

GOVERNMENT POLYTECHNIC, BHUBANESWAR

ସରକାରୀ ପଲିଟେକ୍ନିକ ଭୁବନେଶ୍ୱର



ENGINEERING MECHANICS NOTE

CHAPTER-1

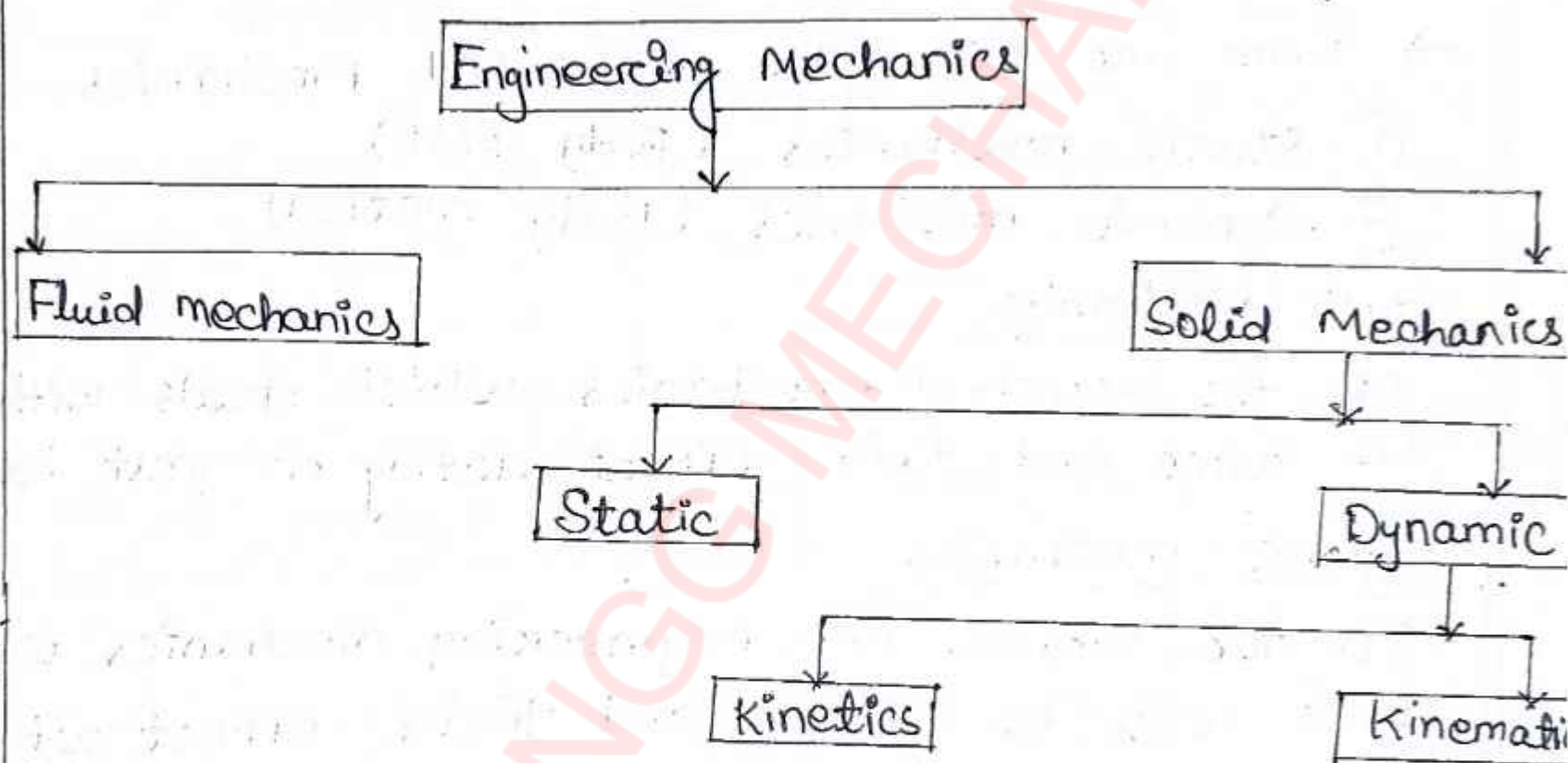
[BASICS OF MECHANICS AND FORCE SYSTEM]

PREPARED BY

ER. RAJKUMAR PRADHAN [RAJ SIR]

Engineering Mechanics

- It's the branch of engineering science which deals with the study of concerned with the state of rest or motion body subjected to the action of motion.
- Father of mechanics - Albert Einstein
- There are various types of engineering mechanics

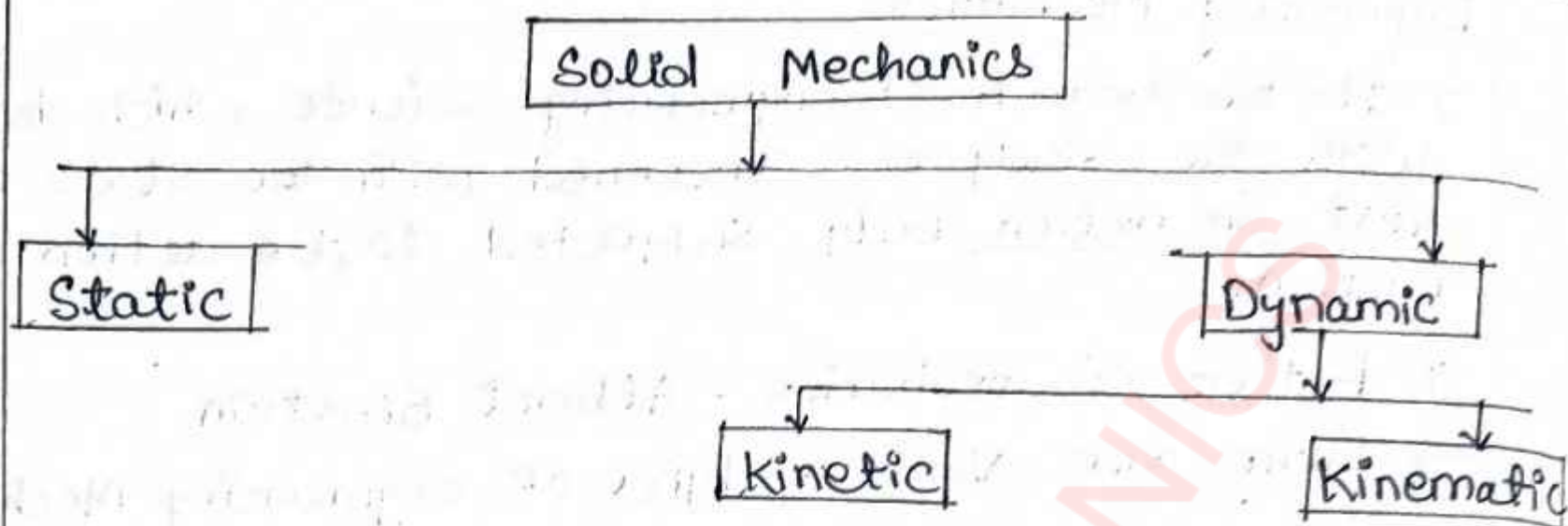


Fluid Mechanics

- It's the branch of engineering science which deals with the study of concerned with the state of rest or motion fluid body subjected to the action of motion.

Solid Mechanics

- It's the branch of engineering science which deals with the study of concerned with the state of rest or motion solid body subjected to the action of motion.



→ There are two types of solid mechanics

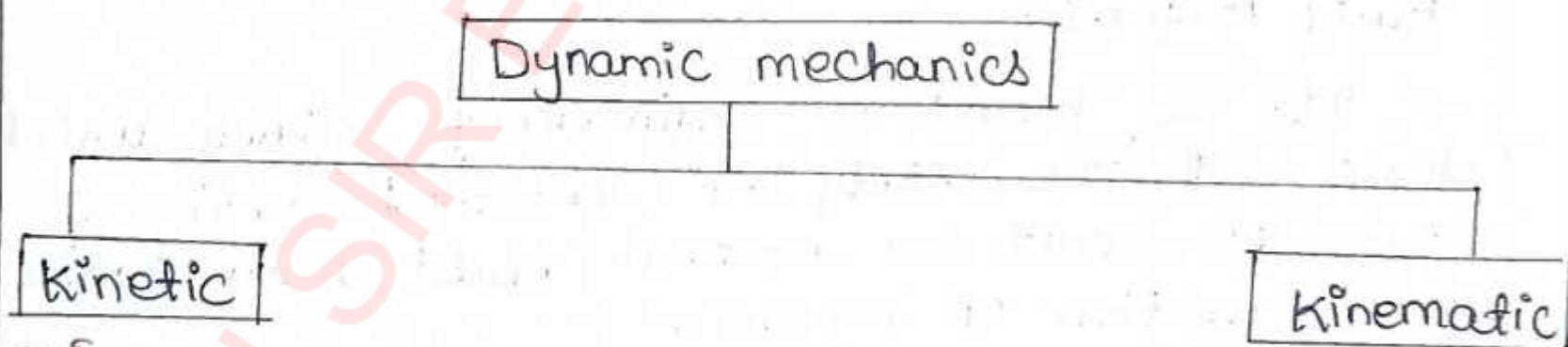
- ① Static mechanics (Body rest)
- ② Dynamic mechanics (Body motion)

Static Mechanics

Its the branch e-mechanics which deals with the force and their effect acting on rest body.

Dynamic mechanics

Its the branch of engineering mechanics which deals with the force and their effect acting on motion body.



V.U.E

Kinetic

Its the branch of dynamics which deals with the motion body with the application of force.

Kinematics

Its the branch of dynamics which deals with the motion body without the application of force.

Body

If any substance contain basic quantities this substance is known as body.

→ There are various types of body.

① Rigid body

② Elastic body etc.

Rigid body

→ A body which do not passes any deformation this body is known as Rigid body.

→ There is no body in this universe is said to be perfect rigid body.

Elastic body

→ A body which passes any deformation by the action of force is known as Elastic body.

→ Ex - steel, rubber, gold etc.

Basic quantity

There are various types of basic quantity
Mass, weight, length, Time

Mass

→ Total quantity of matter contain in a body is known as mass.

→ Its a scalar quantity.

→ mass of a body never 0.

Weight

→ Its the product of mass, and gravitational force due to acceleration.

$$W = mg$$

m = mass of body

$$g = 9.81 \text{ m/s}^2$$

→ Its a vector quantity.

→ Weight of a body can be zero.

Length

Its located the position of the particle and describe size of the physical system.

→ Ex - 1cm, 10mm, 1km etc

Time

Its the measurement of the sequency event.

→ It also basic quantity of dynamic.

Ex - hour, second, minute

Unit

The unit which are used to measure physical quantity of the body.

→ There are various types of unit

① Fundamental unit

② Derived unit

Fundamental unit

The unit which are used for fundamental physical quantity is known as fundamental unit.

Ex - Mass - Kg

Weight - Newton

Time - Second

Length - meter

Derive Unit

The unit which are derived by using fundamental unit are called derive unit.

$$\text{Velocity} \rightarrow \frac{\text{Displacement}}{\text{Time}} = \frac{\text{Length}}{\text{Time}}$$
$$= \text{m/sec}$$

S.I unit

It means system of international unit.

Ex - Mass - Kg

Weight - Newton

Time - Sec

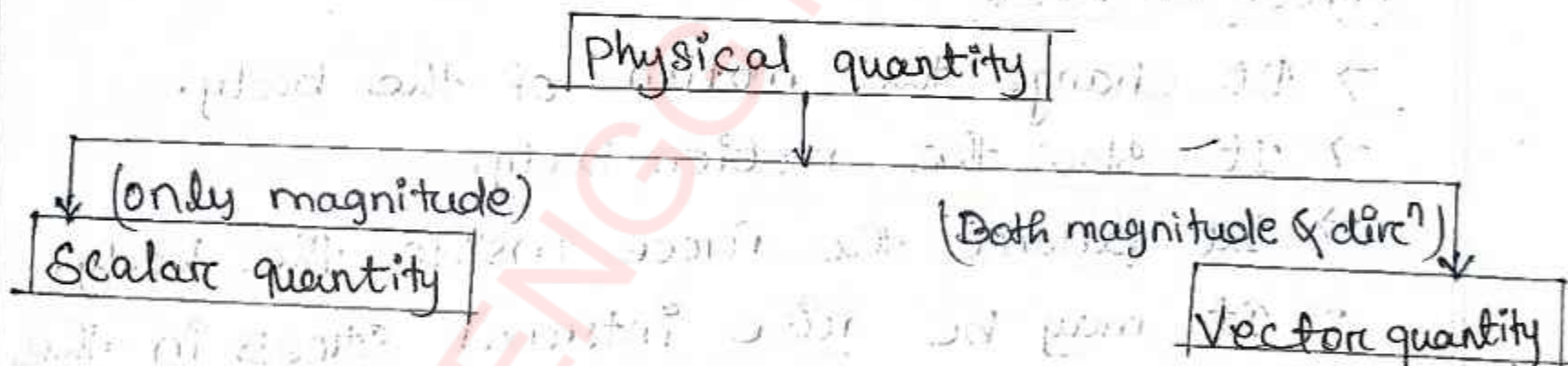
Length - meter

$$\rightarrow \text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{\text{Length}}{\text{Time}} \Rightarrow \text{m/sec}$$

Physical quantities

Physical property that can be quantified and measure by using number this is known as physical quantity.



Scalar quantity

The physical quantities which help only magnitude is known as scalar quantity.

Ex - mass, Volume, Time etc.

Vector quantity

The physical quantities which help both magnitude and direction is known as Vector quantity.

Ex - Velocity, Temperature etc.

Force

It's defined as an external agent of the body which produced or tends to produced, destroy or tends to destroy motion of the body.

→ Product of mass and acceleration of the body is known as force.

→ Mathematically $F = ma$

Where F = force of the body

m = mass of the body

a = acceleration of the body

→ Unit is - Newton.

Effect of force

→ It change the motion of the body.

→ It slow the motion body.

→ It balance the force inside the body.

→ It may be rise internal stress in the body.

Note

Stress :- force per unit area

$$P = \frac{F}{A}$$

F = force

A = area of the body

Strain :- change in length by original length.

Characteristics of force

→ There are 4 characteristics in force.

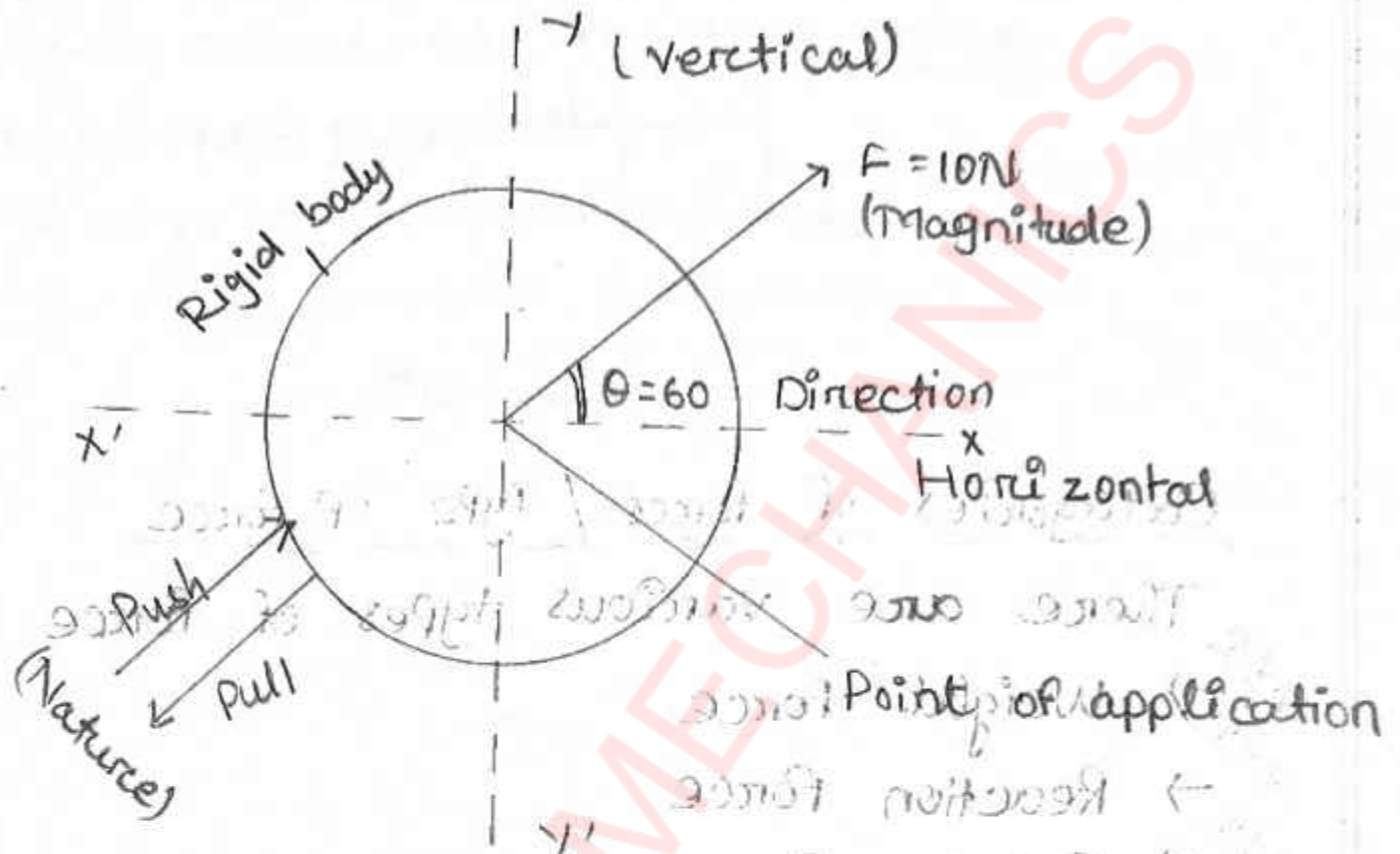
→ Magnitude

→ Direction

→ Nature (Pull or Push)

→ Point of application.

V.V.D



Direction

→ Angular position of the force with the Ref axis.

→ Calculate by anti clock-wise.

Nature :-

Force is apply on a body by the two nature

① Pull

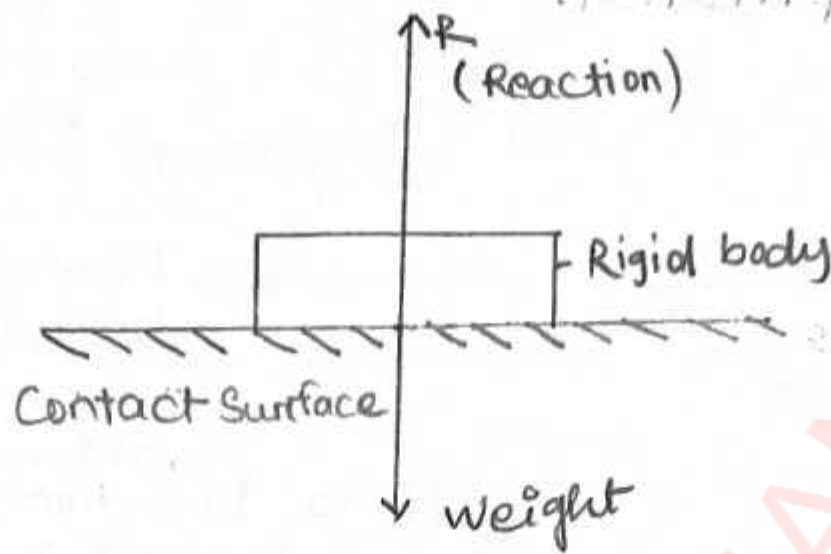
② push

V.V.D

Point of application :-

The force applied on which point this point is also known as point of application.

Action, Reaction, Contact Surface :-



Categories of force / Type of force

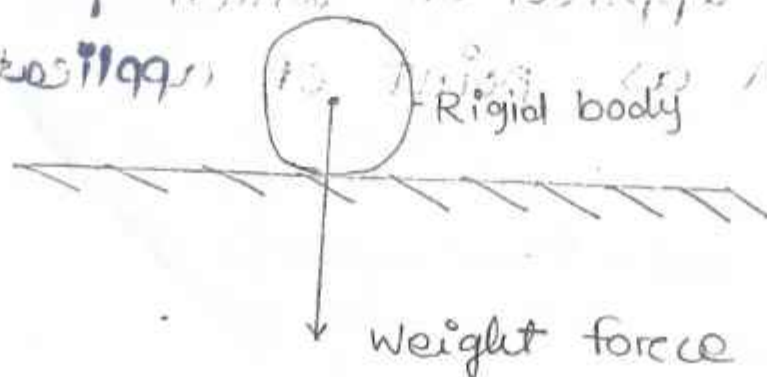
There are various types of force

- N.V.P. → Weight force
- N.V.P. → Reaction force
- N.V.P. → Friction force
- External force
- Tension force
- Compression force

Weight force :-

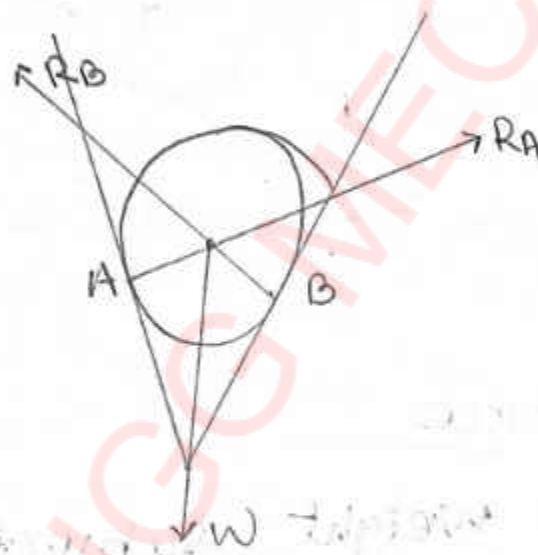
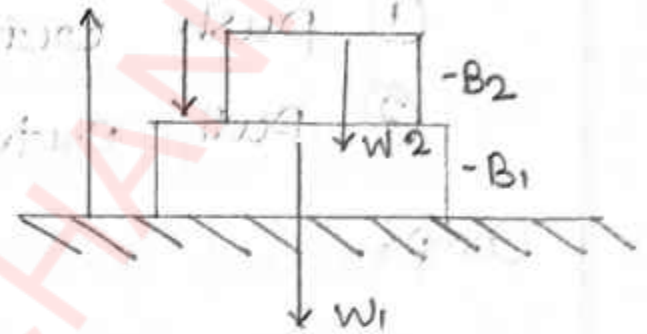
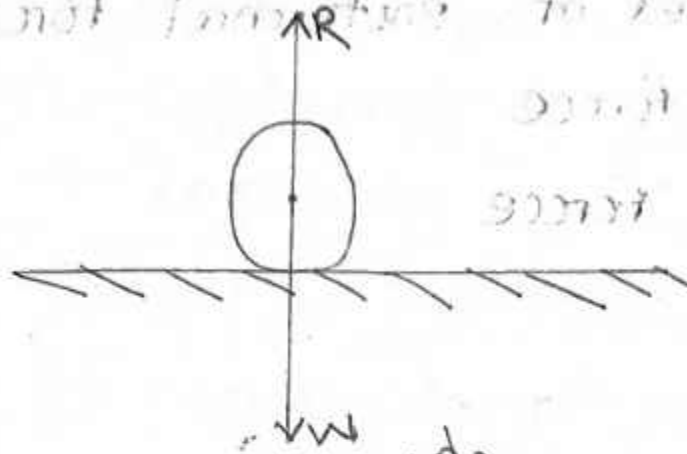
It's the force produced on the body by the acceleration due to gravity.

- It also called as Body force.
- It always down ward.



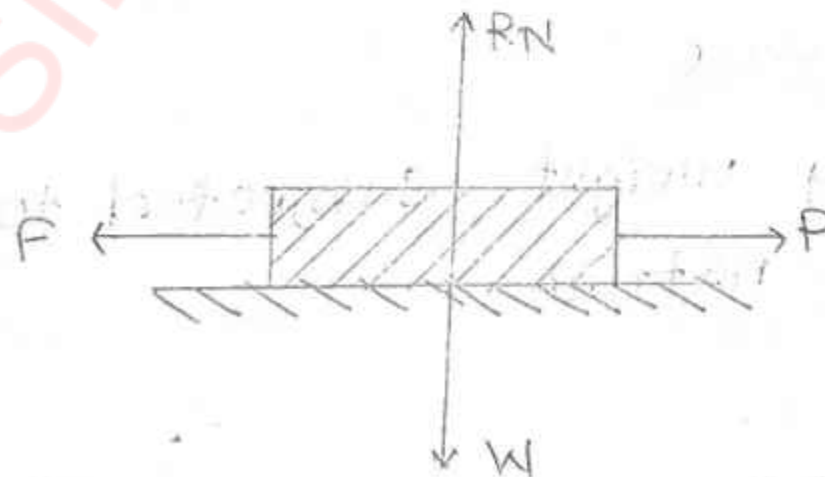
Reaction force:-

- It's the force exerted between two contact surface due to action of force on rigid body.
- It's also called as reactive force.
- It always upward.



Friction force:-

force is acting on opposite direction of body known as friction force.



