{r^2} \quad \text{--- (1)}$$

→ Now let's find out the value of $\sin\theta$, r & dl .

In the ~~triangle~~ ^{triangle} right angle $\triangle OMP$,

$$\alpha + \theta = 90^\circ \Rightarrow \theta = 90^\circ - \alpha \Rightarrow \sin\theta = \sin(90^\circ - \alpha) = \cos\alpha \quad \text{--- (2)}$$

& in the same triangle $\triangle OMP$,

$$\cos\alpha = \frac{MP}{OP} = \frac{r}{u} \Rightarrow u = \frac{r}{\cos\alpha} \Rightarrow u^2 = \frac{r^2}{\cos^2\alpha} \quad \text{--- (3)}$$

& in this triangle, $\tan\alpha = \frac{OM}{PM} = \frac{l}{r}$

$$\Rightarrow l = r \tan\alpha \Rightarrow dl = r \sec^2\alpha d\alpha \quad \text{--- (4)}$$

Using (2), (3) & (4) in eqn (1) we get,

$$\begin{aligned} dB &= \frac{\mu_0}{4\pi} \frac{i (r \sec^2\alpha d\alpha) \cos\alpha}{r^2 / \cos^2\alpha} \\ &= \frac{\mu_0}{4\pi} \frac{i}{r} \cos\alpha d\alpha \quad \text{--- (5)} \end{aligned}$$

Now the net magnetic field intensity at pt. 'p' due to total length of the conductor

is given by, $B = \int dB = \int_0^{\theta_2} \frac{\mu_0}{4\pi} \frac{i}{r} \cos \alpha d\alpha$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{i}{r} \int_{\theta_1}^{\theta_2} \cos \alpha d\alpha$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{i}{r} \left[\sin \alpha \right]_{\theta_1}^{\theta_2}$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{i}{r} (\sin \theta_2 - \sin \theta_1) \quad \text{--- (6)}$$

2) This is the ^{total} magnetic field intensity for a finite length of conductor.
For Infinitely long conductor:-

(1) In this case, the angles

$$\theta_1 \rightarrow -90^\circ \text{ \& } \theta_2 \rightarrow 90^\circ$$

Putting the value of θ_1 & θ_2 in (6) we get,

$$B = \frac{\mu_0}{4\pi} \frac{i}{r} \{ \sin 90^\circ - \sin(-90^\circ) \}$$

$$= \frac{\mu_0}{4\pi} \frac{i}{r} (\sin 90^\circ + \sin 90^\circ)$$

$$= \frac{\mu_0}{4\pi} \frac{i}{r} 2 \sin 90^\circ$$

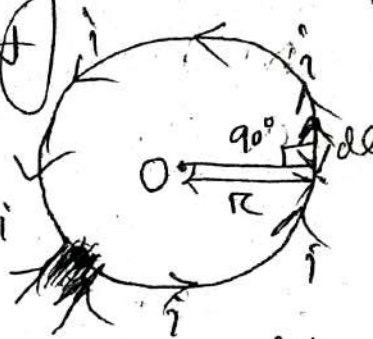
$$= \frac{\mu_0}{4\pi} \frac{i}{r} (2 \times 1) = \frac{\mu_0}{4\pi} \frac{i}{r} \cdot 2$$

$$\Rightarrow \boxed{B = \frac{\mu_0}{2\pi} \frac{i}{r}}$$

Magnetic Field at the centre of a Circular coil

Let's consider a circular coil of radius r carrying current ' i ' & having centre at 'O'.

Let's consider a small element dl of the circular coil & find out the magnetic field intensity at the centre 'O' of the coil (which is at a distance ' r ' from dl & making an angle $\theta = 90^\circ$ with current flow direction through ' dl ').