Where F = frictional force

P = applied force

W = weight

R_N = Normal Reaction

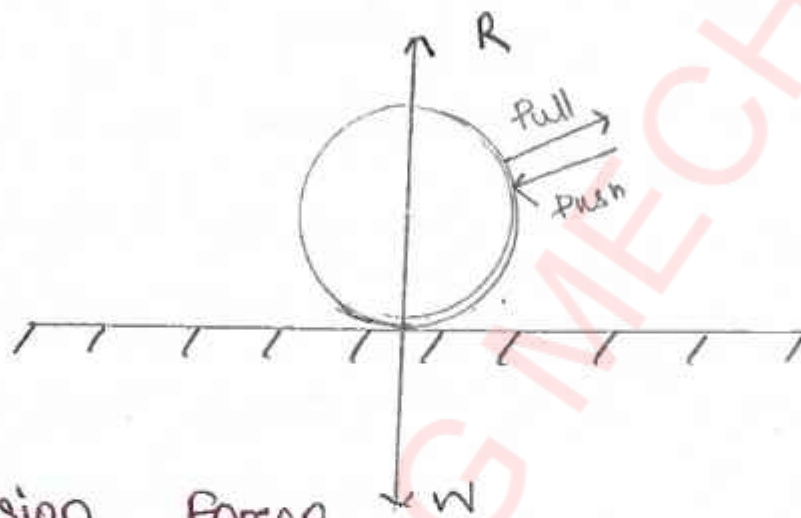
External force

The source produced from external on rigid body due to external means (collision, vibration etc)

There are two types of external force

- ① push external force
- ② pull external force

Compo



Compression Force

The body weight subjected to the force of push nature.



Tension Force

The body weight subjected to the force of pull nature.



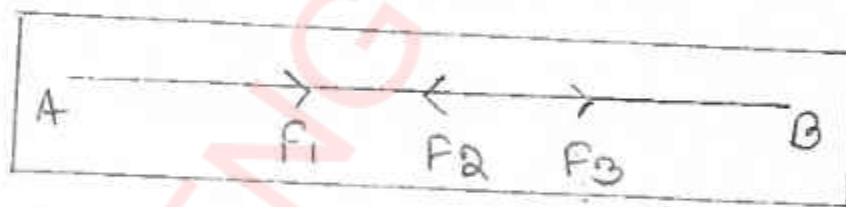
System of force

There are various type of force.

- Collinear force
- Concurrent force
- Co-planer force
- Coplaner parallel force
- Coplaner concurrent force
- Coplaner non concurrent force
- non coplaner concurrent force
- Non coplaner non concurrent force.

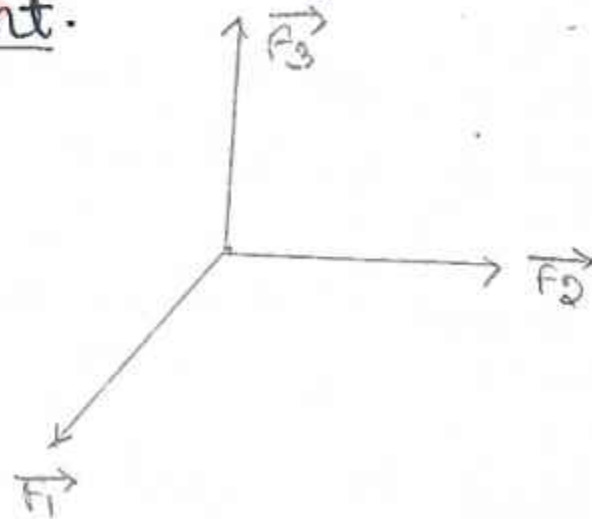
Collinear force:-

The force acting on a rigid body in a common straight line.



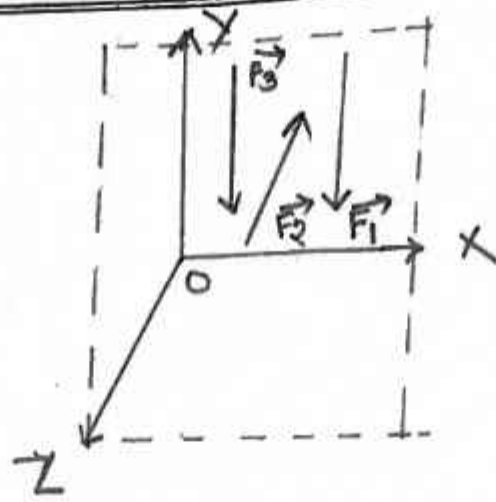
Concurrent force:-

The force acting on a rigid body in a common point.



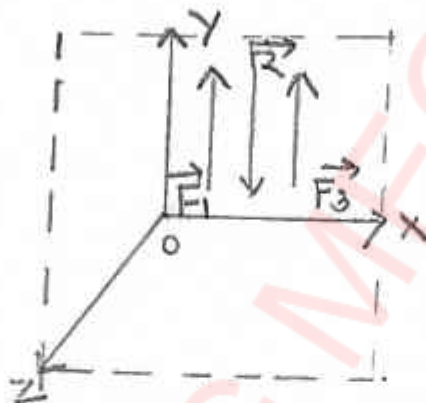
Coplaner force:-

The force acting on a rigid body in a common plane.



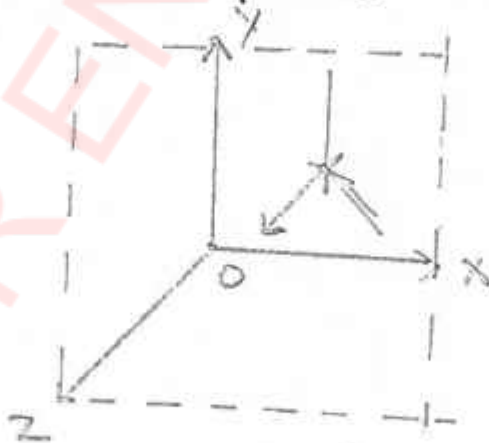
Coplaner parallel force

→ The force acting on a rigid body in a common plane with parallel.



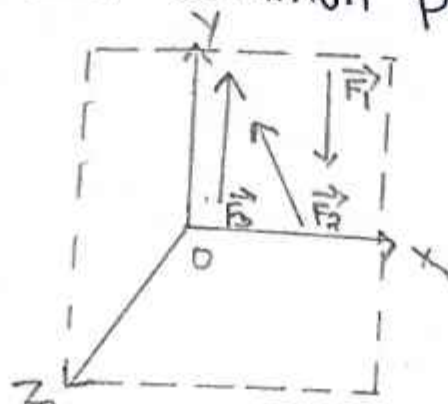
Coplaner concurrent force

→ The force acting on a rigid body in a common plane with common point.



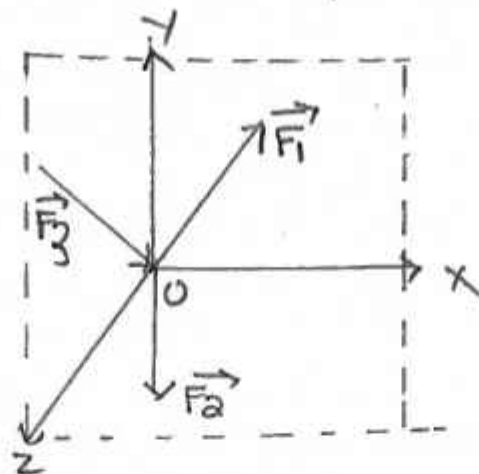
Coplaner non concurrent force

The force acting on a rigid body in a common plane but don't contain common point.



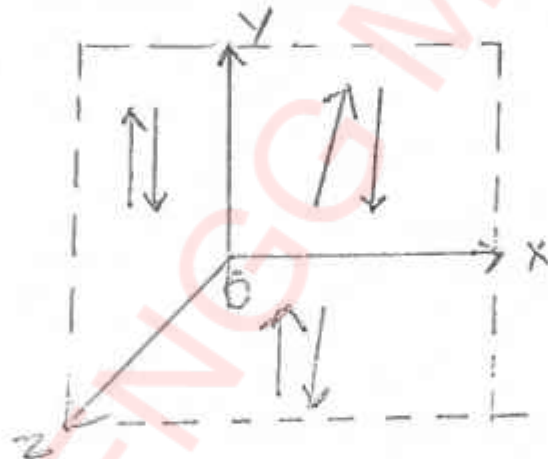
Non coplaner concurrent force

The force acting on a rigid body in different plane with common point.



Non coplaner non concurrent force

→ The force acting on a rigid body in different plane but don't contain any common point.

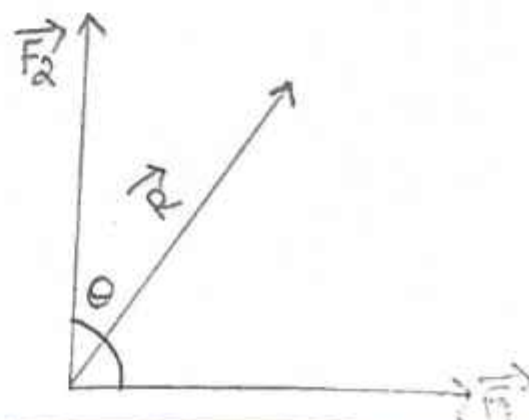


Resultant force

→ Combine effect of force into a single force is known as resultant force.

Combine effect of
force = Single force

→ It's denoted by 'R'.



Solving of Resultant force

Analytical Method

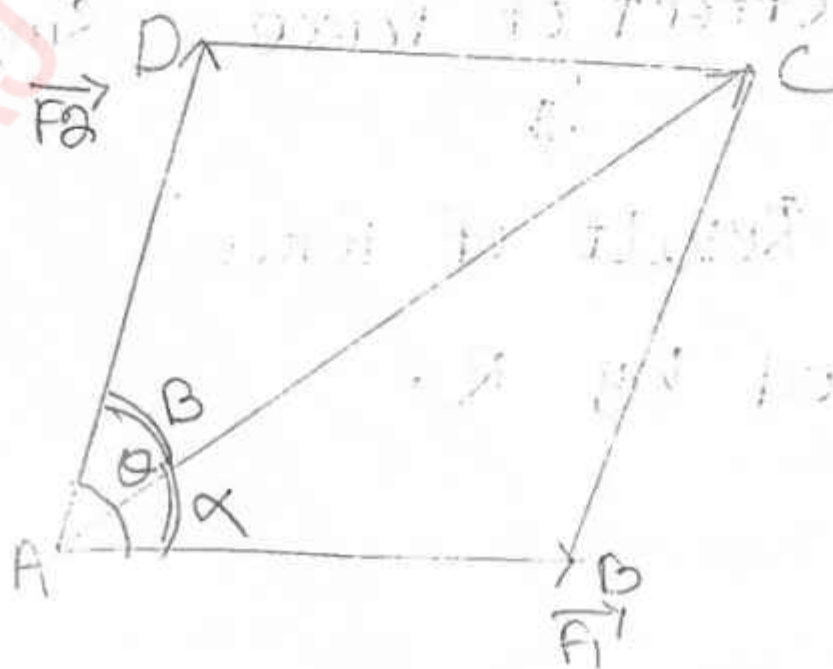
- Parallelogram law of force
- Resolution force
- Composition force

Graphical Method

- Triangle law of force
- polygon law of force

Parallelogram law of force

If two force acting on a body at an angle θ , it represent magnitude and direction by two side of parallelogram, then the diagonal passing between them represent magnitude and direction of the resultant of force.



$$\theta = \alpha + \beta$$

Where.

$\vec{F}_1$ and $\vec{F}_2$ = two force

$\vec{R}$ = R (Resultant force)

θ : Angle between two force F_1 & F_2

α : Horizontal angle with F_1 force

β : Vertical angle with F_2 force

Mathematically,

→ Magnitude of resultant force

$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta}$$

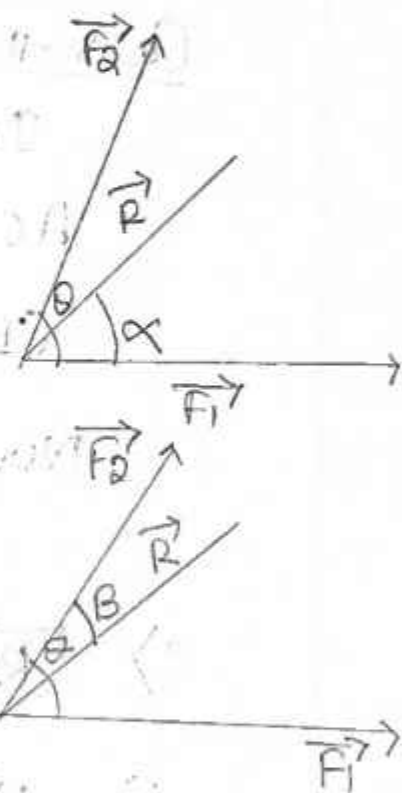
→ Direction

① Horizontal Angle

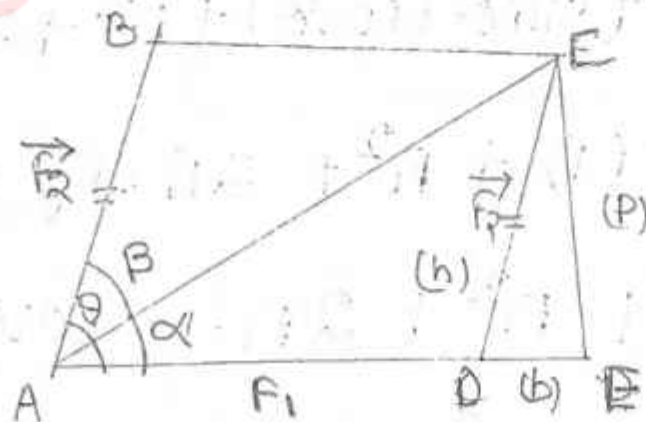
$$\alpha = \tan^{-1} \frac{F_2 \cos \theta}{F_1 + F_2 \sin \theta}$$

② Vertical Angle

$$\beta = \tan^{-1} \frac{F_1 \sin \theta}{F_2 + F_1 \cos \theta}$$



Proove



In this diagram

$\angle BAD = \angle CDE = \theta$ ($\because$ Corresponding Angle)

Now at ΔCDE

$$\sin \theta = \frac{p}{h} = \frac{CE}{CD} = \frac{CE}{F_2}$$

$$\Rightarrow \sin \theta = \frac{CE}{F_2}$$

$$\Rightarrow \boxed{CE = \sin \theta F_2} \quad \text{--- (1)}$$

Again

$$\cos \theta = \frac{b}{h} = \frac{DE}{CD} = \frac{DE}{F_2}$$

$$\Rightarrow \boxed{\cos \theta F_2 = DE} \quad \text{--- (2)}$$

Case-II

In ΔCAE

$$AC = \vec{R} = h \quad AE(b) = AD + DE = \vec{F}_1 + \vec{F}_2 \cos \theta \quad (\because \text{eqn 1})$$

$$CE = p = \vec{F}_2 \sin \theta \quad (\because \text{eqn 1})$$

Now applying Pythagoras theorem

$$h^2 = p^2 + b^2$$

$$\Rightarrow (\vec{R})^2 = (F_2 \sin \theta)^2 + (F_1 + F_2 \cos \theta)^2$$

$$\Rightarrow R^2 = F_2^2 \sin^2 \theta + F_1^2 + F_2^2 \cos^2 \theta + 2 F_1 \cdot F_2 \cos \theta$$

$$\Rightarrow R^2 = F_2^2 (\sin^2 \theta + \cos^2 \theta) + F_1^2 + 2 F_1 \cdot F_2 \cos \theta$$

$$\Rightarrow R^2 = F_2^2 (1) + F_1^2 + 2 F_1 \cdot F_2 \cos \theta$$

$$\Rightarrow R^2 = F_2^2 + F_1^2 + 2 F_1 \cdot F_2 \cos \theta$$

$$\Rightarrow R = \sqrt{F_2^2 + F_1^2 + 2 F_1 \cdot F_2 \cos \theta}$$

$$\Rightarrow \boxed{R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta}}$$

Solving Resolution of force

Quadrant Method

Q-2

$$\sum H = -ve$$

$$\sum V = +ve$$

Q-1

$$\sum H = +ve$$

$$\sum V = +ve$$

-ve ←

Q-3

$$\sum H = -ve$$

$$\sum V = -ve$$

Q-4

$$\sum H = +ve$$

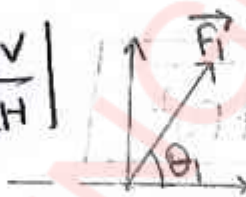
$$\sum V = -ve$$

→ +ve (Horizontal)

↓ -ve

Q-1

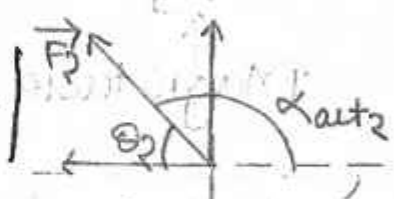
$$\theta_1 = \tan^{-1} \left| \frac{\sum V}{\sum H} \right|$$



$$\alpha_{act1} = \theta_1$$

Q-2

$$\theta_2 = \tan^{-1} \left| \frac{\sum V}{\sum H} \right|$$



$$\alpha_{act2} = 180 - \theta_2$$

-ve ←

Q-3

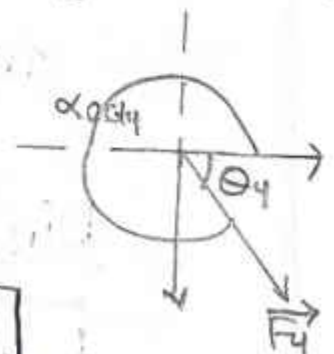
$$\theta_3 = \tan^{-1} \left| \frac{\sum V}{\sum H} \right|$$



$$\alpha_{act3} = 180 + \theta_3$$

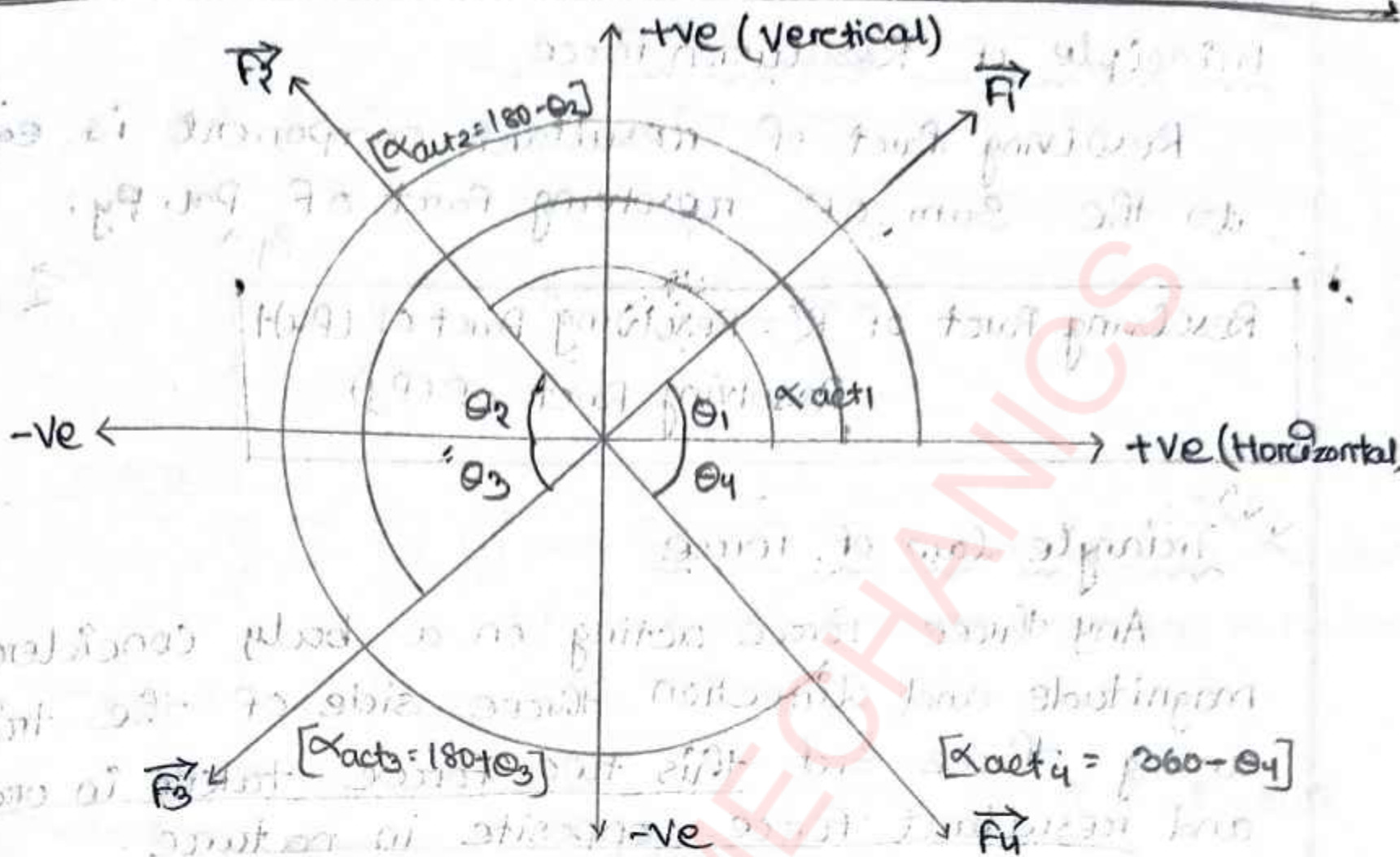
Q-4

$$\theta_4 = \tan^{-1} \left| \frac{\sum V}{\sum H} \right|$$



$$\alpha_{act4} = 360 - \theta_4$$

↓ -ve



Magnitude

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos (180 - \theta_2) + \vec{F}_3 \cos (180 + \theta_3) + \vec{F}_4 \cos (360 - \theta_4)$$

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin (180 - \theta_2) + \vec{F}_3 \sin (180 + \theta_3) + \vec{F}_4 \sin (360 - \theta_4)$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

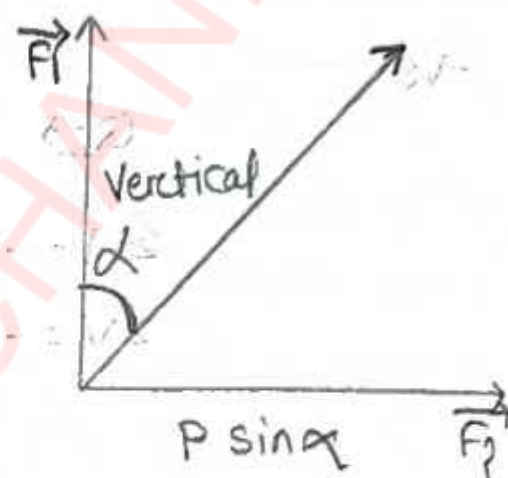
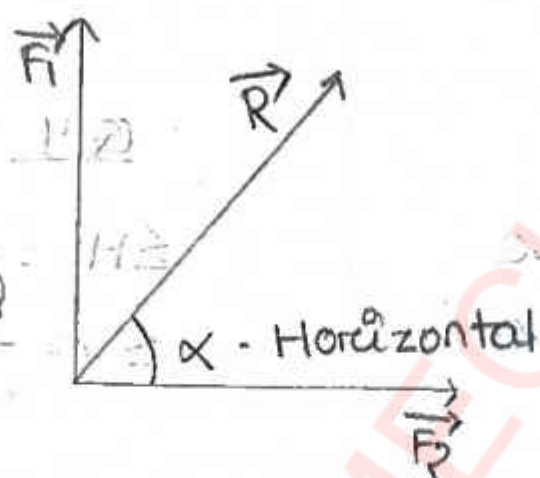
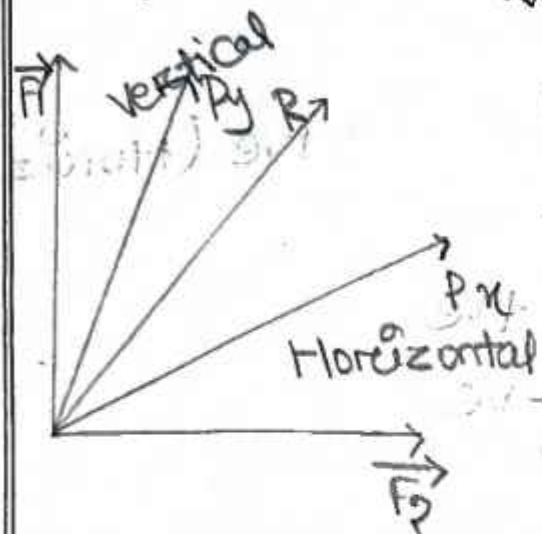
Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

Resolution of force

The process of splitting of a single force into a several force without effect of body. this force is known as resolution of force.

→ Splitting part of the resultant force or single force is known as resolution force.



Horizontal

$$P_x = P \cos \alpha$$

$$P_y = P \sin \alpha$$

Magnitude

$$(R) = \sqrt{P_x^2 + P_y^2}$$

Vertical

$$P_x = P \sin \alpha$$

$$P_y = P \cos \alpha$$

Magnitude

$$(R) = \sqrt{P_x^2 + P_y^2}$$

Direction

$$\alpha = \tan^{-1} \left| \frac{P_y}{P_x} \right|$$

(Horizontal)

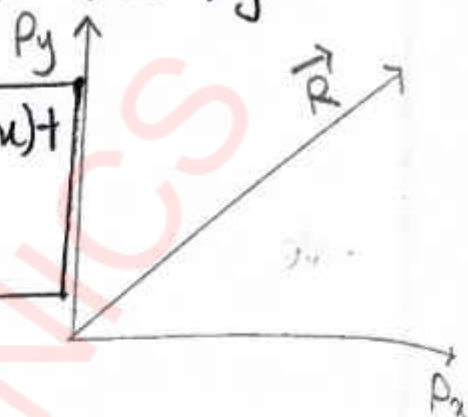
$$\alpha = \tan^{-1} \left| \frac{P_y}{P_x} \right|$$

(Vertical)

Principle of Resolution force

Resolving part of resultant component is equal to the sum of resolving part of P_x, P_y .

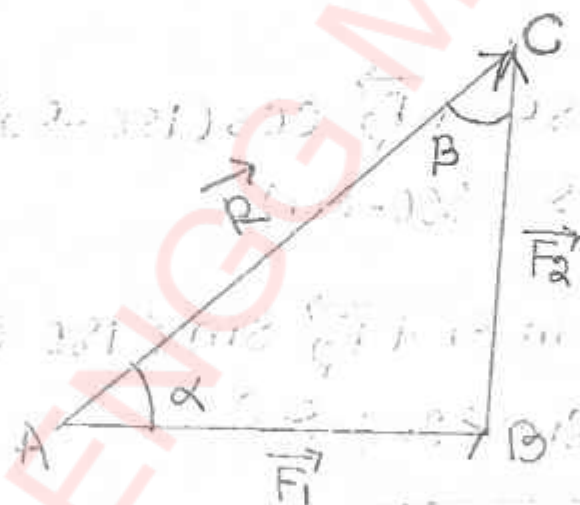
Resolving part of $\vec{R} = \text{Resolving part of } (P_x) + \text{Resolving part of } (P_y)$



V.V.P

Triangle law of force

Any three force acting on a body consider magnitude and direction three side of the triangle among ~~three~~ this two force taken in order and resultant force opposite in nature.



Where

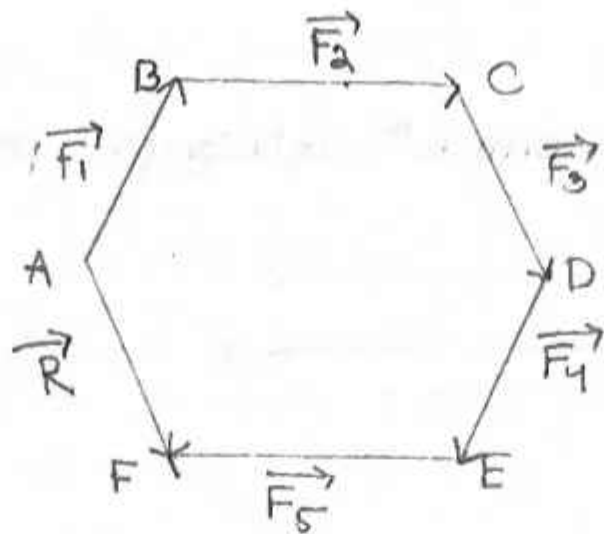
$\vec{F}_1$ and $\vec{F}_2$ = two force (taken in order)

$\vec{R}$ = resultant force opposite in nature

V.V.P

Polygon law of force

Any ~~sin~~ force acting on a body consider magnitude and direction, ~~five~~ ^{sum} side of the polygon among this five force taken in order and resultant force opposite in nature.



Where,

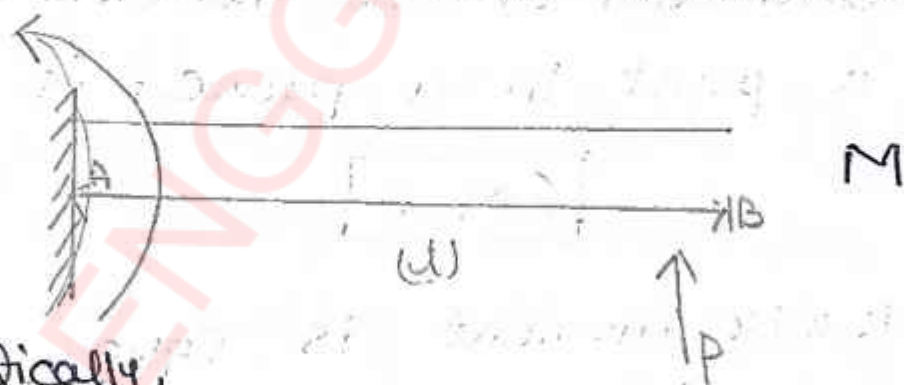
$\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$ and $\vec{F}_5$ = Five force (taken in order)

$\vec{R}$ = resultant force opposite in nature

Moment of force

Turning effect of the force about that point is called moment of force.

→ The product of apply force and length of the body also known as moment of force.



Mathematically,

$$M = P \times l$$

where,

P = applied force

l = length of the body

Unit

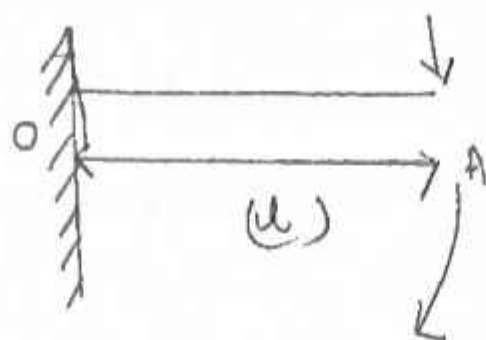
[Newton - Meter]

There are two types of moment

1. Clock wise moment
2. Anticlock wise moment

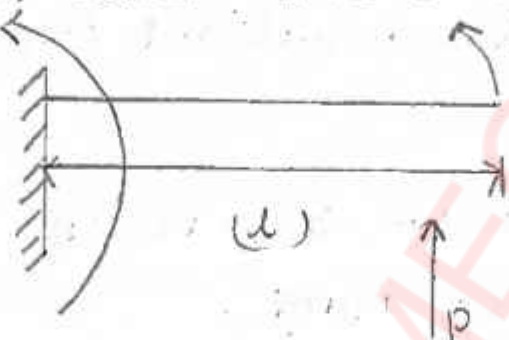
Clock wise moment

Clock-wise moment always positive.



Anti clock-wise moment

Anti clock-wise moment always negative.



V.V.P

Varignon's Theorem (law of moment)

Algebraic sum of the moments of force about a point in a plane is 0.

$$\boxed{\sum M = 0}$$

⇒ Clock wise moment is equal to the anti clock wise moment.

$$\sum M = 0$$

⇒ Clock wise moment + Anti clock wise moment = 0

⇒ Clock wise moment + (- Anti clock wise moment) = 0

$$\Rightarrow (l-u)Q + (-u \times P) = 0$$

$$\Rightarrow (l-u)Q + (-Pu) = 0$$

$$\Rightarrow (l-u)Q = Pu$$

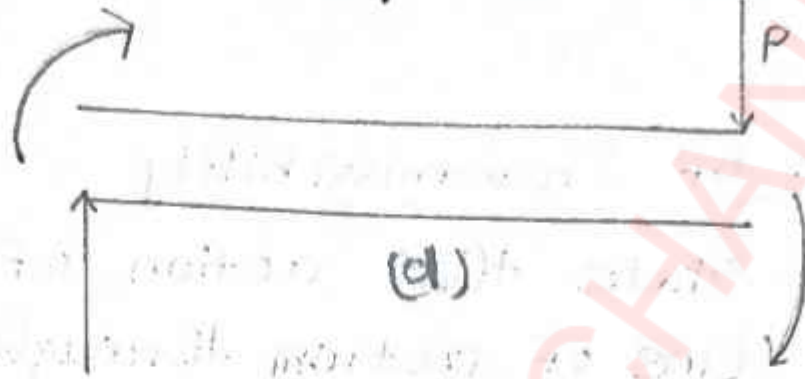
⇒ Clock wise moment = Anticlock wise moment

P_m = Anticlockwise moment

$(C.W.) Q$ = Clockwise moment

Couple

Couple force is defined as a pair of equal parallel and opposite force.



Mathematically,

$$T = P \times d$$

Where,

P = applied force

d = Arm of the force.

Ex- locking or unlocking a lock with a key.
opening or closing of water tap.

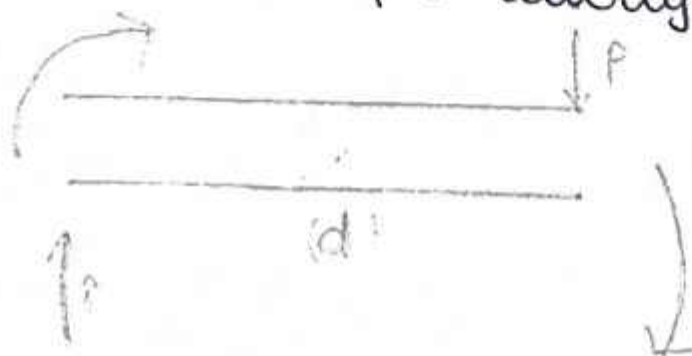
→ There are two types of couple

1. Clockwise couple

2. Anticlockwise couple

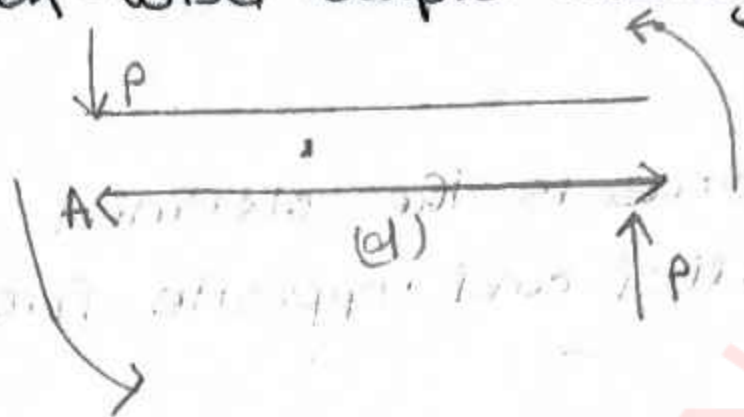
Clockwise couple

Clockwise couple always positive.



Anti clock wise couple

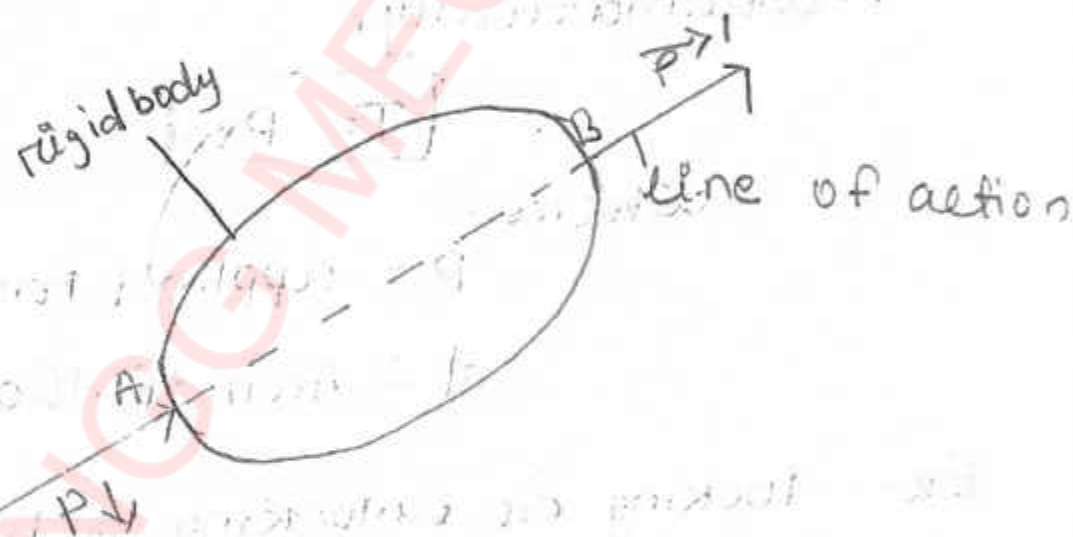
Anti clock-wise couple always negative.



V.V.D

Principle of Transmissibility

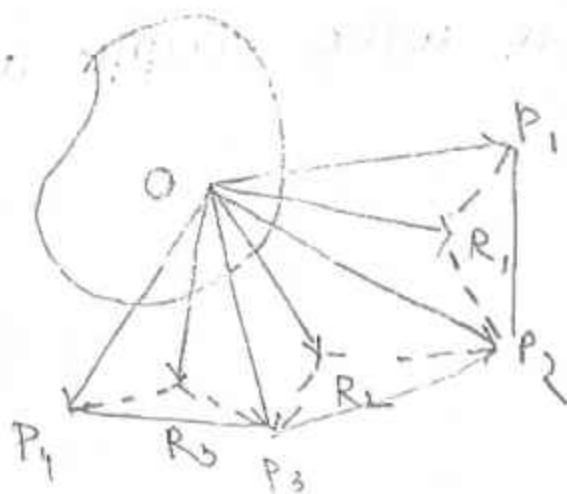
It states that the action of a force is transmitted along its line of action through out the rigid body.



Composition of force

Reduction of force into a single force on a rigid body is known as Composition of force.

→ It rarely uses because its method is very complicated.



Chapter-1

- ① If two force 7N and 8N acting at a point find the resultant force. If the angle between 2 force 60°

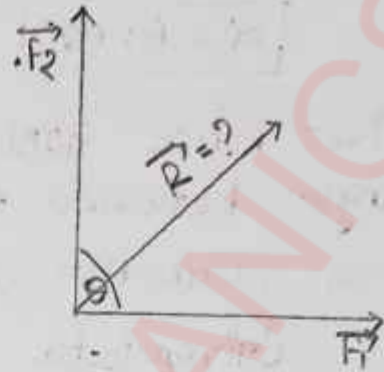
Given data

$$F_1 = 7\text{N}$$

$$F_2 = 8\text{N}$$

$$\theta = 60^\circ$$

$$R = ?$$



Magnitude (Resultant force)

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta}$$
$$= \sqrt{(7)^2 + (8)^2 + 2 \cdot 7 \cdot 8 \cdot \cos(60^\circ)}$$

$$\boxed{R = 13\text{N}}$$

- ② Two force 80N and 70N at a point find the magnitude and direction of resultant force. If the angle between two force 150° .

Given data

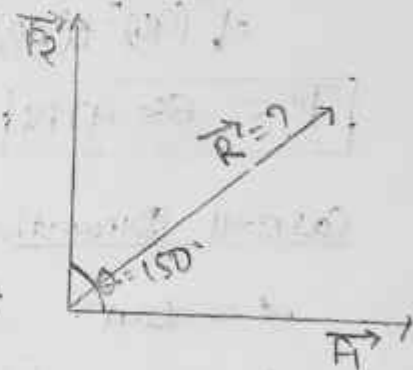
$$F_1 = 80\text{N}$$

$$F_2 = 70\text{N}$$

$$R \text{ (magnitude)} = ?$$

$$\alpha \text{ (Direction)} = ?$$

$$\theta = 150^\circ$$



Case-I (Magnitude)

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta}$$
$$= \sqrt{(80)^2 + (70)^2 + 2 \cdot 80 \cdot 70 \cdot \cos(150^\circ)}$$

$$\boxed{R = 40\text{N}}$$

Case-II (Direction)

$$\alpha = \tan^{-1} \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\alpha = \tan^{-1} \frac{70 \sin 150}{80 + 70 \cos 150}$$

$$\alpha = 61.02^\circ$$

- ③ Two force 90N and 60N at a point. If an angle between two force 130° . Find the magnitude and direction of the resultant force.

Given data

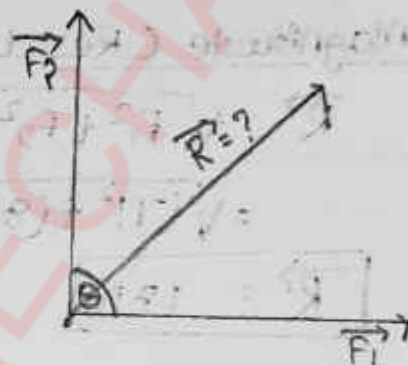
$$F_1 = 90\text{N}$$

$$F_2 = 60\text{N}$$

$$\theta = 130^\circ$$

$$R \text{ (magnitude)} = ?$$

$$\alpha \text{ (Direction)} = ?$$



Case-I

We know resultant force

$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta}$$

$$= \sqrt{(90)^2 + (60)^2 + 2 \cdot 90 \cdot 60 \cdot \cos(130^\circ)}$$

$$R = 68.97\text{N}$$

Case-II (Direction)

$$\alpha = \tan^{-1} \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$= \tan^{-1} \frac{60 \sin 130^\circ}{90 + 60 \cos(130^\circ)}$$

$$\alpha = 41.78^\circ$$

- ④ If the angle between two force 170° . Find the magnitude and direction of the resultant force. If the forces are 25N and 35N.

Given data

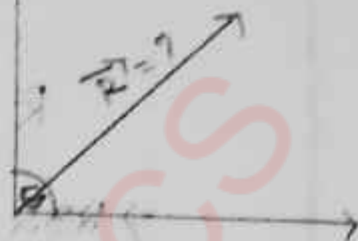
$$F_1 = 25 \text{ N}$$

$$F_2 = 35 \text{ N}$$

$$\theta = 170^\circ$$

$$(2) \vec{R} \text{ (magnitude)} = ?$$

$$\alpha \text{ (Direction)} = ?$$



Case-1

We know resultant force

$$\begin{aligned} R &= \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta} \\ &= \sqrt{(25)^2 + (35)^2 + 2 \cdot 25 \cdot 35 \cdot \cos(170^\circ)} \end{aligned}$$

$$\boxed{R = 11.25 \text{ N}}$$

Case-11 (Direction)

$$\begin{aligned} \alpha &= \tan^{-1} \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta} \\ &= \tan^{-1} \frac{35 \sin 170^\circ}{25 + 35 \cos(170^\circ)} \end{aligned}$$

$$\boxed{\alpha = 32.69^\circ} \text{ (ans)}$$

⑤ The result of two force is 232 N, the angle between two force is 45° . Find the force if $\vec{F}_2$ is 1.5 times of $\vec{F}_1$.

Given data

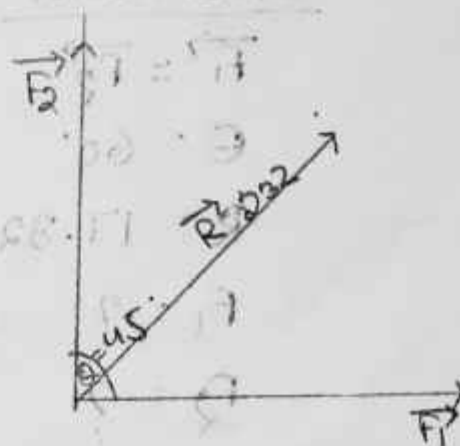
$$R = 232 \text{ N}$$

$$\theta = 45^\circ$$

$$\vec{F}_2 = 1.5 \vec{F}_1$$

$$\vec{F}_1 = ?$$

$$\vec{F}_2 = ?$$



We know

Resultant Force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta}$$

$$\Rightarrow 232 = \sqrt{F_1^2 + (1.5 F_1)^2 + 2F_1 \cdot 1.5 F_1 \cdot \cos(45^\circ)}$$

$$\Rightarrow 232 = \sqrt{F_1^2 + 2.25 F_1^2 + 2 \cdot 1.2 F_1^2}$$

$$\Rightarrow 232 = \sqrt{F_1^2 (5.37)}$$

$$\Rightarrow 232 = \sqrt{F_1^2 (5.37)}$$

$$\Rightarrow 232 = F_1 \sqrt{5.37}$$

$$\Rightarrow F_1 = \frac{232}{\sqrt{5.37}}$$

$$\Rightarrow \boxed{F_1 = 100.11 \text{ N}}$$

Now

$$F_2 = 1.5 F_1$$

$$= 1.5 \times 100.11$$

$$\boxed{F_2 = 150.16 \text{ N}}$$

- ⑥ The resultant of two equal force at a point with an angle 60° between them is 17.32 N . Find the magnitude of each force.

Given data

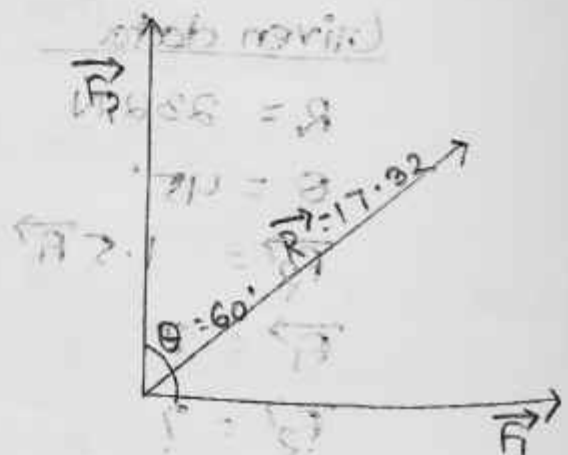
$$\vec{F}_1 = \vec{F}_2$$

$$\theta = 60^\circ$$

$$R = 17.32 \text{ N}$$

$$F_1 = ?$$

$$F_2 = ?$$



We know the Resultant force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta}$$

$$11.32 = \sqrt{F_2^2 + F_2^2 + 2F_2 \cdot F_2 \cos(60^\circ)}$$

$$\Rightarrow 11.32 = \sqrt{F_2^2 + F_2^2 + 2F_2^2 \cos(60^\circ)}$$

$$\Rightarrow 11.32 = \sqrt{F_2^2 (1 + 1 + 2 \cos(60^\circ))}$$

$$\Rightarrow 11.32 = \sqrt{F_2^2 \cdot 3}$$

$$\Rightarrow 11.32 = F_2 \sqrt{3}$$

$$\Rightarrow F_2 = \frac{11.32}{\sqrt{3}}$$

$$\Rightarrow \boxed{F_2 = 9.99 \text{ N}}$$

Now

$$F_1 = F_2$$

$$\text{So } \boxed{F_1 = 9.99 \text{ N}}$$

- ① a force are acting at an angle 60° between then the resultant is 80 N . Find the force if F_1 is twice the F_2 .

Given data

$$\theta = 60^\circ$$

$$\vec{R} = 80 \text{ N}$$

$$\vec{F}_1 = 2\vec{F}_2$$

$$\vec{F}_1 = \vec{F}_2$$

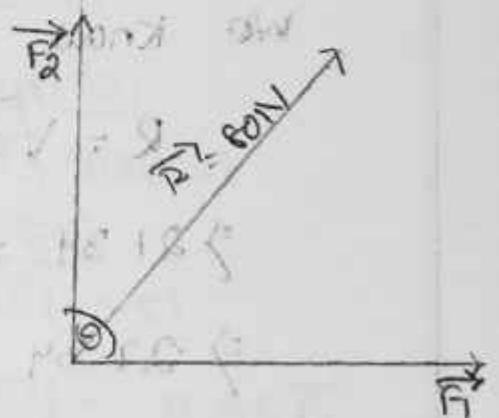
We know

Resultant force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta}$$

$$\Rightarrow 80 = \sqrt{(2F_2)^2 + F_2^2 + 2 \cdot 2F_2 \cdot F_2 \cos(60^\circ)}$$

$$\Rightarrow 80 = \sqrt{4F_2^2 + F_2^2 + 2F_2^2 \cos(60^\circ)}$$



$$\Rightarrow 80 = \sqrt{F_2^2 (4+1+4 \cos 60)}$$

$$\Rightarrow 80 = \sqrt{F_2^2 (4+1+2)}$$

$$\Rightarrow 80 = \sqrt{F_2^2 (7)}$$

$$\Rightarrow (80) = F_2 \sqrt{7}$$

$$\Rightarrow (F_2) = \frac{80}{\sqrt{7}}$$

$$\Rightarrow \boxed{F_2 = 30.23 \text{ N}}$$

Now

$$\vec{F} = 2\vec{F}_2$$

$$\vec{F} = 2 \times 30.23 \text{ N}$$

$$\boxed{\vec{F} = 60.46 \text{ N}}$$

- ⑧ The resultant of 2 equal force acting at a point with an angle 120° between them is 27.34 N . Find the magnitude of each force.

Given data

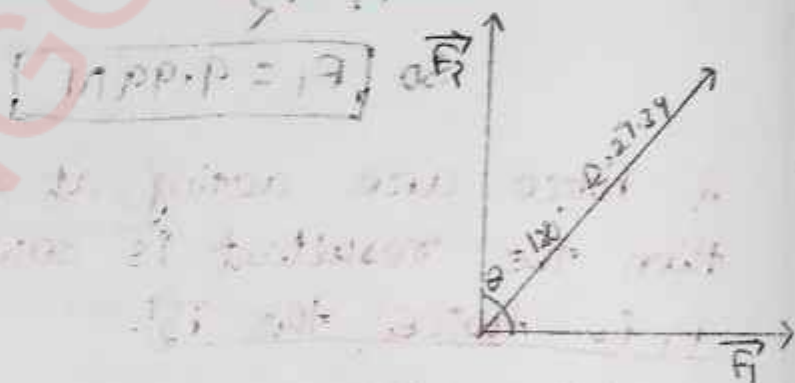
$$\vec{F}_1 = \vec{F}_2$$

$$\theta = 120^\circ$$

$$R = 27.34$$

$$\vec{F}_1 = ?$$

$$\vec{F}_2 = ?$$



We know Resultant force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta}$$

$$\Rightarrow 27.34 = \sqrt{F_2^2 + F_2^2 + 2F_2 \cdot F_2 \cos(120)}$$

$$\Rightarrow 27.34 = \sqrt{2F_2^2 + 2F_2^2 \cos(120)}$$

$$\Rightarrow 27.34 = \sqrt{F_2^2 (2 + 2 \cos(120))}$$

$$\Rightarrow 27.34 = \sqrt{F_2^2 \cdot 1}$$

$$\Rightarrow 27.34 = F_2 \sqrt{1}$$

$$\Rightarrow 27.34 = F_2 \cdot 1$$

$$\Rightarrow \boxed{F_2 = 27.34 \text{ N}}$$

Now

$$\vec{F}_1 = \vec{F}_R$$

$$\boxed{\vec{F}_1 = 27.34 \text{ N}}$$

① Two forces act at an angle of 120° the bigger force is 40 N the resultant force is perpendicular to the smaller one Find the smaller force.