Now according to Biot-Savart's law,

$$|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{i dl \sin\theta}{r^2}$$

But here $\theta = 90^\circ \Rightarrow |d\vec{B}| = \frac{\mu_0}{4\pi} \frac{i dl \sin 90^\circ}{r^2}$

$$\Rightarrow |d\vec{B}| = \frac{\mu_0}{4\pi} \frac{i dl}{r^2}$$

Now the net magnetic field intensity at 'O' due to total length of the circular coil is given by,

$$B = \int_0^B dB = \int \frac{\mu_0}{4\pi} \frac{i dl}{r^2} = \frac{\mu_0}{4\pi} \frac{i}{r^2} \int_0^l dl$$

$$= \frac{\mu_0}{4\pi} \frac{i}{r^2} \times 2\pi r$$

$$= \frac{\mu_0}{4\pi} \frac{2\pi i}{r}$$

$\because \int dl = \text{circumference of the circular coil}$
 $\therefore dl = 2\pi r$

$$\Rightarrow B = \frac{\mu_0 i}{2r}$$

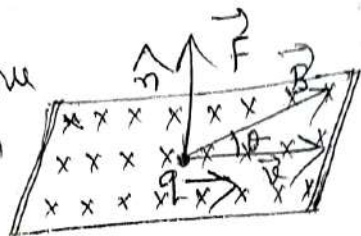
For 'N' no. of turns in coil,

$$B = N \frac{\mu_0 i}{2\pi r}$$

Motion of a charged particle inside a uniform magnetic field:-

Let's consider a charge (q) moving inside a uniform magnetic field ($\vec{B}$) with a velocity $\vec{v}$ such that the dirⁿ of motion of charge makes an angle θ with the dirⁿ of magnetic field as shown in the fig below.

Then the charge ' q ' will experience a force ' $\vec{F}$ ' given by,



$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\Rightarrow \vec{F} = qvB \sin\theta \hat{n}$$

— (1)

$$\& |\vec{F}| = qvB \sin\theta$$

Special case:-

→ Case-1:- If $\theta = 0^\circ/180^\circ \Rightarrow \sin\theta = 0 \Rightarrow |\vec{F}| = 0$
i.e. no force experienced by a charge moving along a line parallel/antiparallel to the dirⁿ of magnetic field.

→ Case-2:- If $\theta = 90^\circ \Rightarrow \sin\theta = 1 \Rightarrow |\vec{F}| = qvB$
i.e. maximum force experienced by a charge particle while moving at 1^{st} dirⁿ to magnetic field.

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19. Electro-Magnetic Induction

Electromagnetic Induction:-

The phenomenon of production/change in electricity due to magnetism is called as electro-magnetic induction. i.e. electricity is induced due to magnetic field.

Faraday's Laws of Electromagnetic Induction:-

There are ~~three~~ two laws such that:-

① Whenever magnetic flux linked with the circuit changes an e.m.f (electromotive force) is induced in it. and

② The induced e.m.f is such that it maintains the circuit so long as the change in magnetic flux linked with it continues.

③ The induced e.m.f is equal to the -ve rate of change of magnetic flux linked with the circuit.

i.e. e.m.f,
$$E = - \frac{d\Phi_B}{dt}$$

Where, $E \rightarrow$ e.m.f or electromotive force,

$\Phi_B \rightarrow$ magnetic flux linked with the circuit.

$\frac{d\Phi_B}{dt} \rightarrow$ Rate of change of magnetic flux

-ve sign $\rightarrow$ Indicating that e.m.f $\propto -\frac{d\Phi_B}{dt}$ tends to oppose the change in ~~flux~~ ~~current~~

If 'N' $\rightarrow$ no. of coils linked with the,
 then,
$$E = -N \frac{d\Phi_B}{dt}$$

NOTE:-

Electromotive Force:- (e.m.f)

Electromotive force is not a force actually but a potential ore voltage measured in volt i.e it is the energy per unit charge.

i.e $E = \frac{\text{work}}{\text{charge}} = \frac{W}{Q} = \frac{\text{Joule}}{\text{coulomb}} = \text{volt}.$

Hence e.m.f is the energy supplied to an electron charge.

Magnetic Flux:- (Φ_B)

The magnetic flux linked with an electric circuit is given by,

$$\Phi_B = \vec{B} \cdot \vec{A}$$

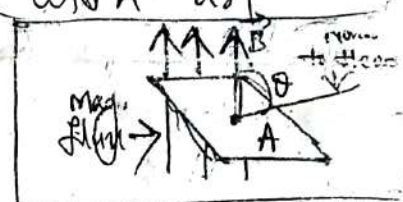
$$\Rightarrow \Phi_B = BA \cos \theta \text{ and for 'N' no. of turns,}$$

where, $\Phi_B \rightarrow$ magnetic flux.

$B \rightarrow$ magnetic field intensity.