Given data

$$\theta = 120^\circ$$

$$\vec{F}_1 = 40 \text{ N}$$

$$\vec{R} \perp \vec{F}_2$$

$$\beta = 90^\circ$$

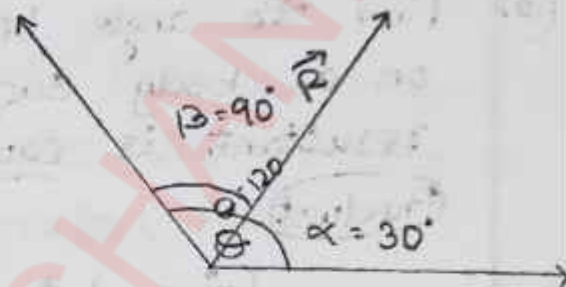
$$\vec{F}_2 = ?$$

$$\theta = \beta + \alpha$$

$$\Rightarrow 120^\circ = 90^\circ + \alpha$$

$$\Rightarrow 120 - 90 = \alpha$$

$$\Rightarrow \alpha = 30^\circ$$



Now we know

$$\alpha = \tan^{-1} \frac{\vec{F}_2 \sin \theta}{\vec{F}_1 + \vec{F}_2 \cos \theta}$$

$$\Rightarrow \tan \alpha = \frac{\vec{F}_2 \sin \theta}{\vec{F}_1 + \vec{F}_2 \cos \theta}$$

$$\Rightarrow \tan 30^\circ = \frac{\vec{F}_2 \sin(120)}{40 + \vec{F}_2 \cos(120)}$$

$$\Rightarrow 0.57 = \frac{\vec{F}_2 (0.86)}{40 + \vec{F}_2 (-0.5)}$$

$$\Rightarrow 0.57 (40 - 0.5 \vec{F}_2) = 0.86 \vec{F}_2$$

$$\Rightarrow (0.57 \times 40) - (0.57 \times 0.5) \vec{F}_2 = 0.86 \vec{F}_2$$

$$\Rightarrow 22.8 - 0.28 \vec{F}_2 = 0.86 \vec{F}_2$$

$$\Rightarrow 22.8 = 0.86 \vec{F}_2 + 0.28 \vec{F}_2$$

$$\Rightarrow 22.8 = \vec{F}_2 (0.86 + 0.28)$$

$$\Rightarrow 22.8 = \vec{F}_2 \cdot 1.14$$

$$\Rightarrow \vec{F}_2 = \frac{22.8}{1.14}$$

$$\Rightarrow \boxed{\vec{F}_2 = 20\text{N}}$$

- (10) Find the angle between two equal force acting on a body such that the square of their resultant is equal to the 3 times of their Product.

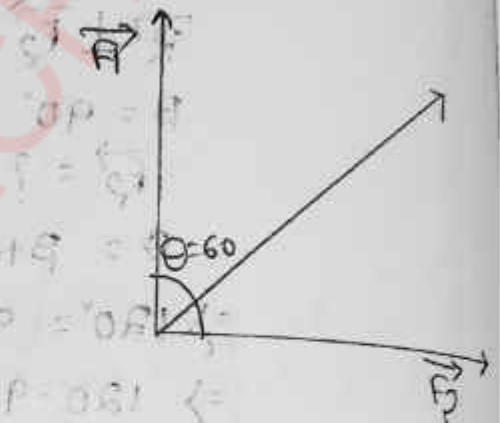
Given data

$$\theta = ?$$

$$\vec{F}_1 = \vec{F}_2 = \vec{F}$$

$$R^2 = 3\vec{F}\vec{F}$$

$$\Rightarrow \boxed{R^2 = 3F^2} \quad \text{--- (1)}$$



Now we know Resultant Force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta}$$

$$\Rightarrow R^2 = F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta$$

$$\Rightarrow R^2 = F^2 + F^2 + 2F \cdot F \cos \theta$$

$$\Rightarrow R^2 = F^2 (1 + 1 + 2 \cos \theta)$$

$$\Rightarrow R^2 = F^2 (2 + 2 \cos \theta)$$

$$\Rightarrow R^2 = 2F^2 (1 + \cos \theta)$$

$$\Rightarrow \boxed{R^2 = 2F^2 (1 + \cos \theta)} \quad \text{--- (2)}$$

Now eqn-① and ② we get

$$3F^2 = 2F^2 (1 + \cos \theta)$$

$$\Rightarrow 3 = 2(1 + \cos \theta)$$

$$\Rightarrow \frac{3}{2} = 1 + \cos \theta$$

$$\Rightarrow 1.5 = 1 + \cos\theta$$

$$\Rightarrow 1.5 - 1 = \cos\theta$$

$$\Rightarrow 0.5 = \cos\theta$$

$$\Rightarrow \theta = \cos^{-1} 0.5$$

$$\Rightarrow \boxed{\theta = 60^\circ}$$

(11)

Two force are act an angle 135° the bigger force is 80N the resultant is perpendicular to the smaller one Find the smaller force.

Given data

$$\theta = 135^\circ$$

$$\vec{F}_1 = 80\text{N}$$

$$\vec{R}_1 \perp \vec{F}_2$$

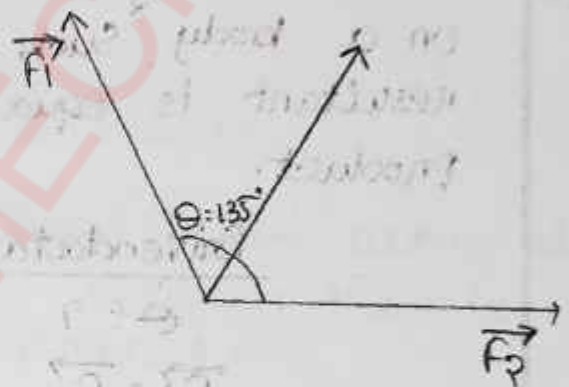
$$\beta = 90^\circ$$

$$\theta = \alpha + \beta$$

$$\Rightarrow 135 = \alpha + 90$$

$$\Rightarrow 135 - 90 = \alpha$$

$$\Rightarrow 45^\circ = \alpha$$



Now we know

$$\alpha = \tan^{-1} \frac{\vec{F}_2 \sin\theta}{\vec{F}_1 + \vec{F}_2 \cos\theta}$$

$$45^\circ = \tan^{-1} \frac{\vec{F}_2 \sin\theta}{\vec{F}_1 + \vec{F}_2 \cos\theta}$$

$$\Rightarrow \tan 45^\circ = \frac{\vec{F}_2 \sin(135^\circ)}{80 + \vec{F}_2 \cos(135^\circ)}$$

$$\Rightarrow 1 = \frac{\vec{F}_2 (0.70)}{80 + \vec{F}_2 (-0.70)}$$

$$\Rightarrow 80 + \vec{F}_2 (-0.70) = \vec{F}_2 (0.70)$$

$$\Rightarrow 80 - (0.70) \vec{F}_2 = \vec{F}_2 (0.70)$$

$$\Rightarrow 80 = 0.70 \vec{F}_2 + 0.70 \vec{F}_2$$

$$\Rightarrow 80 = \vec{F}_2 (0.70 + 0.70)$$

$$\Rightarrow 80 = 1.4 \vec{F}_2$$

$$\Rightarrow \vec{F}_2 = \frac{80}{1.4}$$

$$\Rightarrow \boxed{\vec{F}_2 = 57.14 \text{ N}}$$

(12)

Find the angle between two equal force acting on a body such that the square of their resultant is equal to the four times of their product.

Given data

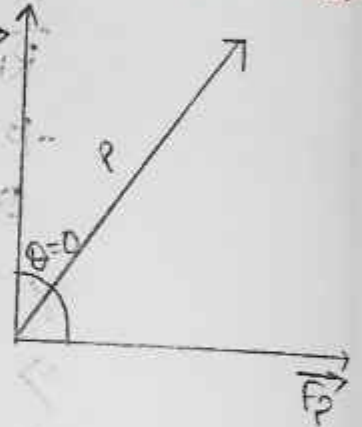
$$\theta = ?$$

$$\vec{F}_1 = \vec{F}_2$$

$$\vec{F}_1 = \vec{F}_2 = F$$

$$R^2 = 4 F F$$

$$\Rightarrow \boxed{R^2 = 4 F^2} \quad \text{--- (1)}$$



Now

$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta}$$

$$\Rightarrow R^2 = F^2 + F^2 + 2 F \cdot F \cos \theta$$

$$\Rightarrow R^2 = F^2 + F^2 + 2 F^2 \cos \theta$$

$$\Rightarrow R^2 = F^2 (1 + 1 + 2 \cos \theta)$$

$$\Rightarrow R^2 = F^2 (2 + 2 \cos \theta)$$

$$\Rightarrow R^2 = 2 F^2 (1 + \cos \theta)$$

$$\Rightarrow \boxed{R^2 = 2 F^2 (1 + \cos \theta)} \quad \text{--- (2)}$$

Now eqⁿ - ① and ② we get

$$4F^2 = 2F^2 (1 + \cos\theta)$$

$$\Rightarrow 4 = 2(1 + \cos\theta)$$

$$\Rightarrow \frac{4}{2} = (1 + \cos\theta)$$

$$\Rightarrow 2 = (1 + \cos\theta)$$

$$\Rightarrow 2 - 1 = \cos\theta$$

$$\Rightarrow 1 = \cos\theta$$

$$\Rightarrow \theta = \cos^{-1} 1$$

$$\Rightarrow \theta = 0^\circ$$

13. The resultant of two equal force acting at a point with an angle 60° between them is 20N . Find the magnitude of force.

Given data

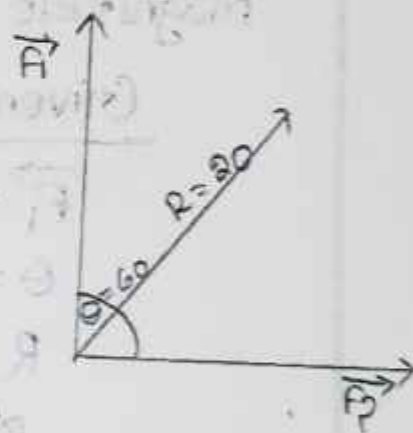
$$\vec{F}_1 = \vec{F}_2 = F$$

$$\theta = 60^\circ$$

$$R = 20\text{N}$$

$$\vec{F}_1 = ?$$

$$\vec{F}_2 = ?$$



We know that

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos\theta}$$

$$\Rightarrow 20 = \sqrt{F^2 + F^2 + 2F \cdot F \cos(60^\circ)}$$

$$\Rightarrow 20 = \sqrt{F^2 + F^2 + 2F^2 \cos(60^\circ)}$$

$$\Rightarrow 20 = \sqrt{F^2 (1 + 1 + 2 \cos 60^\circ)}$$

$$\Rightarrow 20 = \sqrt{F^2 (1 + 1 + 1)}$$

$$\Rightarrow 20 = \sqrt{F^2 \cdot 3}$$

$$20\sqrt{3} = F\sqrt{3}$$

$$\Rightarrow F = \frac{20}{\sqrt{3}}$$

$$\Rightarrow F = \frac{20}{1.73}$$

$$\Rightarrow \boxed{F = 11.56 \text{ N}}$$

We know

$$\vec{F}_1 = \vec{F}_2 = \vec{F}$$

$$\Rightarrow \vec{F}_1 = 11.56 \text{ N}$$

$$\Rightarrow \vec{F}_2 = 11.56 \text{ N}$$

- 14) Two unequal force inclined to one another at an angle of 120° have a resultant force 86.6 N which makes an angle of 30° with one of the force. Find the magnitude of two force.

Given data

$$\vec{F}_1 \neq \vec{F}_2$$

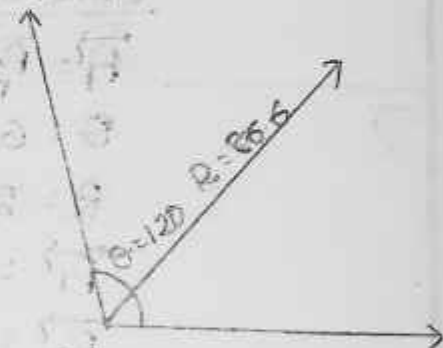
$$\theta = 120^\circ$$

$$R = 86.6 \text{ N}$$

$$\alpha = 30^\circ$$

$$\vec{F}_1 = ?$$

$$\vec{F}_2 = ?$$



Case-1

We know

$$\alpha = \tan^{-1} \left(\frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta} \right)$$

$$\Rightarrow \tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\Rightarrow \tan 30^\circ = \frac{F_2 \sin(120^\circ)}{F_1 + F_2 \cos(120^\circ)}$$

$$\Rightarrow \tan 30^\circ = \frac{F_2 (0.86)}{F_1 + F_2 (-0.5)}$$

$$\Rightarrow 0.57 = \frac{(0.86) F_2}{F_1 + F_2 (-0.5)}$$

$$\Rightarrow 0.57 (F_1 + F_2 (-0.5)) = 0.86 F_2$$

$$\Rightarrow 0.57 F_1 + (0.57 \times 0.5) F_2 = 0.86 F_2$$

$$\Rightarrow 0.57 F_1 - 0.28 F_2 = 0.86 F_2$$

$$\Rightarrow 0.57 F_1 = 0.86 F_2 + 0.28 F_2$$

$$\Rightarrow 0.57 F_1 = F_2 (0.86 + 0.28)$$

$$\Rightarrow 0.57 F_1 = F_2 (1.14)$$

$$\Rightarrow F_1 = \left(\frac{1.14}{0.57} \right) F_2$$

$$\Rightarrow \boxed{F_1 = 2 F_2} \quad \text{--- (1)}$$

Case-II

We know

$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 \cdot F_2 \cos \theta}$$

$$\Rightarrow R = \sqrt{2 F_2^2 + F_2^2 + 2 (2 F_2) \cdot F_2 (\cos \theta)}$$

$$\Rightarrow R = \sqrt{4 F_2^2 + F_2^2 + 4 F_2^2 (\cos \theta)}$$

$$\Rightarrow 86.6 = \sqrt{F_2^2 (4 + 1 + 4 \cos \theta)}$$

$$\Rightarrow 86.6 = \sqrt{F_2^2 (1.73)}$$

$$\Rightarrow 86.6 = F_2 (1.73)$$

$$\Rightarrow F_2 = \frac{86.6}{1.73}$$

$$\boxed{F_2 = 50.05 \text{ N}}$$

Now F_2 value putting eqⁿ ①

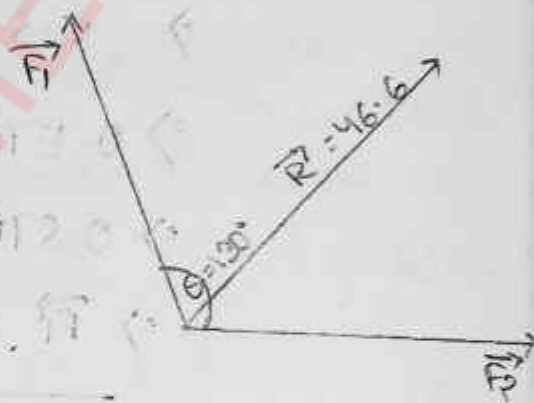
$$\begin{aligned} \vec{F}_1 &= 2\vec{F}_2 \\ &= 2 \times 50.05 \text{ N} \end{aligned}$$

$$\boxed{\vec{F}_1 = 100.1 \text{ N}}$$

- 15) Two equal forces inclined to one another at an angle of 130° have a resultant force 46.6 N which makes an angle of 70° with one of the forces. Find the magnitude of two forces.

Given data

$$\begin{aligned} \vec{F}_1 &= \vec{F}_2 \\ \theta &= 130^\circ \\ R &= 46.6 \text{ N} \\ \alpha &= 70^\circ \\ \vec{F}_1 &= ? \quad \vec{F}_2 = ? \end{aligned}$$



Case-I)

We know

$$\alpha = \tan^{-1} \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\Rightarrow \tan \alpha = \frac{F_2 \sin (130)}{F_1 + F_2 \cos (130)}$$

$$\Rightarrow \tan 70 = \frac{F_2 (0.76)}{F_1 + F_2 (-0.64)}$$

$$\Rightarrow 2.74 = \frac{F_2 (0.76)}{F_1 + F_2 (-0.64)}$$

$$\Rightarrow 2.74(F_1 + F_2 (-0.64)) = F_2 (0.76)$$

$$\Rightarrow 2.74 F_1 + (2.74 \times -0.64) F_2 = F_2 (0.76)$$

$$\Rightarrow 2.74 \vec{F}_1 + (-1.75 \vec{F}_2) = 0.76 \vec{F}_2$$

$$\Rightarrow 2.74 \vec{F}_1 - 1.75 \vec{F}_2 = 0.76 \vec{F}_2$$

$$\Rightarrow 2.74 \vec{F}_1 = 0.76 \vec{F}_2 + 1.75 \vec{F}_2$$

$$\Rightarrow 2.74 \vec{F}_1 = \vec{F}_2 (0.76 + 1.75)$$

$$\Rightarrow 2.74 \vec{F}_1 = 2.51 \vec{F}_2$$

$$\Rightarrow \vec{F}_1 = \left(\frac{2.51}{2.74} \right) \vec{F}_2$$

$$\Rightarrow \boxed{\vec{F}_1 = 0.91 \vec{F}_2}$$

Case-II

We know

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta}$$

$$\Rightarrow 46.6 = \sqrt{(0.91)^2 F_2^2 + F_2^2 + 2 \cdot 0.91 F_2 \cdot F_2 \cos(130^\circ)}$$

$$\Rightarrow 46.6 = \sqrt{0.82 F_2^2 + F_2^2 + 1.82 F_2^2 (-0.64)}$$

$$\Rightarrow 46.6 = \sqrt{F_2^2 (0.82 + 1 + 1.82 (-0.64))}$$

$$\Rightarrow 46.6 = \sqrt{F_2^2 (1.82 - 1.16)}$$

$$\Rightarrow 46.6 = \sqrt{F_2^2 (0.66)}$$

$$\Rightarrow 46.6 = F_2 \sqrt{0.66}$$

$$\Rightarrow 46.6 = F_2 \cdot 0.81$$

$$\Rightarrow F_2 = \frac{46.6}{0.81}$$

$$\Rightarrow \boxed{F_2 = 57.53 \text{ N}}$$

Now $\vec{F}_1$

$$= 0.91 \vec{F}_2$$

$$= 0.91 \times 57.53 \text{ N}$$

$$\boxed{\vec{F}_1 = 52.35 \text{ N}}$$

The resultant of two equal force acting at a point with an angle 80° between them is 30N . Find the magnitude of force.

Grivendata

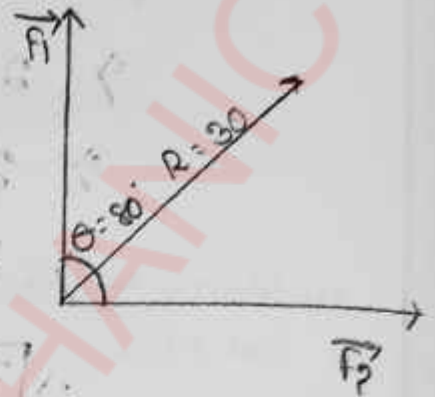
$$\vec{F}_1 = \vec{F}_2 = \vec{F}$$

$$\theta = 80^\circ$$

$$R = 30\text{N}$$

$$\vec{F}_1 = ?$$

$$\vec{F}_2 = ?$$



We know that

Resultant force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos \theta}$$

$$\Rightarrow 80 = \sqrt{F^2 + F^2 + 2F \cdot F \cos(80)}$$

$$\Rightarrow 80 = \sqrt{F^2 + F^2 + 2F^2 \cos(80)}$$

$$\Rightarrow 80 = \sqrt{F^2 + F^2 (0.17)}$$

$$\Rightarrow 80 = \sqrt{F^2 (1 + 1 \times 0.17)}$$

$$\Rightarrow 80 = \sqrt{F^2 (1 + 1 + 0.34)}$$

$$\Rightarrow 80 = \sqrt{F^2 (2.34)}$$

$$\Rightarrow 80 = F \sqrt{2.34}$$

$$\Rightarrow F = \frac{80}{\sqrt{2.34}}$$

$$\Rightarrow \boxed{F = 19.61\text{N}}$$



11

If two force acting the same line the resultant is 12N. If they act at right angle their resultant is $\sqrt{12}$ N Find the two force.

Given data

$$\theta_1 = 0$$

$$\theta_2 = 90^\circ$$

$$R_1 = 12\text{N}$$

$$R_2 = \sqrt{12}\text{N}$$

$$\vec{F}_1 = ?$$

$$\vec{F}_2 = ?$$



Case-I

$$\theta_1 = 0$$

$$R_1 = 12\text{N}$$

We know

$$R_1 = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot \cos \theta}$$

$$\Rightarrow 12 = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot \cos(0)}$$

$$\Rightarrow 12 = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot 1}$$

$$\Rightarrow 12 = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2}$$

$$\Rightarrow 12 = \sqrt{(F_1 + F_2)^2}$$

$$\Rightarrow 12 = F_1 + F_2$$

$$\Rightarrow \boxed{F_1 = 12 - F_2} \quad \text{--- (1)}$$

Case-II

$$\theta_2 = 90^\circ$$

$$R_2 = \sqrt{12}$$

We know

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot \cos \theta}$$

$$\Rightarrow \sqrt{12} = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot \cos 90^\circ}$$

$$\Rightarrow 72 = F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot 0$$

$$\Rightarrow 72 = F_1^2 + F_2^2$$

$$\Rightarrow 72 = (12 - F_2)^2 + F_2^2$$

$$\Rightarrow 72 = (12)^2 + F_2^2 - 2 \cdot 12 \cdot F_2 + F_2^2$$

$$\Rightarrow 72 = 144 + F_2^2 - 24F_2 + F_2^2$$

$$\Rightarrow 144 - 72 + F_2^2 + F_2^2 - 24F_2 = 0$$

$$\Rightarrow 72 + 2F_2^2 - 24F_2 = 0$$

$$\Rightarrow \frac{72}{2} + \frac{2F_2^2}{2} - \frac{24F_2}{2} = 0$$

$$\Rightarrow 36 + F_2^2 - 12F_2 = 0$$

$$\Rightarrow (6)^2 + (F_2)^2 - 2 \cdot 6 \cdot F_2 = 0$$

$$\Rightarrow (6 - F_2)^2 = 0$$

$$\Rightarrow 6 - F_2 = 0$$

$$\Rightarrow 6 = F_2$$

$$\Rightarrow \boxed{F_2 = 6 \text{ N}}$$

Now F_2 value putting eqⁿ - ①

$$F_1 = 12 - F_2$$

$$= 12 - 6 \text{ N}$$

$$\boxed{F_1 = 6 \text{ N}}$$

(18)

If two force act along the same line the resultant is 8N if they at right angle their resultant is $\sqrt{32}$ N. Find the two force.

Given data

$$\theta_1 = 0$$

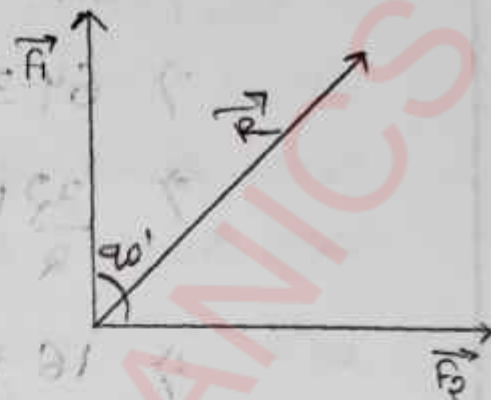
$$R_1 = 8 \text{ N}$$

$$R_2 = \sqrt{32} \text{ N}$$

$$\theta_2 = 90$$

$$F_1 = ?$$

$$F_2 = ?$$



Case-I

We know

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot \cos \theta}$$

$$\Rightarrow 8 = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot \cos(0)}$$

$$\Rightarrow 8 = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot 1}$$

$$\Rightarrow 8 = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2}$$

$$\Rightarrow 8 = \sqrt{(F_1 + F_2)^2}$$

$$\Rightarrow 8 = F_1 + F_2$$

$$\Rightarrow \boxed{F_1 = 8 - F_2} \quad \text{--- (1)}$$

Case-II

We know

$$\theta_2 = 90$$

$$R_2 = \sqrt{32} \text{ N}$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot \cos \theta}$$

$$R = \sqrt{(8 - F_2)^2 + F_2^2 + 2(8 - F_2) \cdot F_2 \cdot \cos(90)}$$

$$\Rightarrow \sqrt{32} = \sqrt{(8 - F_2)^2 + F_2^2 + 2(8 - F_2) \cdot F_2 \cdot 0}$$

$$\Rightarrow 32 = \sqrt{(8 - F_2)^2 + F_2^2 + 2(8 - F_2) \cdot 0}$$

$$\Rightarrow 32 = (8 - F_2)^2 + F_2^2$$

$$\Rightarrow 32 = 64 + F_2^2 - 16F_2 + F_2^2$$

$$\Rightarrow 32 = 64 + 2F_2^2 - 16F_2$$

$$\Rightarrow 64 - 32 + 2F_2^2 - 16F_2 = 0$$

$$\Rightarrow \frac{32}{2} + \frac{2F_2^2}{2} - \frac{16F_2}{2} = 0$$

$$\Rightarrow 16 + F_2^2 - 8F_2 = 0$$

$$\Rightarrow (4)^2 + (F_2)^2 - 2 \cdot 4 \cdot F_2 = 0$$

$$\Rightarrow (4 - F_2)^2 = 0$$

$$\Rightarrow 4 - F_2 = \sqrt{0}$$

$$\Rightarrow 4 - F_2 = 0$$

$$\Rightarrow 4 = F_2$$

$$\Rightarrow \boxed{F_2 = 4\text{ N}}$$

Now F_2 value putting eqn-①

$$F_1 = 8 - F_2$$

$$\Rightarrow F_1 = 8 - 4$$

$$\Rightarrow \boxed{F_1 = 4\text{ N}}$$

(CA-2) Find the magnitude and direction of the resultant force if

- (1) 14N making 30° towards North-east
- (2) 50N making 45° towards West-North
- (3) 60N making 40° towards South-West
- (4) 70N making 60° towards South-east.

Given data

$$R = ?$$

$$\alpha_{act} = ?$$

$$\vec{F}_1 = 14\text{N}$$

$$\theta_1 = 30^\circ$$

$$\vec{F}_2 = 50\text{N}$$

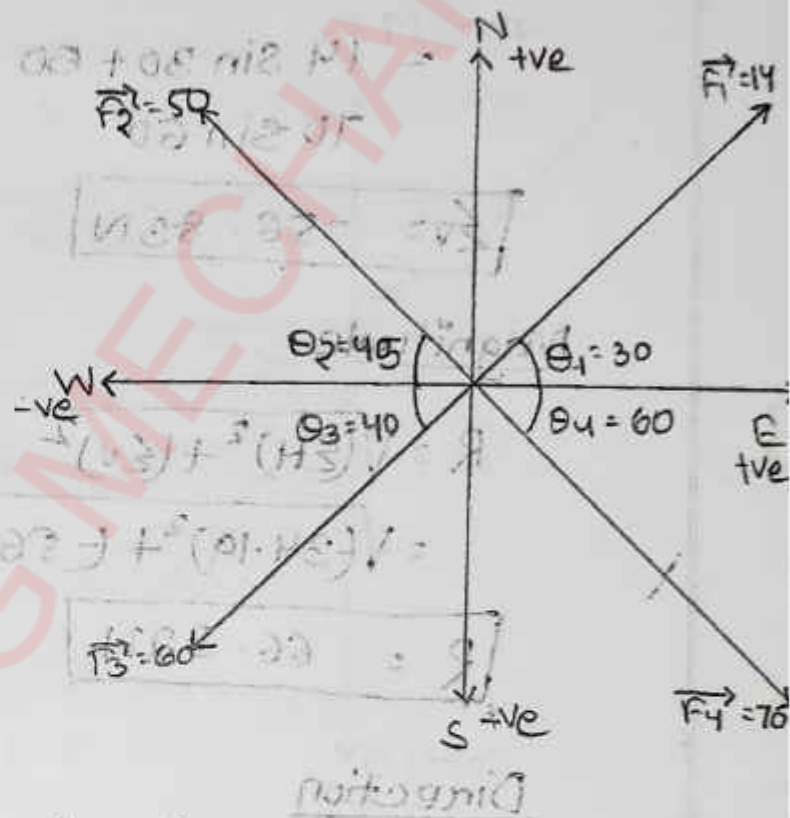
$$\theta_2 = 45^\circ$$

$$\vec{F}_3 = 60\text{N}$$

$$\theta_3 = 40^\circ$$

$$\vec{F}_4 = 70\text{N}$$

$$\theta_4 = 60^\circ$$



Case-1 (Horizontal)

$$\sum H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos(180 - \theta_2) + \vec{F}_3 \cos(180 + \theta_3) + \vec{F}_4 \cos(360 - \theta_4)$$

$$= 14 \cos 30 + 50 \cos(180 - 45) + 60 \cos(180 + 40) + 70 \cos(360 - 60)$$

$$= 14 \cos 30 + 50 \cos 135 + 60 \cos 220 + 70 \cos 300$$

$$\boxed{\sum H = -34.19\text{N}}$$

Case-1) (Vertical)

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin (180 - \theta_2) + \vec{F}_3 \sin (180 + \theta_3) + \vec{F}_4 \sin (360 - \theta_4)$$

$$= \vec{F}_1 \sin 30 + \vec{F}_2 \sin (180 - 45) + \vec{F}_3 \sin (180 + 40) + \vec{F}_4 \sin (360 - 60)$$

$$= 14 \sin 30 + 50 \sin 135^\circ + 60 \sin 220 + 70 \sin 60$$

$$\boxed{\Sigma V = -56.83 \text{ N}}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(-34.19)^2 + (-56.83)^2}$$

$$\boxed{R = 66.32 \text{ N}}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$\theta = \tan^{-1} \left| \frac{-56.83}{-34.19} \right|$$

$$\boxed{\theta = 58.96}$$

Now

$$\Sigma H = -ve$$

$$\Sigma V = -ve$$

$$\alpha_{act} = 180 + \theta_3$$
$$= 180 + 58.96$$

$$\boxed{\alpha_{act} = 238.96^\circ}$$

(20)

The following force act at a point

- (1) 30KN inclined a 35° towards north-East
- (2) 22KN towards north
- (3) 30KN inclined a 30° towards North-west
- (4) 33KN inclined 25° towards South-west

Find the magnitude and direction of the resultant force.

Grivendata

$$\vec{F}_1 = 30 \text{ KN}$$

$$\theta_1 = 35^\circ$$

$$\vec{F}_2 = 22 \text{ KN}$$

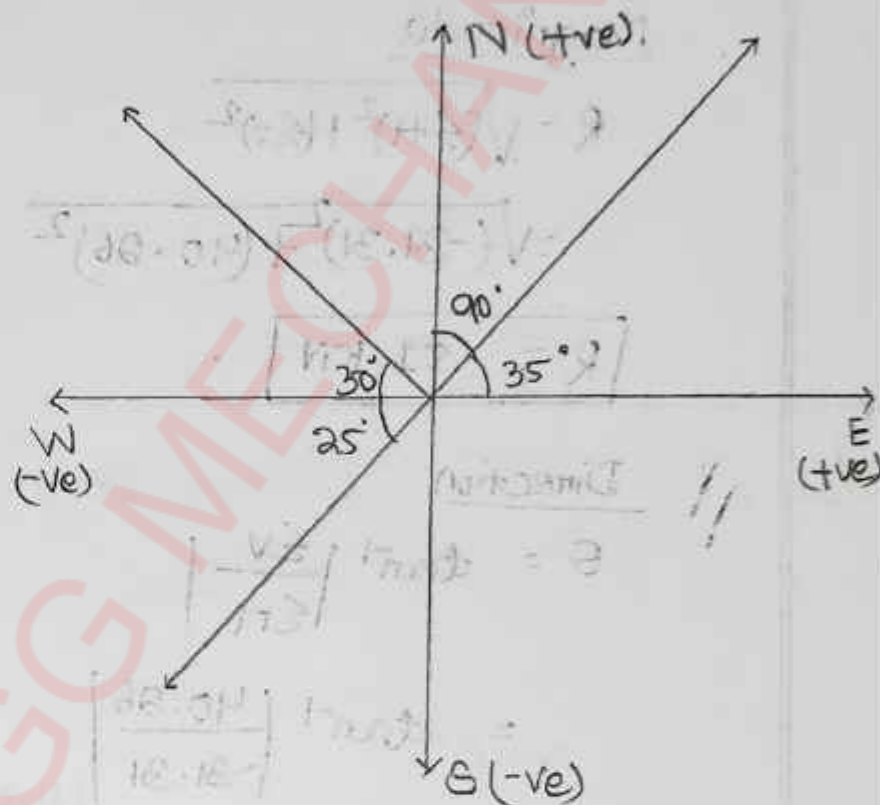
$$\theta_2 = 90^\circ$$

$$\vec{F}_3 = 30 \text{ KN}$$

$$\theta_3 = 30^\circ$$

$$\vec{F}_4 = 33 \text{ KN}$$

$$\theta_4 = 25^\circ$$



Case-I (Horizontal)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos (180 + \theta_3) + \vec{F}_4 \cos (360 - \theta_4)$$

$$= 30 \cos 35^\circ + 22 \cos 90^\circ + 30 \cos (180 + 30) + 33 \cos (180 + 25)$$

$$= 30 \cos 35^\circ + 22 \cos 90^\circ + 30 \cos 150^\circ + 33 \cos 205^\circ$$

$$\boxed{\Sigma H = -31.31 \text{ KN}}$$

Case-II (Vertical)

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin (180 - \theta_3) + \vec{F}_4 \sin (180 + \theta_4)$$

$$= 30 \sin 35 + 22 \sin 90 + 30 \cos (180-30) + 33 \sin (180+25)$$

$$= 30 \sin 35 + 22 \sin 90 + 30 \cos 150 + 33 \sin 205$$

$$\boxed{\sum V = 40.26 \text{ kN}}$$

Magnitude

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

$$= \sqrt{(-31.31)^2 + (40.26)^2}$$

$$\boxed{R = 51 \text{ kN}}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\sum V}{\sum H} \right|$$

$$= \tan^{-1} \left| \frac{40.26}{-31.31} \right|$$

$$= 52.12$$

$$\boxed{\theta = 52.12}$$

Now

$$\sum H = -31.31 \text{ kN (-ve)}$$

$$\sum V = 40.26 \text{ kN (+ve)}$$

$$\alpha_{act} = 180 - \theta$$

$$= 180 - 52.12$$

$$= 127.88$$

② The following forces acting at a point.

- (1) 15N inclined at 30° towards North-East
- (2) 45N towards West
- (3) 25N towards South-West
- (4) 30N inclined at 50° towards South-East

Given data

$$\vec{F}_1 = 15\text{N}$$

$$\theta_1 = 30^\circ$$

$$\vec{F}_2 = 45\text{N}$$

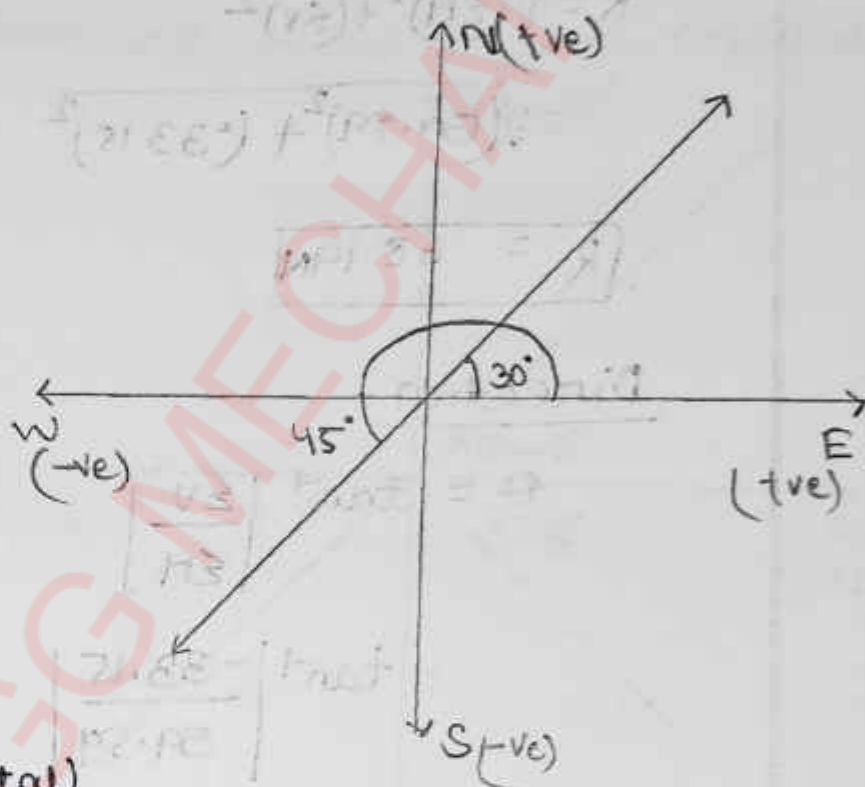
$$\theta_2 = 180^\circ$$

$$\vec{F}_3 = 25\text{N}$$

$$\theta_3 = 45^\circ$$

$$\vec{F}_4 = 30\text{N}$$

$$\theta_4 = 50^\circ$$



Case-I (Horizontal)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos (180 - \theta_2) + \vec{F}_3 \cos (180 + \theta_3) + \vec{F}_4 \cos (360 - \theta_4)$$

$$= 15 \cos 30 + 45 \cos 0 + 25 \cos (180 + 45) + 30 \cos (360 - 50)$$

$$= 15 \cos 30 + 45 \cos 0 + 25 \cos (225) + 30 \cos (310)$$

$$\boxed{\Sigma H = 59.59\text{N}}$$

Case-II (Vertical)

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin (180 - \theta_2) + \vec{F}_3 \sin (180 + \theta_3) + \vec{F}_4 \sin (360 - \theta_4)$$

$$= 15 \sin 30 + 45 \sin (180-180) + 25 \sin (180+95) + 30 \sin (360-50)$$

$$= 15 \sin 30 + 45 \sin 0 + 25 \sin 225 + 30 \sin (310)$$

$$\boxed{\Sigma V = -33.15 \text{ N}}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(59.59)^2 + (-33.15)^2}$$

$$\boxed{R = 68.19 \text{ N}}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{-33.15}{59.59} \right|$$

$$= 29.11$$

$$\boxed{\theta = 29.11^\circ}$$

Now

$$\Sigma H = + 59.59 \text{ (} +ve \text{)}$$

$$\Sigma V = -33.15 \text{ (-ve)}$$

$$\alpha = 360 - \theta$$

$$\alpha_{\text{act}} = 360 - 29.11$$

$$= 330.92^\circ$$

$$\boxed{\alpha_{\text{act}} = 330.92^\circ}$$

23

Following force

- (1) 20N at 45° toward S North-East
- (2) 50N towards north
- (3) 60N towards South-West
- (4) 35N towards South-East

Find magnitude and direction of the resultant

Given data

$$F_1 = 20\text{N}$$

$$\theta_1 = 45^\circ$$

$$F_2 = 50$$

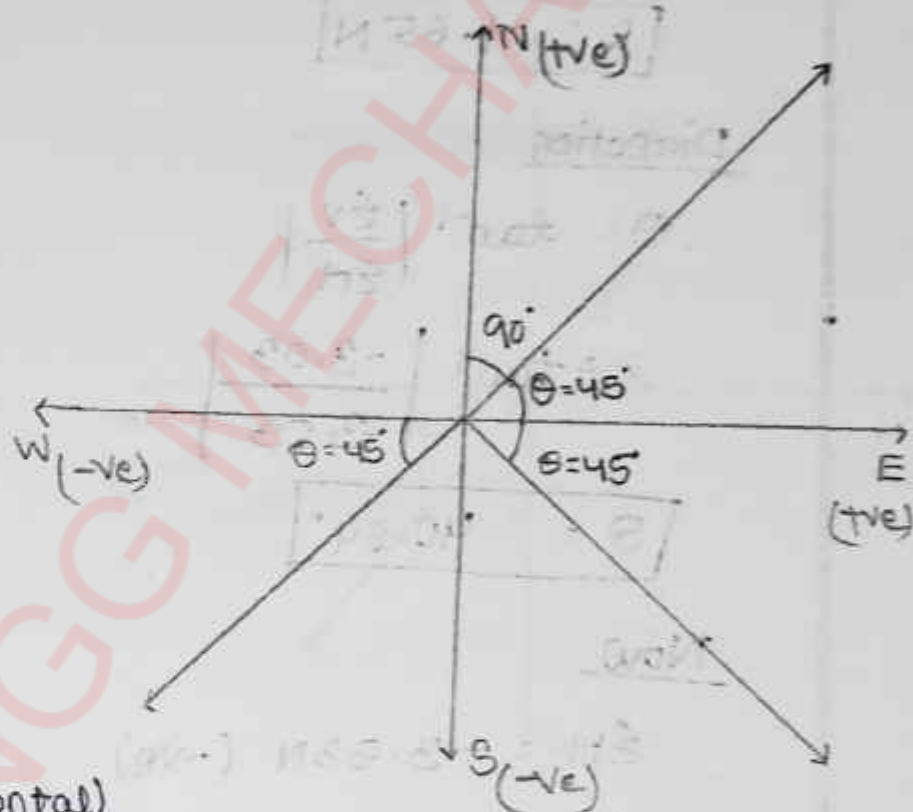
$$\theta_2 = 90^\circ$$

$$F_3 = 60\text{N}$$

$$\theta_3 = 45^\circ$$

$$F_4 = 35$$

$$\theta_4 = 45^\circ$$



Case-I (Horizontal)

$$\begin{aligned} \Sigma H &= F_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos (180 + \theta) + F_4 \cos (360 - \theta) \\ &= 20 \cos 45 + 50 \cos 90 + 60 \cos (180 + 45) + 35 \cos (360 - 45) \\ &= 20 \cos 45 + 50 \cos 90 + 60 \cos (225) + 35 \cos (215) \end{aligned}$$

$$\boxed{\Sigma H = -56.95\text{N}} \\ \boxed{-3.53}$$

Case-II (Vertical)

$$\begin{aligned} \Sigma V &= F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin (180 + \theta) + F_4 \sin (360 - \theta) \\ &= 20 \sin 45 + 50 \sin 90 + 60 \sin (180 + 45) + 35 \sin (360 - 45) \end{aligned}$$

$$= 20 \sin 45 + 50 \sin 90 + 60 \sin 225 + 35 \sin 315$$

$$\Sigma V = -3.03 \text{ N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(-3.53)^2 + (-3.03)^2}$$

$$R = 4.65 \text{ N}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{-3.03}{-3.53} \right|$$

$$\theta = 40.64^\circ$$

Now

$$\Sigma H = -3.53 \text{ N } (-ve)$$

$$\Sigma V = -3.03 \text{ N } (-ve)$$

$$\alpha_{act} = 180 + \theta_3$$

$$= 180 + 40.64$$

$$\alpha_{act} = 220.64^\circ$$

(23)

Following force

(1) 95N towards north-east

(2) 37N towards west

(3) 65N at 55° towards south west

(4) 97N towards south

Find magnitude and direction of the resolution force.

Grivendata

$$\vec{F}_1 = 95\text{N}$$

$$\theta_1 = 45^\circ$$

$$\vec{F}_2 = 37\text{N}$$

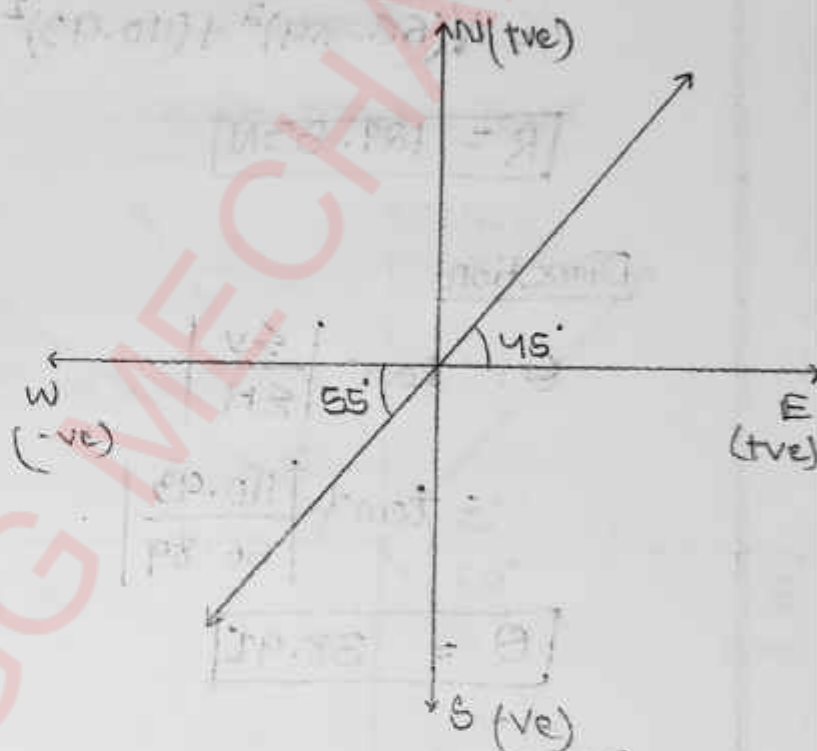
$$\theta_2 = 180^\circ$$

$$\vec{F}_3 = 65\text{N}$$

$$\theta_3 = 55^\circ$$

$$\vec{F}_4 = 97\text{N}$$

$$\theta_4 = 270^\circ$$

Case-1 (Horizontal)

$$\begin{aligned}
 \Sigma H &= \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos (180 - \theta_2) + \vec{F}_3 \cos (180 + \theta_3) + \vec{F}_4 \cos (180 + \theta_4) \\
 &= 95 \cos 45 + 37 \cos (180 - 180) + 65 \cos (180 + 55) + 97 \cos (180 + 270) \\
 &= 95 \cos 45 + 37 \cos 0 + 65 \cos (235) + 97 \cos (450)
 \end{aligned}$$

$$\Sigma H = 66.89\text{N}$$

Case-1) (Vertical)

$$\begin{aligned}
 \Sigma V &= \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin (180 - \theta_2) + \vec{F}_3 \sin (180 + \theta_3) \\
 &\quad + \vec{F}_4 \sin (180 + \theta_4)
 \end{aligned}$$

$$= 95 \sin 45 + 37 \sin (180-180) + 65 \cos (180+55) + 97 \cos (180+270)$$

$$= 95 \sin 45 + 37 \sin 0 + 65 \cos 235 + 97 \cos (450)$$

$$\boxed{\Sigma V = 110.93 \text{ N}}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(66.89)^2 + (110.93)^2}$$

$$\boxed{R = 129.53 \text{ N}}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{110.93}{66.89} \right|$$

$$\boxed{\theta = 58.91^\circ}$$

Now

$$\Sigma H = 66.89 \text{ N (tve)}$$

$$\Sigma V = 110.93 \text{ (tve)}$$

$$\alpha_{act} = \theta$$

$$\boxed{\alpha_{act} = 58.91^\circ}$$

Q

Following force

Find the magnitude and direction of the resultant force if

- (1) 20N towards East
- (2) 75N at 35° towards north-east
- (3) 135N towards north-west
- (4) 60N at 35° towards north-west
- (5) 25N at 60° towards South-East

Given data

$$F_1 = 20N$$

$$\theta_1 = 0^\circ$$

$$F_2 = 75N$$

$$\theta_2 = 35^\circ$$

$$F_3 = 135N$$

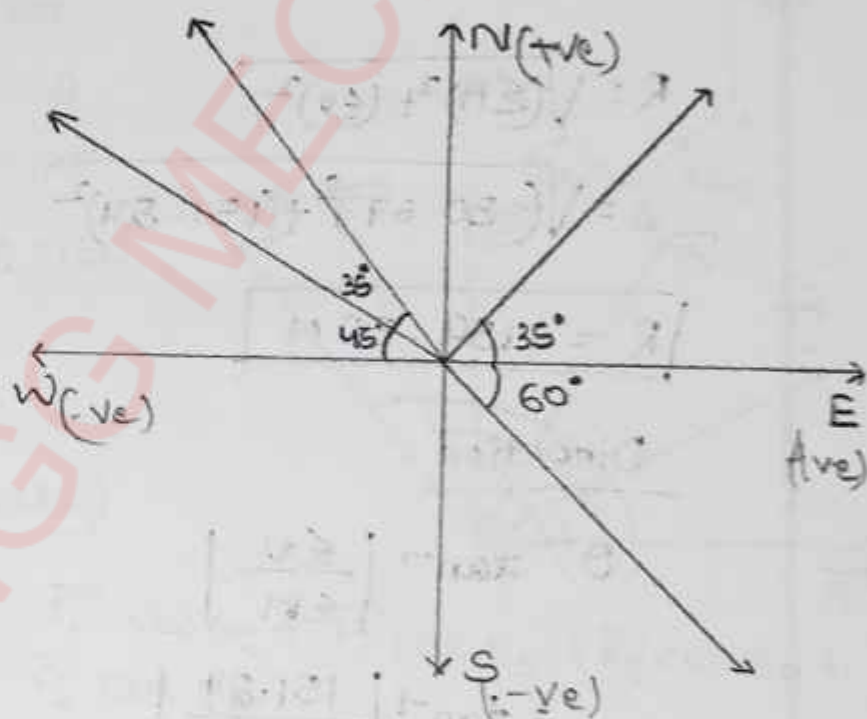
$$\theta_3 = 45^\circ$$

$$F_4 = 60N$$

$$\theta_4 = 35^\circ$$

$$F_5 = 25N$$

$$\theta_5 = 60^\circ$$



Case-1 (Horizontal)

$$\begin{aligned} \sum H &= F_1 \cos \theta_1 + F_2 \cos (180 - \theta_2) + F_3 \cos (180 - \theta_3) + \\ &F_4 \cos (180 - 35^\circ) + F_5 \cos (360 - 60^\circ) \\ &= 20 \cos 0 + 75 \cos 35 + 135 \cos (180 - 45) + \\ &60 \cos (180 - 35) + 25 \cos (360 - 60) \end{aligned}$$

$$\boxed{\sum H = -50.67N}$$

Case-II (Vertical)

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin (180 - \theta_3) + \vec{F}_4 \sin (180 - \theta_4) + \vec{F}_5 \sin (360 - \theta_5)$$

$$= 20 \sin 0 + 75 \sin 35 + 135 \cos (180 - 45) + 60 \cos (180 - 35) + 25 \cos (360 - 60)$$

$$= 20 \sin 0 + 75 \sin 35 + 135 \cos 135 + 60 \cos 145 + 25 \cos 300$$

$$\Sigma V = 151.24 \text{ N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(-50.67)^2 + (151.24)^2}$$

$$R = 159.50 \text{ N}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{151.24}{-50.67} \right|$$

$$\theta = 71.47^\circ$$

Now

$$\Sigma H = -50.67 \text{ N (-ve)}$$

$$\Sigma V = 151.24 \text{ (+ve)}$$

$$\alpha_{\text{act}} = 180 - \theta$$

$$= 180 - 71.47$$

$$\alpha_{\text{act}} = 108.53^\circ$$

(25)

Force 10N, 30N, 40N, 60N, 50N are acting on one angular point of a regular hexagon towards the other 5 angular point taken in order. Find the direction and magnitude of the resultant force. Find the magnitude and direction of the resultant force as shown in figure.