$A \rightarrow$ Area of the electric circuit or the area of the conductor/loop.

$\theta \rightarrow$ Angle betⁿ the magnetic field and normal to the area of the conductor.



$$\Phi_B = NBA \cos \theta$$

case-1

$$\text{If } \theta = 0^\circ \Rightarrow \Phi_B = BA$$

$$\text{If } \theta = 180^\circ \Rightarrow \Phi_B = -BA$$

case-2

$$\text{If } \theta = 90^\circ$$

$$\Rightarrow \Phi_B = 0 \text{ i.e no mag. flux link}$$

Law:-

statement:- It states that "the direction of induced e.m.f. is such that it tends to oppose the very cause which produce it".

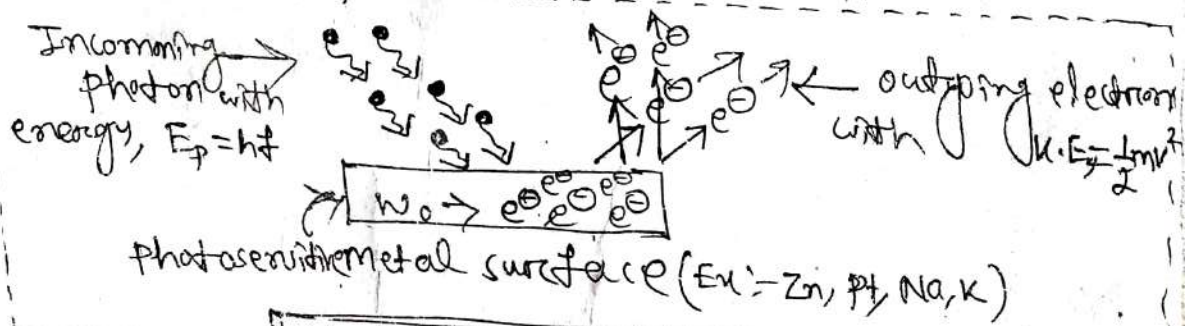
i.e.
$$E = - \frac{d\phi_B}{dt}$$

- Hence the '-ve' sign indicating that ~~both~~ e.m.f. and ~~rate~~ rate of change of magnetic flux both have opposite sign.
- i.e. one is opposing other or the e.m.f. opposing the rate of change of magnetic flux.
- As we all know due to current flow in any region there is influence of magnetic field in this region. Since here the e.m.f./voltage will create its own magnetic field due to flow of this induced current.
- So this ~~induced~~ magnetic field created by the induced e.m.f./voltage/induced current will try to oppose the original magnetic field linked with the original magnetic flux which is the cause of this induced e.m.f.

10. MODERN PHYSICS

Photoelectric Effect

Definition:- The phenomenon in which the emission of electrons occurs from the surface of a photosensitive metal due to incidence of light of shorter wavelength, is called as photoelectric effect or photoelectric emission.



Photoelectric Emission

Some terms connected with photoelectric emission:-

① Photoelectric current/photocurrent (I):-

The emitted electrons in the photoelectric process are called as photoelectrons & the current produced due to motion of these electrons is called as photocurrent.

② Stopping Potential (V_0):-

It is the minimum potential/voltage below which photoelectric emission stops or below which there is no flow of electrons.
 " " the value of photocurrent, $I=0$.

③ Threshold Frequency (f_0):- It is the certain minimum frequency of incident light, below which photoelectric emission stops or photocurrent, $I=0$.

(4) Work Function (W_0):

It is the minimum energy required to break the binding energy of the metal to produce free electrons & below this minimum energy photoelectric emission stops. i.e. photocurrent, $I = 0$.

→ The work function value is different for different metal as the binding energy for different metal is different.

Laws of photoelectric Effect:-

(1) It is an instantaneous process. (as it can occur within time $\cdot 10^{-9}$ sec)

(2) The photoelectric current (I) is directly proportional to intensity of incident light & independent of frequency of incident light.

i.e. $I \propto \text{Intensity of incident light}$
& I is independent of frequency of incident light

(3) The stopping potential (V_0) is directly proportional to frequency of incident light & independent of intensity of incident light.

i.e. $V_0 \propto \text{Frequency of incident light}$
& V_0 is independent of intensity of incident light

(4) The photoelectric emission stops below a certain minimum freq. of incident light, it is called as threshold frequency (f_0).