Given data

$$F_1 = 50\text{N}, \theta_1 = 0$$

$$F_2 = 55\text{N}, \theta_2 = 30$$

$$F_3 = 60\text{N}, \theta_3 = 60$$

$$F_4 = 65\text{N}, \theta_4 = 90$$

$$F_5 = 70\text{N}, \theta_5 = 120$$

$$F_6 = 75\text{N}, \theta_6 = 150$$

$$R = ?$$

$$\alpha_{\text{act}} = ?$$

Case-I (Horizontal)

$$\sum H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 + \vec{F}_4 \cos \theta_4 + \vec{F}_5 \cos \theta_5 + \vec{F}_6 \cos \theta_6$$

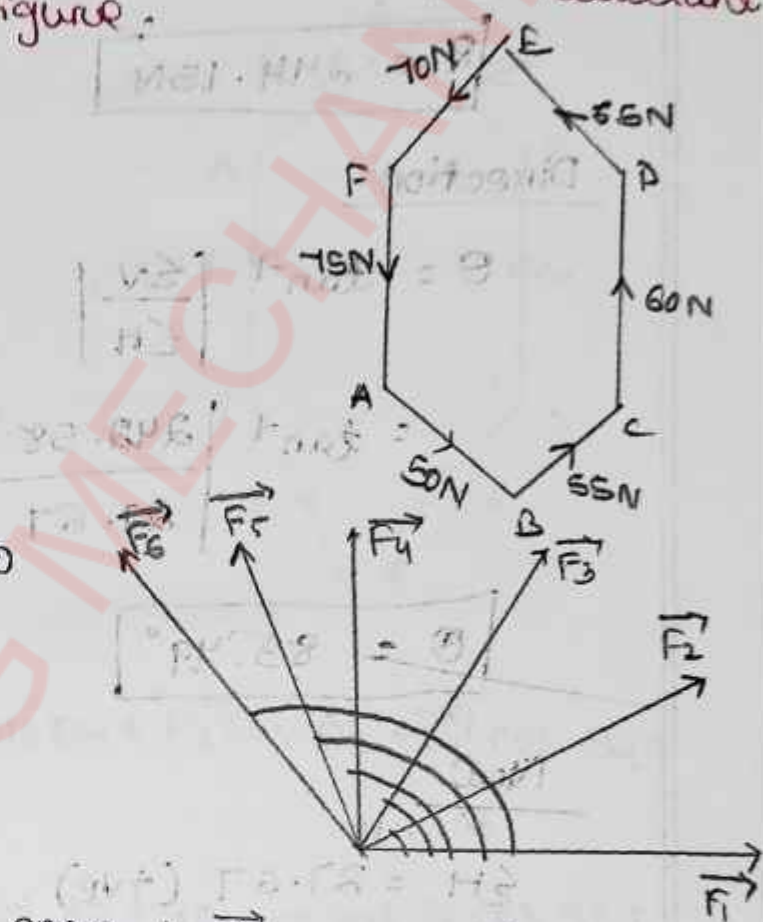
$$= 50 \cos 0 + 55 \cos 30 + 60 \cos 60 + 65 \cos 90 + 70 \cos 120 + 75 \cos 150$$

$$\boxed{\sum H = 27.67\text{N}}$$

Case-II (Vertical)

$$\sum V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 + \vec{F}_4 \sin \theta_4 + \vec{F}_5 \sin \theta_5 + \vec{F}_6 \sin \theta_6$$

$$= 50 \sin 0 + 55 \sin 30 + 60 \sin 60 + 65 \sin 90 + 70 \sin 120 + 75 \sin 150$$



$$\Sigma v = 242.58 \text{ N}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma v)^2}$$

$$= \sqrt{(27.67)^2 + (242.58)^2}$$

$$R = 244.15 \text{ N}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma v}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{242.58}{27.67} \right|$$

$$\theta = 83.49^\circ$$

Now

$$\Sigma H = 27.67 \text{ (+ve)}$$

$$\Sigma v = 242.58 \text{ (+ve)}$$

$$\alpha_{act} = \theta$$

$$\alpha_{act} = 83.49^\circ$$

Force 10N, 30N, 40N, 60N, 80N are acting on one angular point of a regular hexagon towards the other 5 angular point taken in order. Find the direction and magnitude of the resultant force.

Given data

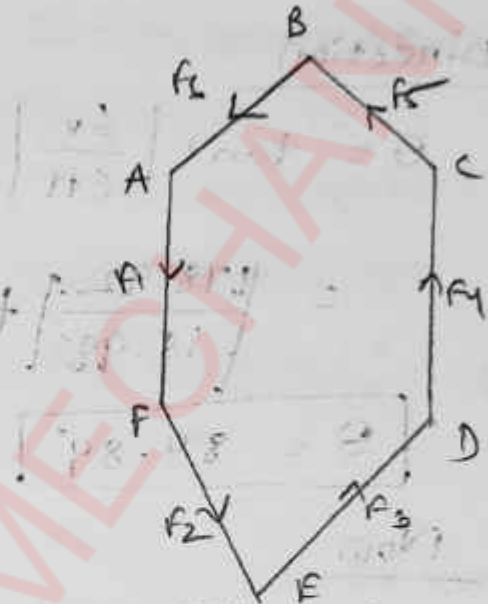
$$\vec{F}_1 = 10\text{N}, \theta_1 = 0^\circ$$

$$\vec{F}_2 = 30\text{N}, \theta_2 = 30^\circ$$

$$\vec{F}_3 = 40\text{N}, \theta_3 = 60^\circ$$

$$\vec{F}_4 = 60\text{N}, \theta_4 = 90^\circ$$

$$\vec{F}_5 = 80\text{N}, \theta_5 = 120^\circ$$



Case-I (Horizontal)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 + \vec{F}_4 \cos \theta_4 + \vec{F}_5 \cos \theta_5 + \vec{F}_6$$

$$= 10 \cos 0 + 30 \cos 30 + 40 \cos 60 + 60 \cos 90 + 80 \cos 120$$

$$\boxed{\Sigma H = 15.98\text{N}}$$

Case-II (Vertical)

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 + \vec{F}_4 \sin \theta_4 + \vec{F}_5 \sin \theta_5$$

$$= 10 \sin 0 + 30 \sin 30 + 40 \sin 60 + 60 \sin 90 + 80 \sin 120$$

$$\boxed{\Sigma V = 178.92\text{N}}$$

Magnitude

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$
$$= \sqrt{(15.98)^2 + (178.92)^2}$$

$$R = 179.63 \text{ N}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\sum V}{\sum H} \right|$$

$$= \left| \frac{178.92}{15.98} \right| \tan^{-1}$$

$$\theta = 84.89^\circ$$

Now

$$\sum V = 178.92 \text{ (+ve)}$$

$$\sum H = 15.98 \text{ (+ve)}$$

$$\alpha_{\text{act}} = \theta$$

$$\alpha_{\text{act}} = 84.89^\circ$$

- (21) Find the magnitude and direction of the resultant force as shown in figure:

Given data

$$\vec{F}_1 = 20, \theta_1 = 0$$

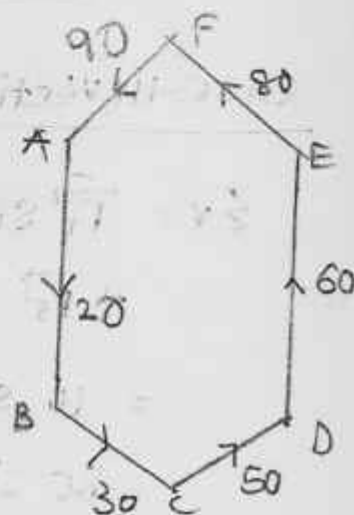
$$\vec{F}_2 = 30, \theta_2 = 30$$

$$\vec{F}_3 = 50, \theta_3 = 60$$

$$\vec{F}_4 = 60, \theta_4 = 90$$

$$\vec{F}_5 = 80, \theta_5 = 120$$

$$\vec{F}_6 = 90, \theta_6 = 150$$



Case-1 (Horizontal)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 + \vec{F}_4 \cos \theta_4 + \vec{F}_5 \cos \theta_5 + \vec{F}_6 \cos \theta_6$$

$$= 20 \cos 0 + 30 \cos 30 + 50 \cos 60 + 60 \cos 90 + 80 \cos 120 + 90 \cos 150$$

$$\boxed{\Sigma H = -46.96 \text{ N}}$$

Case-II

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 + \vec{F}_4 \sin \theta_4 + \vec{F}_5 \sin \theta_5 + \vec{F}_6 \sin \theta_6$$

$$= 20 \sin 0 + 30 \sin 30 + 50 \sin 60 + 60 \sin 90 + 80 \sin 120 + 90 \sin 150$$

$$\boxed{\Sigma V = 232.58 \text{ N}}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(-46.96)^2 + (232.58)^2}$$

$$\boxed{R = 237.27 \text{ N}}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{232.58}{-46.96} \right|$$

$$\boxed{\theta = 78.58^\circ}$$

Now

$$\Sigma V = +232.58 \text{ (+ve)}$$

$$\Sigma H = -46.96 \text{ (-ve)}$$

$$\alpha_{\text{act}} = \theta$$

$$= 78.58^\circ$$

$$\boxed{\alpha_{\text{act}} = 78.58^\circ}$$

- 26) 6 force 25N, 35N, 45N, 55N, 65N, 75N are acting on angular point with regular hexagon towards the other 6 force angular point taken in order. Find magnitude and direction of the resultant force.

Given data

$$\vec{F}_1 = 25, \theta_1 = 0$$

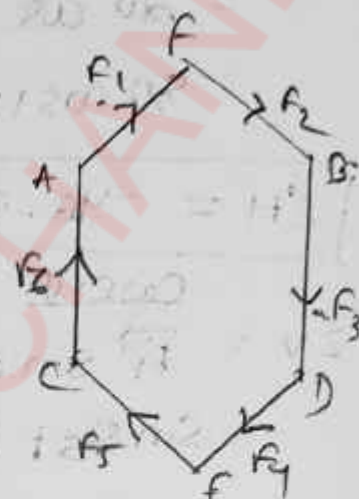
$$\vec{F}_2 = 35N, \theta_2 = 30$$

$$\vec{F}_3 = 45N, \theta_3 = 60$$

$$\vec{F}_4 = 55N, \theta_4 = 90$$

$$\vec{F}_5 = 65N, \theta_5 = 120$$

$$\vec{F}_6 = 75N, \theta_6 = 150$$



Case-I (Horizontal)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 + \vec{F}_4 \cos \theta_4 + \vec{F}_5 \cos \theta_5 + \vec{F}_6 \cos \theta_6$$

$$= 25 \cos 0 + 35 \cos 30 + 45 \cos 60 + 55 \cos 90 + 65 \cos 120 + 75 \cos 150 =$$

$$\boxed{\Sigma H = -19.64N}$$

Case-II

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 + \vec{F}_4 \sin \theta_4 + \vec{F}_5 \sin \theta_5 + \vec{F}_6 \sin \theta_6$$

$$= 25 \sin 0 + 35 \sin 30 + 45 \sin 60 + 55 \sin 90 + 65 \sin 120 + 75 \sin 150$$

$$\boxed{\Sigma V = 205.26N}$$

$$R = \sqrt{(\sum V)^2 + (\sum H)^2}$$

$$= \sqrt{(205.26)^2 + (-19.64)^2}$$

$$R = 206.19 \text{ N}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\sum V}{\sum H} \right|$$

$$= \tan^{-1} \left| \frac{205.26}{-19.64} \right|$$

$$\theta = 84.53^\circ$$

Now

$$\sum H = -19.64$$

$$\sum V = 205.26$$

$$\alpha_{\text{act}} = \theta$$

$$= 84.53^\circ$$

$$\alpha_{\text{act}} = 84.53^\circ$$

Q9) The practice is at by the force P , $2P$, $3\sqrt{3}P$ and $4P$. The angle between first force and second force, Second force and third force, Third force and fourth force are 60° , 90° , 150° . Respectly find the magnitude and direction of the resultant force.

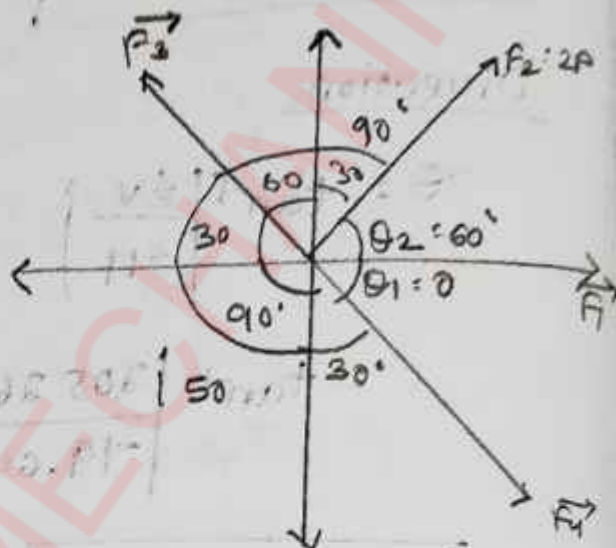
Given data

$$\vec{F}_1 = P \quad \theta_1 = 0$$

$$\vec{F}_2 = 2P \quad \theta_2 = 60$$

$$\vec{F}_3 = 3\sqrt{3}P \quad \theta_3 = 30$$

$$\vec{F}_4 = 4P \quad \theta_4 = 60$$



Case-I (H)

$$\sum H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos (180 - \theta_3) + \vec{F}_4 \cos (360 - \theta_4)$$

$$= P \cos 0 + 2P \cos 60 + 3\sqrt{3}P \cos (180 - 30) + 4P \cos (360 - 60)$$

$$= P (\cos 0 + 2 \cos 60 + 3\sqrt{3} \cos 150 + 4 \cos 300)$$

$$\boxed{\sum H = -0.5P}$$

Case-II (V)

$$\sum V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin (180 - \theta_3) + \vec{F}_4 \sin (360 - \theta_4)$$

$$= P \sin 0 + 2P \sin 60 + 3\sqrt{3}P \sin (180 - 30)$$

$$+ 4P \sin (360 - 60)$$

$$= P (\sin 0 + 2 \sin 60 + 3\sqrt{3} \sin 150 + 4 \sin 300)$$

$$\boxed{\sum V = 0.86P}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(-0.5)^2 + (0.86)^2}$$
$$= 0.99P$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$
$$= \tan^{-1} \left| \frac{0.86}{-0.5} \right|$$

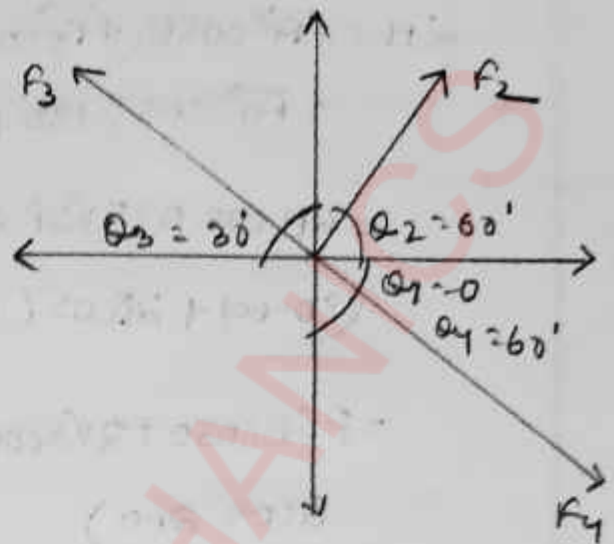
$$\theta = 59.82^\circ$$

Now

$$\Sigma H = -ve$$

$$\Sigma V = +ve$$

$$\alpha_{act} = 180 - \theta =$$
$$= 180 - 59.82 = 120.18^\circ$$



- ③ Particle is act by the force $6P$, $2\sqrt{3}P$, $8P$, $2P$ the angle between them 1st and 2nd force, 2nd and 3rd force, 3rd and 4th force are 70° , 50° , 140° respectively find the magnitude and direction of the resultant force.

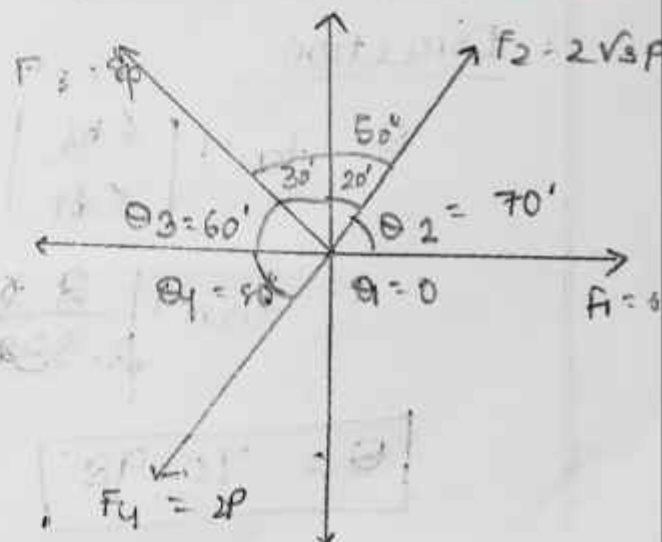
Given data

$$\vec{F}_1 = 6P \quad \theta = 0$$

$$\vec{F}_2 = 2\sqrt{3}P \quad \theta_2 = 70$$

$$\vec{F}_3 = 8P \quad \theta_3 = 50$$

$$\vec{F}_4 = 2P \quad \theta_4 = 140$$



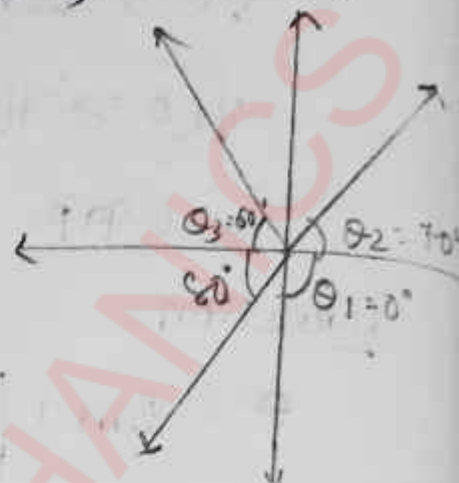
Case-1 (H)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos (180 - \theta_3) + \vec{F}_4 \cos (180 + \theta_3)$$

$$= 6P \cos 0 + 2\sqrt{3}P \cos 70 + 8P \cos (180 - 60) + 2P \cos (180 + 80)$$

$$= P(6 \cos 0 + 2\sqrt{3} \cos 70 + 8 \cos 120 + 2 \cos 260)$$

$$\boxed{\Sigma H = 2.83P}$$



$$\begin{aligned} \Sigma V &= \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin (180 - \theta_3) + \vec{F}_4 \sin (180 + \theta_3) \\ &= 6P \sin 0 + 2\sqrt{3}P \sin 70 + 8P \sin (180 - 60) + 2P \sin (180 + 80) \\ &= P(6 \sin 0 + 2\sqrt{3} \sin 70 + 8 \sin 120 + 2 \sin 260) \end{aligned}$$

$$\boxed{\Sigma V = 8.21P}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(2.83)^2 + (8.21)^2}$$

$$\boxed{R = 8.68P}$$

Direction

$$\begin{aligned} \theta &= \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right| \\ &= \tan^{-1} \left| \frac{8.21}{2.83} \right| \end{aligned}$$

$$\boxed{\theta = 70.98^\circ}$$

Now

$$\Sigma H = \text{tve}$$

$$\Sigma V = \text{tve}$$

$$\alpha_{\text{act}} = \theta$$

$$\boxed{\alpha_{\text{act}} = 70.98^\circ}$$

31) A triangle abc has its sides $ab = 40\text{mm}$ along positively x-axis and side $bc = 30\text{mm}$ along positively y-axis. Three forces of 40N , 50N & 30N act along the sides of ab , bc and c respectively. Find the magnitude and direction of the resultant.

Given data

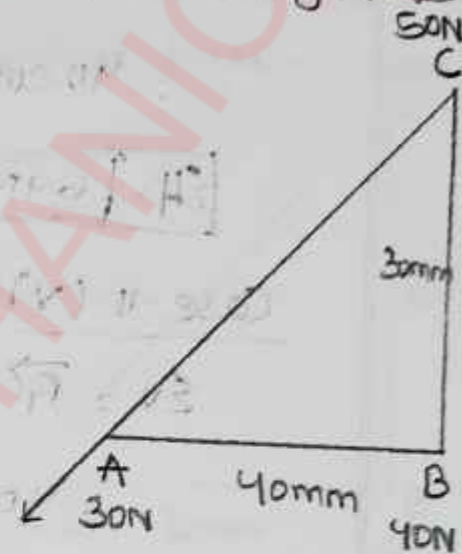
$$AB = 40\text{mm}$$

$$BC = 30\text{mm}$$

$$\vec{F}_1 = AB = 40\text{N} \text{ (tve)} \quad \theta_1 = 0$$

$$\vec{F}_2 = BC = 30\text{N} \text{ (tve)} \quad \theta_2 = 90$$

$$\vec{F}_3 = CA = 30\text{N} \text{ (tve)} \quad \theta_3 = \theta$$



→ Now ΔABC

$$h = \sqrt{p^2 + b^2}$$

$$= \sqrt{(30)^2 + (40)^2}$$

$$= \sqrt{900 + 1600}$$

$$= \sqrt{2500}$$

$$h = 50\text{mm}$$

$$\rightarrow \sin \theta = \frac{p}{h}$$

$$= \frac{30}{50}$$

$$\sin \theta = 0.6$$

$$\rightarrow \cos \theta = \frac{b}{h}$$

$$= \frac{40}{50}$$

$$\cos \theta = 0.8$$

Case-I (H)

$$\begin{aligned}\Sigma H &= \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 \\ &= 40 \cos 0 + 50 \cos 90 + 30 \cos \theta \\ &= 40 \cos 0 + 50 \cos 90 + 30 \cdot 0.8\end{aligned}$$

$$\boxed{\Sigma H = 64 \text{ N}}$$

Case-II (V)

$$\begin{aligned}\Sigma V &= \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 \\ &= 40 \sin 0 + 50 \sin 90 + 30 \sin \theta \\ &= 40 \sin 0 + 50 \sin 90 + 30 \cdot 0.6\end{aligned}$$

$$\boxed{\Sigma V = 68 \text{ N}}$$

Magnitude

$$\begin{aligned}R &= \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \\ &= \sqrt{(64)^2 + (68)^2}\end{aligned}$$

$$\boxed{R = 93.38 \text{ N}}$$

Direction

$$\begin{aligned}\theta &= \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right| \\ &= \tan^{-1} \left| \frac{68}{64} \right|\end{aligned}$$

$$\boxed{\theta = 46.73^\circ}$$

Now,

$$\Sigma H = +ve$$

$$\Sigma V = +ve$$

$$\boxed{\theta_{act} = \theta = 46.73^\circ}$$

Q2

A triangle ABC has its side AB = 50mm along positively x-axis and side BC = 30mm along positive y-axis. Three forces are 60N, 70N, 80N at along the side of AB, BC and CA respectively find the magnitude and direction of the resultant force.

Given data

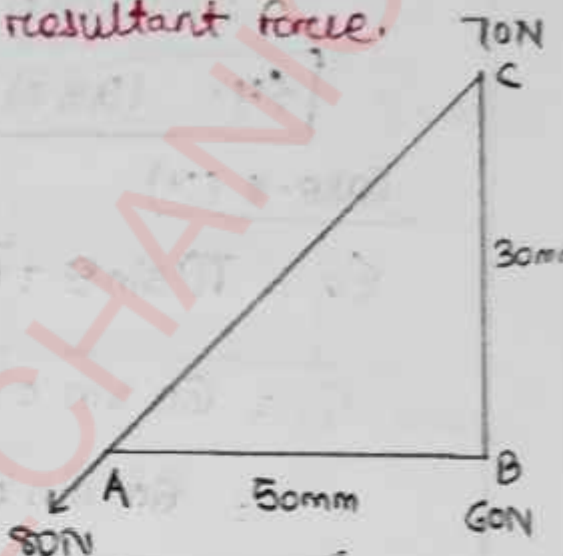
$$AB = 50\text{mm}$$

$$BC = 30\text{mm}$$

$$\vec{F}_1 = AB = 60\text{N}$$

$$\vec{F}_2 = BC = 70\text{N}$$

$$\vec{F}_3 = CA = 80\text{N}$$



→ Now ΔABC

$$h = \sqrt{p^2 + b^2}$$

$$= \sqrt{(30)^2 + (50)^2}$$

$$= \sqrt{900 + 2500}$$

$$\boxed{h = 58.30\text{mm}}$$

$$\rightarrow \sin \theta = \frac{p}{h}$$

$$= \frac{30}{58.30}$$

$$\boxed{\sin \theta = 0.51}$$

$$\rightarrow \cos \theta = \frac{b}{h}$$

$$= \frac{50}{58.30}$$

$$\boxed{\cos \theta = 0.85}$$

Case-I (H)

$$\Sigma H = \vec{F}_1 \cos \theta + \vec{F}_2 \cos \theta + \vec{F}_3 \cos \theta$$

$$= 60 \cos 0 + 70 \cos 90 + 80 \cos \theta$$

$$= 60 \cos 0 + 70 \cos 90 + 80 \cdot 0.85$$

$$\boxed{\Sigma H = 128 \text{ N}}$$

Case-II (V)

$$\Sigma V = \vec{F}_1 \sin \theta + \vec{F}_2 \sin \theta + \vec{F}_3 \sin \theta$$

$$= 60 \sin 0 + 70 \sin 90 + 80 \sin \theta$$

$$= 60 \sin 0 + 70 \sin 90 + 80 \cdot 0.51$$

$$\boxed{\Sigma V = 110.8 \text{ N}}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(128)^2 + (110.8)^2}$$

$$\boxed{R = 169.29 \text{ N}}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{110.8}{128} \right|$$

$$\boxed{\theta = 40.88^\circ}$$

Now,

$$\Sigma H = +ve$$

$$\Sigma V = +ve$$

$$\boxed{\alpha_{act} = \theta = 40.88^\circ}$$

(33)

A system of force are acting at the block as shown in the figure determine the magnitude and direction of the resultant force.

Given data

$$\vec{F}_1 = 20 \text{ kN } (-ve) (H)$$

$$\theta_1 = 0^\circ$$

$$\vec{F}_2 = 35 \text{ kN } (-ve) (V)$$

$$\theta_2 = 90^\circ$$

$$\vec{F}_3 = 25 \text{ kN } (+ve) (H)$$

$$\theta_3 = 0^\circ$$

$$\vec{F}_4 = 50 \text{ kN } (+ve) (V)$$

$$\theta_4 = 90^\circ$$



Case-I (H)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_3 \cos \theta_3$$

$$= -20 \cos 0 + 25 \cos 0$$

$$\boxed{\Sigma H = 5 \text{ kN}}$$

Case-II (V)

$$\Sigma V = \vec{F}_2 \sin \theta_2 + \vec{F}_4 \sin \theta_4$$

$$= -35 \sin 90 + (-50) \sin 90$$

$$= -35 \sin 90 - 50 \sin 90$$

$$\boxed{\Sigma V = -85 \text{ kN}}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(5)^2 + (85)^2}$$

$$\boxed{R = 85.14 \text{ kN}}$$

Direction

$$\theta = \tan^{-1} \left(\frac{\sum V}{\sum H} \right)$$
$$= \tan^{-1} \left(\frac{85}{5} \right)$$

$$\theta = 86.63^\circ$$

Now,

$$\sum H = +ve$$

$$\sum V = -ve$$

$$\alpha_{\text{act}} = 360 - \theta$$

$$= 360 - 86.63 = 273.37^\circ$$

3. A system of force are acting at the block as shown in figure determine the magnitude and direction of the resultant force.

Given data

$$\vec{F}_1 = 60 \text{ kN } (-ve)$$

$$\theta_1 = 0$$

$$\vec{F}_2 = 40 \text{ kN } (-ve)$$

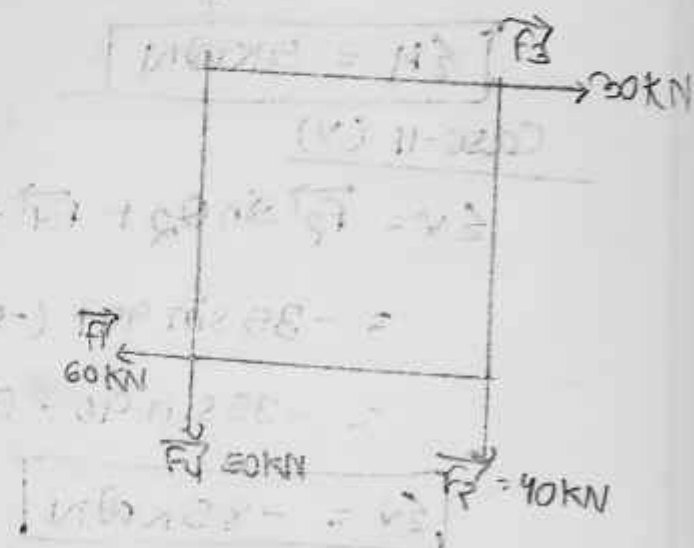
$$\theta_2 = 90$$

$$\vec{F}_3 = 30 \text{ kN } (+ve) (+)$$

$$\theta_3 = 0$$

$$\vec{F}_4 = 50 \text{ kN } (-ve) (-)$$

$$\theta_4 = 90$$



Case-1 (H)

$$\sum H = F_1 \cos \theta_1 + F_3 \cos \theta_3$$

$$= -60 \cos 0 + 30 \cos 0$$

$$\sum H = -30 \text{ kN}$$

Case-II (v)

$$\Sigma v = F_2 \cos \theta_R + F_4 \cos \theta_4$$

$$= -40 \cos 90 + (50) \cos 90$$

$$\boxed{\Sigma v = -90 \text{ kN}}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma v)^2}$$

$$= \sqrt{(90)^2 + (90)^2}$$

$$= \sqrt{(8100)}$$

$$\boxed{R = 90.80 \text{ kN}}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma v}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{-90}{-90} \right|$$

$$\boxed{\theta = 71.56}$$

Now

$$\Sigma H = -ve$$

$$\Sigma v = -ve$$

$$\alpha_{act} = 180 + \theta$$

$$= 180 + 71.56$$

$$\boxed{\alpha_{act} = 251.56}$$

2. Find the magⁿ and dirⁿ of the resultant force.

- (i) A force 20N acting at east
- (ii) The force of 25N acting at 40° to the north-east
- (iii) A force of 30N acting at vertically upwards
- (iv) The force of 10N acting at vertically downwards.

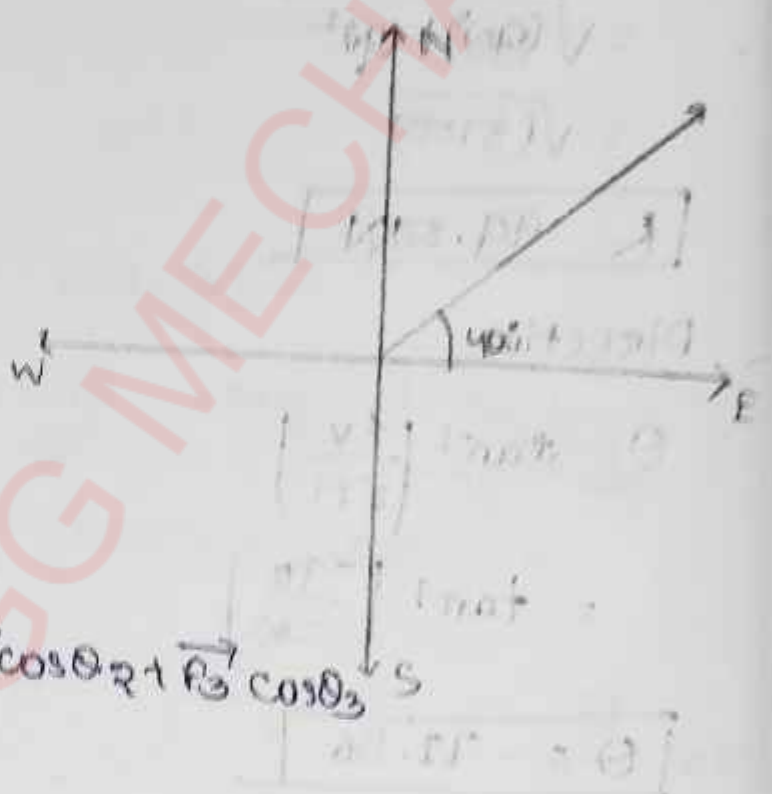
Given data

$$\vec{F}_1 = 20\text{N} \quad \theta_1 = 0$$

$$\vec{F}_2 = 25\text{N} \quad \theta_2 = 40$$

$$\vec{F}_3 = 30\text{N} \quad \theta_3 = 90$$

$$\vec{F}_4 = 10\text{N} \quad \theta_4 = 270$$



Case-I (H)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 + \vec{F}_4 \cos \theta_4$$

$$= 20 \cos 0 + 25 \cos 40 + 30 \cos 90 + 10 \cos 270$$

$$\boxed{\Sigma H = 39.15\text{N}}$$

Case-II (V)

$$\Sigma V = \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 + \vec{F}_4 \sin \theta_4$$

$$= 20 \sin 0 + 25 \sin 40 + 30 \sin 90 + 10 \sin 270$$

$$\boxed{\Sigma V = 36.06\text{N}}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(39.15)^2 + (36.06)^2} = 53.22\text{N}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{36.06}{39.15} \right|$$

$$\boxed{\theta = 42.64^\circ}$$

Now,

$$\Sigma V = +ve$$

$$\Sigma H = +ve$$

$$\boxed{\theta = 42.64^\circ}$$

36

Moment

Two like parallel forces 20N and 10N are acting between a distance of 160mm find the magnitude and point of action of the resultant force.

Given data

$$F_1 = 20N$$

$$F_2 = 10N$$

$$l = 160 \text{ mm}$$

$$R = ?$$

$$x = ? \text{ (Point of action)}$$

Magnitude (like parallel)

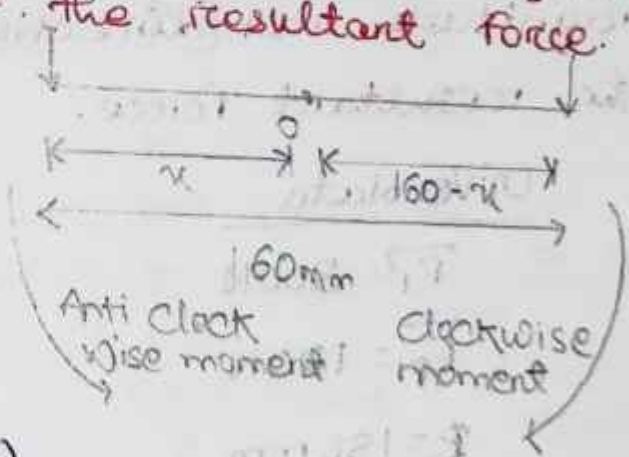
$$R = F_1 + F_2$$

$$= 20 + 10$$

$$\boxed{R = 30N}$$

Point of action

→ According to the law of moment



→ Clockwise moment + Anticlockwise moment = 0

$$\Rightarrow \vec{F}_2 \times (160 - x) + [-x \vec{F}_1] = 0$$

$$\Rightarrow 10 \times (160 - x) + (-x \cdot 20) = 0$$

$$\Rightarrow 1600 - 10x - 20x = 0$$

$$\Rightarrow 1600 - 30x = 0$$

$$\Rightarrow 1600 = 30x$$

$$\Rightarrow \frac{1600}{30} = x$$

$$\Rightarrow x = \frac{1600}{30}$$

$$\Rightarrow \boxed{x = 53.33 \text{ mm}}$$

③ To unlike parallel force 200N and 120N are acting between a distance of 180mm find magnitude and [direction] point of action of the resultant force.

Given data

$$\vec{F}_1 = 200\text{N}$$

$$\vec{F}_2 = 120\text{N}$$

$$L = 180 \text{ mm}$$

$$R = ?$$

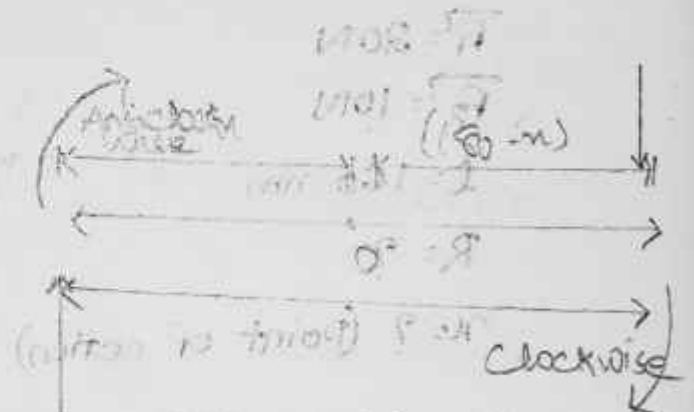
$$x = ? \text{ (Point of action)}$$

Magnitude

$$\vec{R} = \vec{F}_1 - \vec{F}_2$$

$$= 200 - 120$$

$$\boxed{R = 80\text{N}}$$



Point of action

According to the law of moment

$$\boxed{\sum m = 0}$$

$$\Rightarrow \vec{F}_1 + \vec{F}_2 (180 - x) = 0$$

$$\Rightarrow 200x + 120(180 - x) = 0$$

$$\Rightarrow 200x + 21600 - 120x = 0$$

$$\Rightarrow 21600 + 80x = 0$$

$$\Rightarrow 80x = -21600$$

$$\Rightarrow x = \frac{-21600}{80}$$

$$\Rightarrow \boxed{x = -270 \text{ mm}}$$

38

To like parallel force of 6N and 12N are acting a distance of 60mm find the magnitude and point of action of the force.

Given data

$$\vec{F}_1 = 6\text{N}$$

$$\vec{F}_2 = 12\text{N}$$

$$L = 60\text{mm}$$

$$R = ?$$

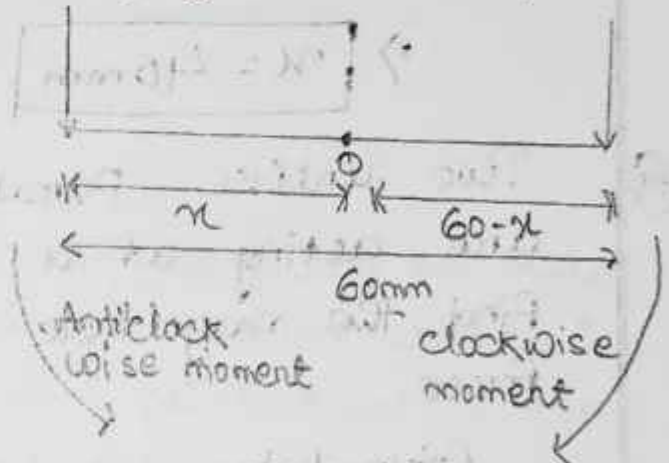
$$x = ? \text{ (Point of action)}$$

Magnitude

$$R = \vec{F}_1 + \vec{F}_2$$

$$= 6 + 12$$

$$\boxed{R = 18\text{N}}$$



Point of action

→ According to the law of moment

$$\boxed{\sum m = 0}$$

→ Clockwise moment + Anticlockwise moment = 0

$$\Rightarrow \vec{F}_2 (60 - x) + [-x \vec{F}_1] = 0$$

$$\Rightarrow 12 (60 - x) + [-x \times 6] = 0$$

$$\Rightarrow 720 - 12x - 6x = 0$$

$$\Rightarrow 720 - 18x = 0$$

$$\Rightarrow 720 = 18x$$

$$\Rightarrow 18x = 720$$

$$\Rightarrow x = \frac{720}{18}$$

$$\Rightarrow \boxed{x = 40 \text{ mm}}$$

39) Two unlike parallel force of 40N and 30N are acting at a point of distance of 30mm. Find the magnitude and point of action of the force.

Given data

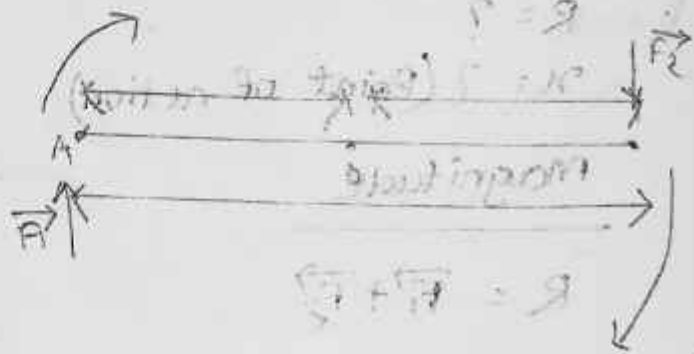
$$\vec{F}_1 = 40 \text{ N}$$

$$\vec{F}_2 = 30 \text{ N}$$

$$l = 30 \text{ mm}$$

$$R = ?$$

$$x = ? (\text{Point of action})$$



$$\vec{F}_1 + \vec{F}_2 = R$$

$$40 + 30 = R$$

$$\boxed{R = 70 \text{ N}}$$

Magnitude

$$\vec{R} = \vec{A} - \vec{B}$$

$$= 40 - 30$$

$$\boxed{R = 10\text{N}}$$

Point of action

According to the law of moment

$$\boxed{\sum m_i = 0}$$

$$\Rightarrow F_1 x + F_2 (300 - x) = 0$$

$$\Rightarrow 40x + 30(300 - x) = 0$$

$$\Rightarrow 40x + 9000 - 30x = 0$$

$$\Rightarrow 9000 + 10x = 0$$

$$\Rightarrow 10x = -9000$$

$$\Rightarrow x = \frac{-9000}{10}$$

$$\Rightarrow \boxed{x = -900\text{mm}}$$

40

Three force of 8P, 11P and 14P act along the three side of equilateral triangle, 200 mm taken in order. Find the magnitude, direction of the resultant force and find the position of the resultant force.

Given data

$$\vec{F}_1 = 8\text{P}$$

$$\vec{F}_2 = 11\text{P}$$

$$\vec{F}_3 = 14\text{P}$$

$$L: AB = BC = AC = 200\text{ mm}$$

$$\theta_1 = 0$$

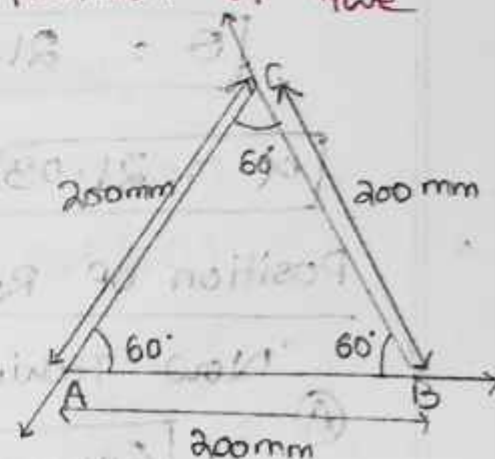
$$\theta_2 = 60^\circ$$

$$\theta_3 = 60^\circ$$

$$R = ?$$

$$x = ?$$

$$y = ?$$



Case-1 (Horizontal)

$$\begin{aligned}\Sigma H &= \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 \\ &= 8P \cos(0) + 11P \cos(60) + 14P \cos(60) \\ &= P(8 \cos(0) + 11 \cos(60) + 14 \cos(60))\end{aligned}$$

$$\boxed{\Sigma H = 6.5P}$$

Case-II (Vertical)

$$\begin{aligned}\Sigma V &= \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 \\ &= 8P \sin(0) + 11P \sin(60) + 14P \sin(60) \\ &= P(8 \sin(0) + 11 \sin(60) + 14 \sin(60))\end{aligned}$$

$$\boxed{\Sigma V = -2.5P}$$

Magnitude

$$\begin{aligned}R &= \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \\ &= \sqrt{(6.5P)^2 + (-2.5P)^2}\end{aligned}$$

$$\boxed{R = 6.96P}$$

Direction

$$\theta = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$

$$= \tan^{-1} \left| \frac{-2.5P}{6.5P} \right|$$

$$\boxed{\theta = 21.03^\circ}$$

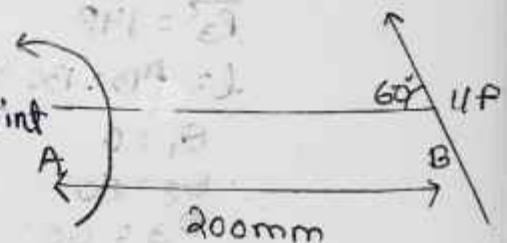
$$\boxed{\alpha = 21.03^\circ}$$

Position of Resultant

Now Taking moment at point

(A)

$$\boxed{\Sigma M_A = 0}$$



→ Clock wise moment + Anti clock wise moment = 0

$$\rightarrow (6.96P \times x) + [-200 \times 11P \times \sin(60)] = 0$$

$$\rightarrow 6.96P \times x - 200 \times 11P \times \sin(60) = 0$$

$$\rightarrow 6.96P \times x = 200 \times 11P \times \sin(60)$$

$$\rightarrow x = \frac{200 \times 11P \times \sin(60)}{6.96P}$$

$$\rightarrow \boxed{x = 273.7 \text{ mm}}$$

(41)

Three forces of $2P$, $3P$, $4P$ act along the three sides of equilateral triangle 100 mm taken in order. Find the magnitude and direction of the resultant force and find the position of resultant force.

Given data

$$\vec{F}_1 = 2P$$

$$\vec{F}_2 = 3P$$

$$\vec{F}_3 = 4P$$

$$L = AB = BC = AC = 100 \text{ mm}$$

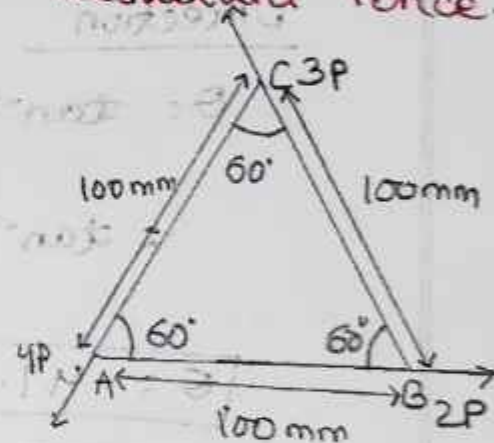
$$\theta_1 = 0$$

$$\theta_2 = 60$$

$$\theta_3 = 60$$

$$R = ?$$

$$x = ?$$



Case-1 (Horizontal)

$$\Sigma H = \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3$$

$$= 2P \cos(0) + 3P \cos(60) + (-4P) \cos(60)$$

$$= P(2 \cos(0) + 3 \cos(60) - 4 \cos(60))$$

$$\boxed{\Sigma H = 1.5P}$$

Case - II (V)

$$\begin{aligned}\Sigma V &= F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3 \\ &= 2P \sin 0 + 3P \sin 60 + (-4P) \sin 60 \\ &= P(2 \sin 0 + 3 \sin 60 - 4 \sin 60) \\ &= -0.86P\end{aligned}$$

$$\boxed{\Sigma V = -0.86P}$$

magnitude,

$$\begin{aligned}R &= \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \\ &= \sqrt{(1.5)^2 + (-0.86)^2} \\ &= 1.72P\end{aligned}$$

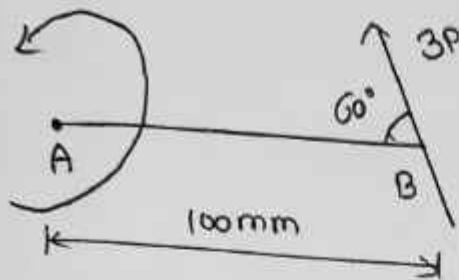
$$\boxed{R = 1.72P}$$

Direction,

$$\begin{aligned}\alpha &= \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right| \\ &= \tan^{-1} \left| \frac{-0.86P}{1.5P} \right| \\ &= 29.82\end{aligned}$$

$$\alpha = 29.82^\circ$$

Position of Resultant:-



→ Now taking moment at point A

$$\sum M_A = 0$$

⇒ Clock wise moment + Anti clock wise moment = 0

$$\Rightarrow (1.72P \times x) + [-100 \times 3P \sin 60^\circ] = 0$$

$$\Rightarrow 1.72P \times x - 100 \times 3P \sin 60 = 0$$

$$\Rightarrow 1.72P \times x = 100 \times 3P \sin 60$$

$$\Rightarrow x = \frac{100 \times 3P \sin 60}{1.72P}$$

$$\Rightarrow x = 87.2 \text{ mm}$$

42. Four force equal to $6P, 8P, 10P$ and $12P$ are acting along the four side of square ABCD taken in order. Find the magnitude and direction of the resultant force and position of the force.

Given data,

$$\vec{F}_1 = 6P \text{ (ve) } \textcircled{H}$$

$$\vec{F}_2 = 8P \text{ (ve) } \textcircled{V}$$

$$\vec{F}_3 = 10P \text{ (-ve) } \textcircled{H}$$

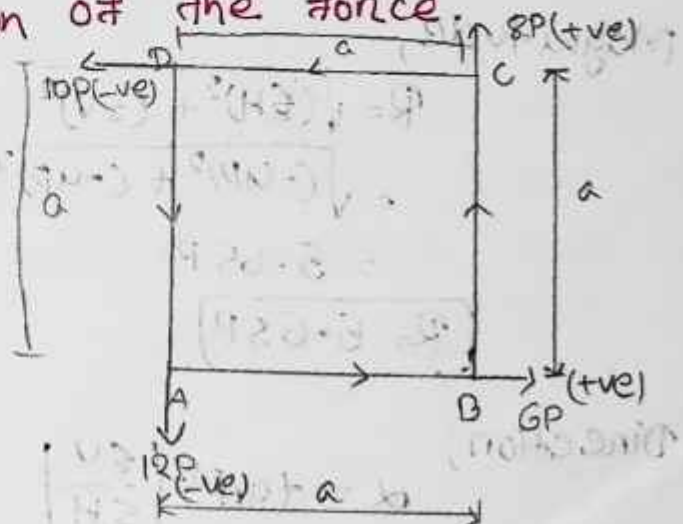
$$\vec{F}_4 = 12P \text{ (-ve) } \textcircled{V}$$

$$\theta_1 = 0^\circ$$

$$\theta_2 = 90^\circ$$

$$\theta_3 = 0^\circ$$

$$\theta_4 = 90^\circ$$



$$R = ?$$

$$\alpha = ?$$

$$n = ?$$

$$l = AB = BC = CD = DA = a$$

Case - I (H)

$$\begin{aligned}\Sigma H &= \vec{F}_1 \cos \theta_1 + \vec{F}_3 \cos \theta_3 \\ &= 6P \cos 0 + (-10P) \cos 0 \\ &= 6P \cos 0 - 10P \cos 0 \\ &= P (6 \cos 0 - 10 \cos 0) \\ &= -4P\end{aligned}$$

$$\boxed{\Sigma H = -4P}$$

Case - II (V)

$$\begin{aligned}\Sigma V &= \vec{F}_2 \sin \theta_2 + \vec{F}_4 \sin \theta_4 \\ &= 8P \sin 90 + (-12P) \sin 90 \\ &= 8P \sin 90 - 12P \sin 90 \\ &= P (8 \sin 90 - 12 \sin 90) \\ &= -4P\end{aligned}$$

$$\boxed{\Sigma V = -4P}$$

Magnitude,

$$\begin{aligned}R &= \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \\ &= \sqrt{(-4P)^2 + (-4P)^2} \\ &= 5.65P\end{aligned}$$

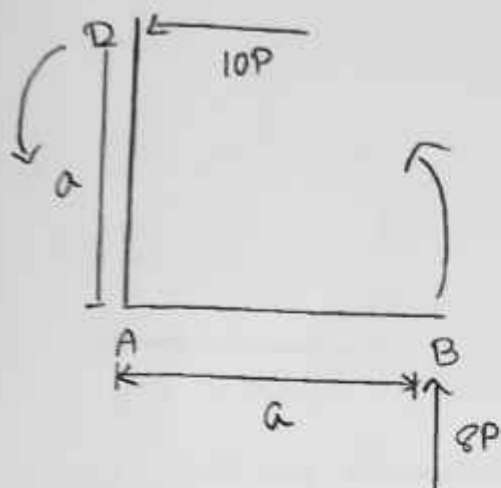
$$\boxed{R = 5.65P}$$

Direction,

$$\begin{aligned}\alpha &= \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right| \\ &= \tan^{-1} \left| \frac{-4P}{-4P} \right| \\ &= 45^\circ\end{aligned}$$

$$\alpha = 45^\circ$$

Position of Resultant:-



Now taking moment at Point (A)

$$\sum M_A = 0$$

Clock wise Moment + Anti Clock wise Moment = 0

$$\Rightarrow 5.65P \times \pi + (-a \times 8P) - (a \times 10P) = 0$$

$$\Rightarrow 5.65P \times \pi - (a \times 8P) - (a \times 10P) = 0$$

$$\Rightarrow 5.65P \times \pi = (a \times 8P) + (a \times 10P)$$

$$\Rightarrow 5.65P \times \pi = aP(8 + 10)$$

$$\Rightarrow 5.65\pi = 18a$$

$$\Rightarrow \pi = \frac{18a}{5.65}$$

$$\Rightarrow \pi = 3.18a \text{ mm}$$

13. Four force equal to P , $2P$, $3P$ and $4P$ are acting along the four sides of square ABCD taken in order. Find the magnitude, direction of the resultant force and position of the force.