Einstein's Equation of photoelectric Effect

Einstein explain the photoelectric effect on the basis of planck's theory & acc. to planck's theory, ~~the~~ a beam of light is the collection of discrete wave packets called as photon, each with energy,

$$E_p = h f \quad \text{--- (1)}$$

where, $E_p \rightarrow$ Energy of photon.

$h \rightarrow$ Planck's constant.

$f \rightarrow$ Frequency of incident light.



When we incident light ^{of short wavelength (Ex: UV rays)} on the surface of a photosensitive metal, then the energy absorbed from the incident light is utilised in two ways. ~~such as:~~ i.e. the minimum energy or the work function to break the binding energy of the metal i.e. ' W_0 ' & the maximum K.E. for electron to emit ~~from the surface~~ ^{instantly} i.e. $\frac{1}{2} m v_{max}^2$ (where, $v_{max} \rightarrow$ max^m velocity).

i.e. the total energy for electron,

$$E_{electron} = W_0 + \frac{1}{2} m v_{max}^2 \quad \text{--- (2)}$$

According to conservation law of energy,
 Energy of incoming photon = Energy of outgoing e⁻s.

$$\Rightarrow E_p = E_{electron}$$

Using ① & ② we get,

$$hf = W_0 + \frac{1}{2}mv_{\text{max}}^2 \quad \text{--- (3)}$$

Again, for minimum f_{near} to the minimum energy, $W_0 = hf_0$ ($\because$ Energy, $E = hf$)

Using this in ③ we get,

$$hf = hf_0 + \frac{1}{2}mv_{\text{max}}^2$$

$$\Rightarrow \frac{1}{2}mv_{\text{max}}^2 = hf - hf_0$$

$$\Rightarrow \boxed{\frac{1}{2}mv_{\text{max}}^2 = h(f - f_0)} \quad \text{--- (4)}$$

This equation is called as Einstein's equation of photoelectric effect which shows that the p.e of emitted electrons is directly proportion to frequency of incident light, as 'h' is constant.

Photoelectric cell / photocell:-

It consists of an evacuated bulb whose inner side is coated with an alkali metal with a window. The bulb is made up of glass if it is to be used for white light & is made up of quartz if it is to be used for UV light.

→ It has ~~coated~~ with an electrode which is given a the potential with the help of a battery.

→ Light from a source is focused into the metal with the help of a convex lens & an ammeter connected in the circuit indicates the photoelectric current due to emission of photoelectrons.

Applications of photocell or Applⁿ of photoelectric Effect:-

- ① It plays an important role in television studio.
- ② It is used for reproduction of sound in films.
- ③ It is used for triggering fire alarm.
- ④ It is used in operating burglar's alarm.
- ⑤ It is used for automatic switching of street light.
- ⑥ It is used in electronic counter to count automatically the no. of persons entering or leaving a hall.

LASER :- It is the abbreviation of Light Amplification by Stimulating Emission of Radiation.

L → Light

A → Amplification by

S → Stimulating

E → Emission of

R → Radiation.

Properties / characteristic of LASER :-

- ① It is highly directional in nature i.e. a relatively narrow beam in a specific direction.
- ② It is coherent in nature i.e. it is highly coherent in space & time.
- ③ It is monochromatic in nature i.e. it has a single wavelength.

- ④ It is a highly intense beam as it can be focus over a very small area i.e. 10^{-6} m.
- ⑤ It give huge light power approximately equal to 10 watt/cm^2 .

Application / Use of LASER :-

- ① In medical field it is used in eye surgery like retina surgery.
- ② It can cut the flesh & can seal the oozing cells in a human body.
- ③ Laser beam is using to cut, drills & melt the metals in industry.
- ④ For high speed photography & as printer also laser is using.
- ⑤ In astronomy, it can control/guide the spaceship upto 10 billion of km.
- ⑥ Fusion is being tried using laser.

LASER PRINCIPLE :-

- A laser system consists of an active medium & this active medium is placed inside a resonating cavity having two reflectors at its both ends.
- Out of these two reflectors, one is total reflector (100%) & the other is partial reflector (90-95%).
- An electrical or optical pump is there within the resonating cavity to excite the atoms of the medium, as shown in the give fig.

By, Anurag Behara, Sr. Engg. Mar 8-16