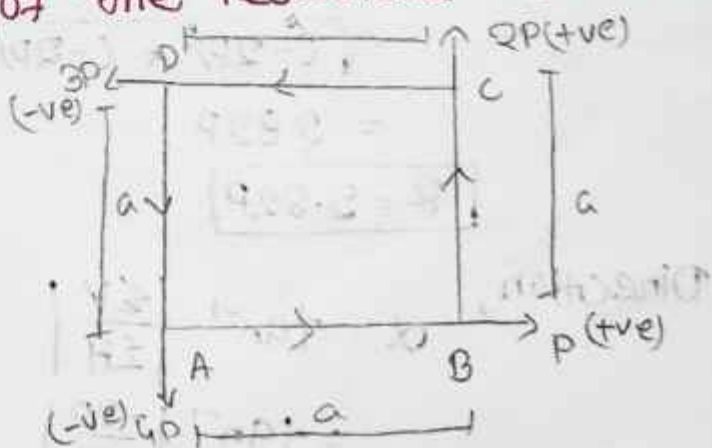
Given data,

$$F_1 = P \text{ (ve)} \text{ (H)}$$

$$F_2 = 2P \text{ (ve)} \text{ (V)}$$

$$F_3 = 3P \text{ (-ve)} \text{ (H)}$$

$$F_4 = 4P \text{ (ve)} \text{ (V)}$$



$$\theta_1 = 0^\circ$$

$$\theta_2 = 90^\circ$$

$$\theta_3 = 0^\circ$$

$$\theta_4 = 90^\circ$$

$$R = ?$$

$$\alpha = ?$$

$$\lambda = ?$$

$$l = AB = BC = CD = DA = a$$

Case-I (H)

$$\begin{aligned}\sum H &= \vec{F}_1 \cos \theta_1 + \vec{F}_3 \cos \theta_3 \\ &= P \cos 0 + (-3P) \cos 0\end{aligned}$$

$$= P \cos 0 - 3P \cos 0$$

$$0 = P (\cos 0 - 3 \cos 0)$$

$$= -2P$$

$$\boxed{\sum H = -2P}$$

Case-II (V)

$$\begin{aligned}\sum V &= \vec{F}_2 \sin \theta_2 + \vec{F}_4 \sin \theta_4 \\ &= 2P \sin 90 + (-4P) \sin 90\end{aligned}$$

$$= 2P \sin 90 - 4P \sin 90$$

$$= P (2 \sin 90 - 4 \sin 90)$$

$$= -2P$$

$$\boxed{\sum V = -2P}$$

Magnitude,

$$\begin{aligned}R &= \sqrt{(\sum H)^2 + (\sum V)^2} \\ &= \sqrt{(-2P)^2 + (-2P)^2} \\ &= 2.82P\end{aligned}$$

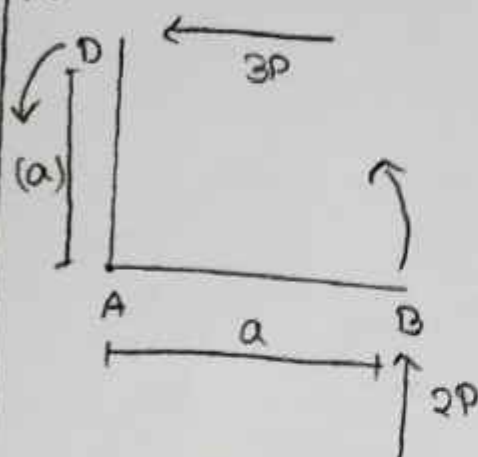
$$\boxed{R = 2.82P}$$

Direction,

$$\begin{aligned}\alpha &= \tan^{-1} \left| \frac{\sum V}{\sum H} \right| \\ &= \tan^{-1} \left| \frac{-2P}{-2P} \right|\end{aligned}$$

$$\alpha = 45^\circ$$

Position of Resultant



→ Now taking moment at point A

$$\sum M_A = 0$$

Clock wise Moment + Anti clock wise Moment = 0

$$\Rightarrow 2.82P \times x + (-a \times 2P) - (a \times 3P) = 0$$

$$\Rightarrow 2.82P \times x - (a \times 2P) - (a \times 3P) = 0$$

$$\Rightarrow 2.82P \times x = (a \times 2P) + (a \times 3P)$$

$$\Rightarrow 2.82P \times x = aP (2 + 3)$$

$$\Rightarrow 2.82x = 5a$$

$$\Rightarrow x = \frac{5a}{2.82}$$

$$\Rightarrow x = 1.77 a \text{ mm}$$

(14)

A Horizontal force line P, Q, R, S is 12cm long where $PQ = QR = RS = 4\text{cm}$, the force of 1000N, 1500N, again 1000N, 500N at a point A, Q, R, S with down ward direction, the line of action of the angle is $90^\circ, 60^\circ, 45^\circ, 30^\circ$ respectively with PS. Find the magnitude, direction and position of the resultant force.

Given data

$$PS = 12\text{cm}$$

$$PQ = QR = RS = 4\text{cm}$$

$$\vec{F}_1 = 1000\text{N}$$

$$\vec{F}_2 = 1500\text{N}$$

$$\vec{F}_3 = 1000\text{N}$$

$$\vec{F}_4 = 500\text{N}$$

$$\theta_1 = 90^\circ$$

$$\theta_2 = 60^\circ$$

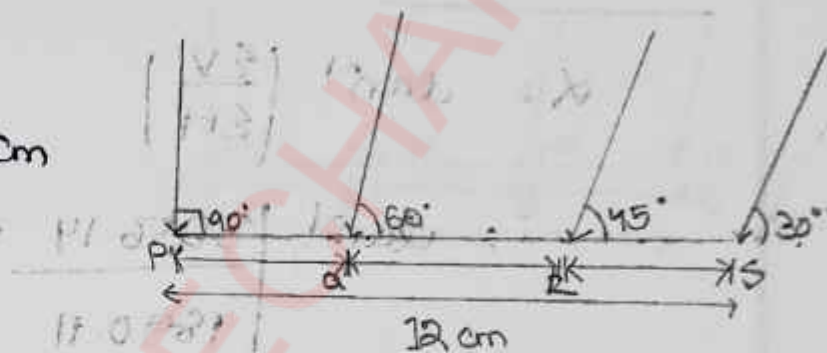
$$\theta_3 = 45^\circ$$

$$\theta_4 = 30^\circ$$

$$R = ?$$

$$\alpha = ?$$

$$x = ?$$



$$12 \cdot PS = x$$

position of resultant

distance from point P

(9) taking moments

$$0 = 9m^2$$

Case-I (Horizontal)

$$\begin{aligned} \Sigma H &= \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 + \vec{F}_4 \cos \theta_4 \\ &= 1000 \cos(90) + 1500 \cos(60) + 1000 \cos 45 + 500 \cos 30 \end{aligned}$$

$$\boxed{\Sigma H = 1840.11\text{N}}$$

Case-II (Vertical)

$$\begin{aligned} \Sigma V &= \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 + \vec{F}_4 \sin \theta_4 \\ &= 1000 \sin 90 + 1500 \sin 60 + 1000 \sin 45 + 500 \sin 30 \end{aligned}$$

$$\boxed{\Sigma V = 3256.14\text{N}}$$

Magnitude

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$
$$= \sqrt{(1890.11)^2 + (3256.14)^2}$$

$$R = 3764.96 \text{ N}$$

Direction

$$\alpha = \tan^{-1} \left(\frac{\sum V}{\sum H} \right)$$

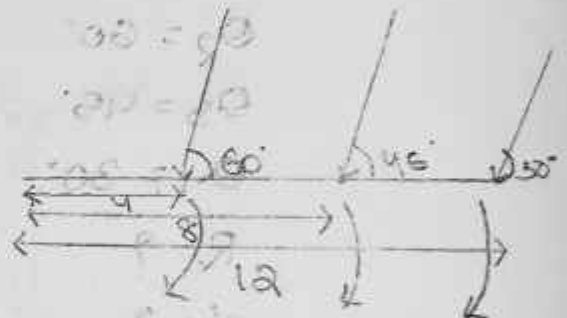
$$= \tan^{-1} \left| \frac{3256.14}{1890.11} \right|$$

$$\alpha = 59.86^\circ$$

Position of resultant

Now taking moment about point (P)

$$\sum m_p = 0$$



$$\Rightarrow 3764.96 \times x = 12 \times 500 \times \sin 30^\circ + 8 \times 1000 \times \sin 45^\circ + 4 \times 1500 \times \sin 60^\circ$$

$$\Rightarrow 3764.96 \times x = 13853$$

$$\Rightarrow x = \frac{13853}{3764.96}$$

$$\Rightarrow x = 3.67 \text{ cm}$$

Q. 48. A horizontal line PQRS is 15cm long where $PQ = QR = RS = 5\text{cm}$, force of 100N, 150N, 200N, 250N act at a point P, Q, R, S with downward direction. The force making angle 90° , 45° , 60° and 70° respectively. Find the magnitude, direction and point of the resultant force.

Given data

$$PS = 5\text{cm}$$

$$PQ = QR = RS = 5\text{cm}$$

$$\vec{F}_1 = 100\text{N}$$

$$\vec{F}_2 = 150\text{N}$$

$$\vec{F}_3 = 200\text{N}$$

$$\vec{F}_4 = 250\text{N}$$

$$\theta_1 = 90^\circ$$

$$\theta_2 = 45^\circ$$

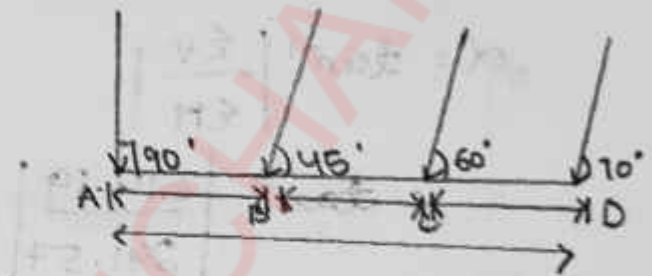
$$\theta_3 = 60^\circ$$

$$\theta_4 = 70^\circ$$

$$R = ?$$

$$\alpha = ?$$

$$m = ?$$



Case-I (Horizontal)

$$\begin{aligned}\Sigma H &= \vec{F}_1 \cos \theta_1 + \vec{F}_2 \cos \theta_2 + \vec{F}_3 \cos \theta_3 + \vec{F}_4 \cos \theta_4 \\ &= 100 \cos 90 + 150 \cos 45 + 200 \cos 60 + 250 \cos 70\end{aligned}$$

$$\boxed{\Sigma H = 291.57\text{N}}$$

Case-II (Vertical)

$$\begin{aligned}\Sigma V &= \vec{F}_1 \sin \theta_1 + \vec{F}_2 \sin \theta_2 + \vec{F}_3 \sin \theta_3 + \vec{F}_4 \sin \theta_4 \\ &= 100 \sin 90 + 150 \sin 45 + 200 \sin 60 + 250 \sin 70\end{aligned}$$

$$\boxed{\Sigma V = 614.19\text{N}}$$

Magnitude

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(291.57)^2 + (614.19)^2}$$

$$R = 679.88 \text{ N}$$

Direction

$$\alpha = \tan^{-1} \left| \frac{\Sigma V}{\Sigma H} \right|$$
$$= \tan^{-1} \left| \frac{614.19}{291.57} \right|$$

$$\alpha = 64.60^\circ$$

Position of Resultant

Now taking moment of point (P)

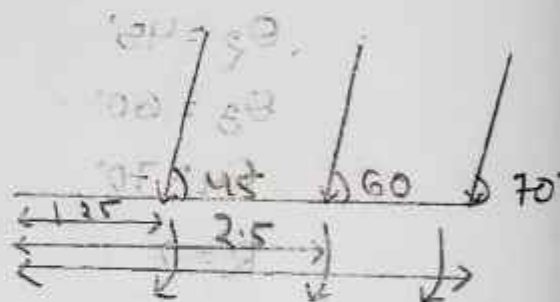
$$\Sigma m_p = 0$$

$$\Rightarrow 679.88 \times n = 15 \times 250 \times \sin 70^\circ + 10 \times 200 \times 60 + 5 \times 150 \times 45$$

$$\Rightarrow 679.88 \times n = 5786$$

$$\Rightarrow n = \frac{5786}{679.88}$$

$$\Rightarrow n = 8.51 \text{ cm}$$



GOVERNMENT POLYTECHNIC, BHUBANESWAR

ସରକାରୀ ପଲିଟେକ୍ନିକ ଭୁବନେଶ୍ୱର



ENGINEERING MECHANICS NOTE

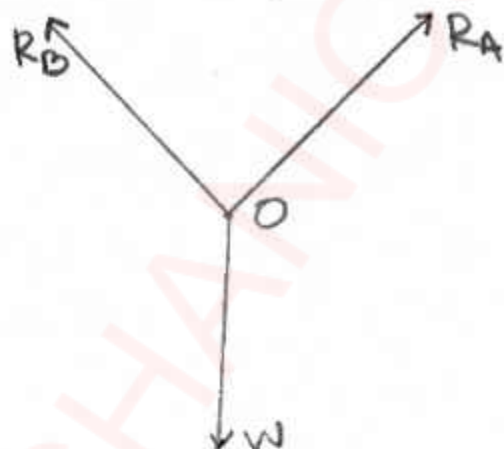
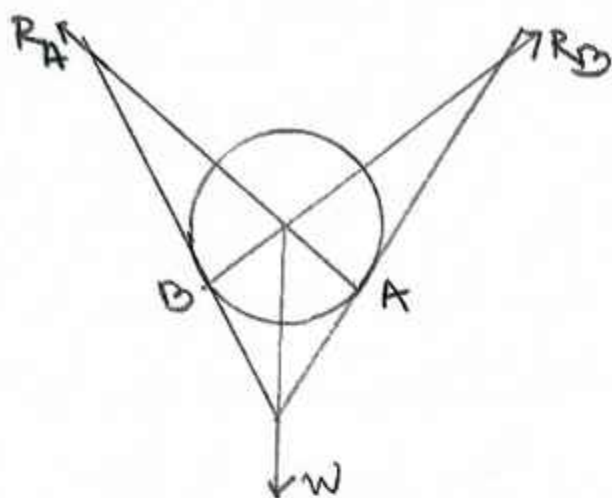
CHAPTER- 2 [EQUILIBRIUM]

PREPARED BY

ER. RAJKUMAR PRADHAN [RAJ SIR]

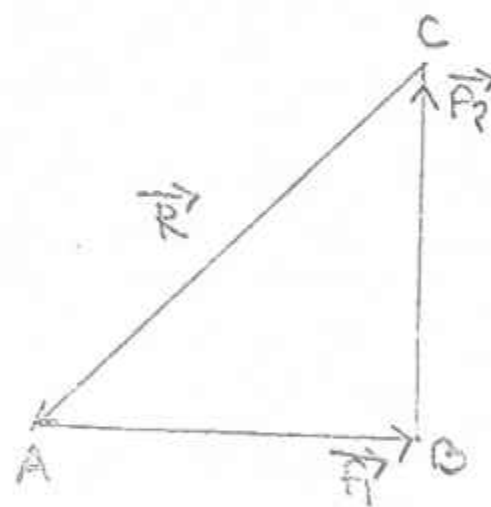
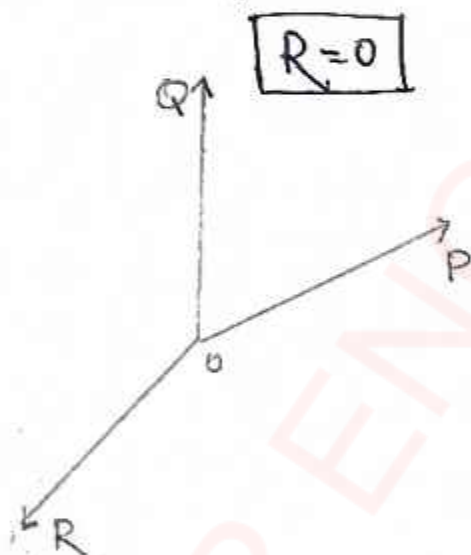
Free body diagram

Its a sketch of the body which represented isolated form, this known as free body diagram.



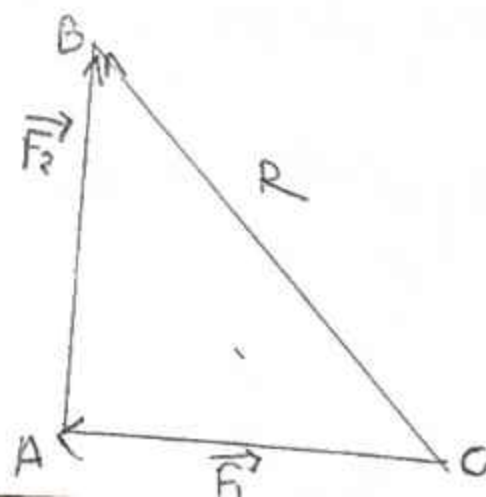
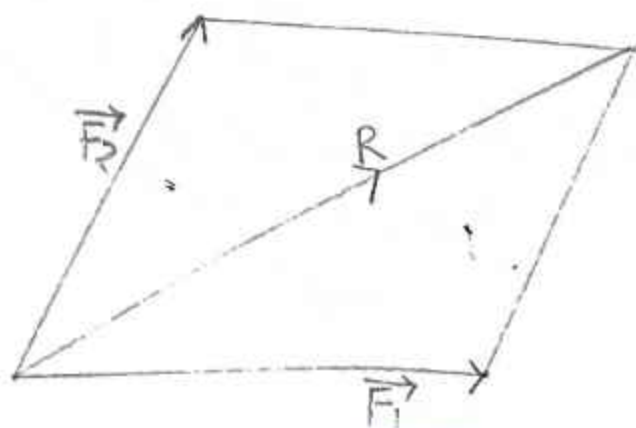
Equilibrium

Equilibrium is the condition of system of force acting on a body, where resultant is zero.



Equilibrant

Equilibrant is the condition of system of force acting on a body, where resultant is not equal to zero.



N.V.S Principle of equilibrium

→ There are two types of principle

① Two force principle

② Three force principle

Two force principle

If a body is in equilibrium under the action of two forces among one force must be equal and opposite with the other force.

Three force principle

If a body is in equilibrium under the action of three forces among ~~or two~~ ^{one} force must be equal and opposite with the other ² force.

N.V.S Lami's Theorem

Three coplanar forces acting at a point be in equilibrium then each force is directly proportional to the sine of the angle between the other two forces.

Where,

P, R, Q = Three forces in equilibrium

γ = Angle between P & Q force

β = Angle between P & R force

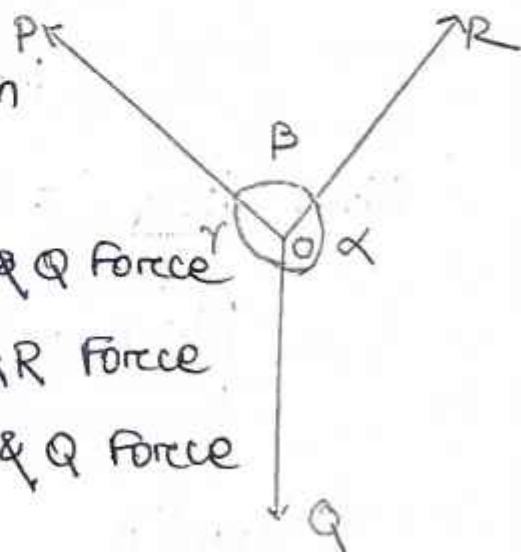
α = Angle between R & Q force

Mathematically,

$$P \propto \sin \alpha$$

$$P = k \sin \alpha$$

$$\Rightarrow \boxed{k = \frac{P}{\sin \alpha}}$$



$$R \propto \sin \gamma$$

$$\Rightarrow R = k \sin \gamma$$

$$\Rightarrow \boxed{k = \frac{R}{\sin \gamma}}$$

$$Q \propto \sin \beta$$

$$\Rightarrow Q = k \sin \beta$$

$$\Rightarrow \boxed{k = \frac{Q}{\sin \beta}}$$

eqn-①, ②, ③ we get

$$k = k = k$$

v.v.f

$$\boxed{\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}}$$

Lami's Theorem proof

Where,

P, Q and R = Apply Force $\vec{P}$

α, β, γ = Force angle

$PO \parallel SR$

Now $\angle POQ = \angle TSO = \gamma$ ($\because$ corresponding Angle)

$$\boxed{\angle QSR = 180 - \gamma} \quad \text{--- ①}$$

$\rightarrow$ Again

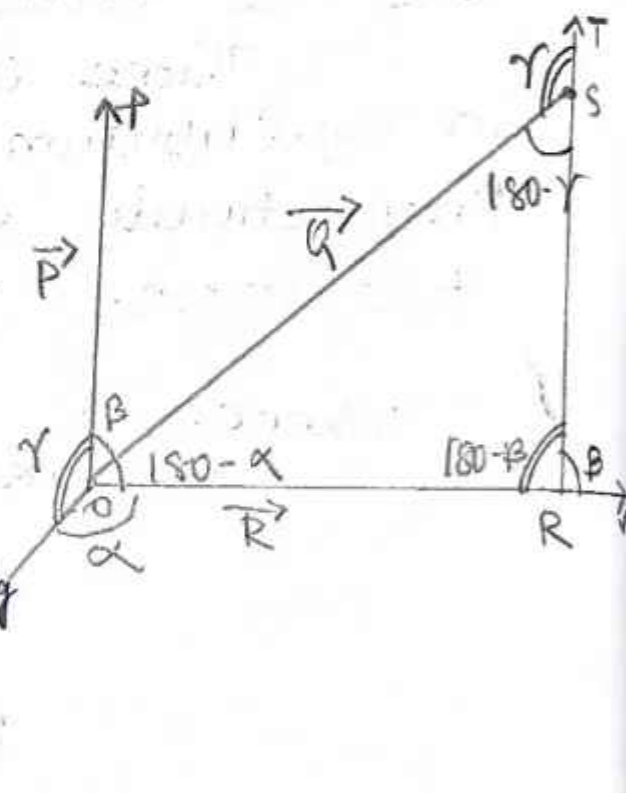
$\angle POR = \angle SRV = \beta$ ($\because$ corresponding Angle)

$$\boxed{\angle SRO = 180 - \beta} \quad \text{--- ②} \quad (\because PO \parallel SR)$$

Now

$$\angle QOR = \alpha$$

$$\angle SOR = 180 - \alpha \quad (RO \perp QS)$$



Now Δ SOR Apply sine rule

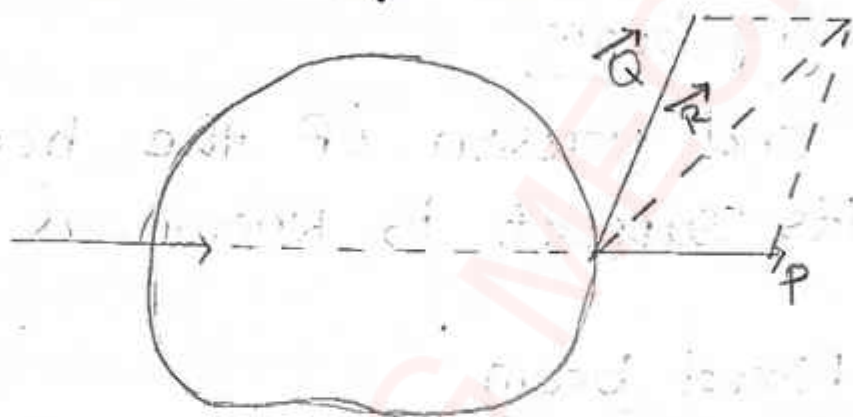
$$\frac{Q}{\sin(180-\beta)} = \frac{P}{\sin(180-\alpha)} = \frac{R}{\sin(180-\gamma)}$$

$$\Rightarrow \boxed{\frac{Q}{\sin\beta} = \frac{P}{\sin\alpha} = \frac{R}{\sin\gamma}}$$

V.V.D

Principle of Superposition

The action of force on a rigid body will no change if we added and subtracted another force force in equilibrium.



Beam

A structural member which is act upon by a system of external load at right angle to its axis is known as Beam.

→ There are various types of beam.

- V.V.D
- ① cantilever Beam
 - ② Simply Supported Beam
 - ③ over hanging beam
 - ④ Rigidity Fixed beam
 - ⑤ Continuous Beam

Cantilever Beam

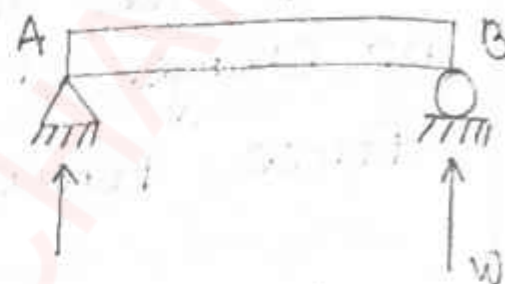
A beam which is fixed at one end and other end free is known as cantilever beam.

→ It's no rotation.



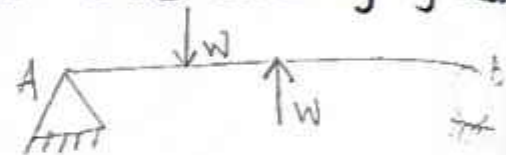
Simply supported Beam

A beam which is freely on the support end both end.



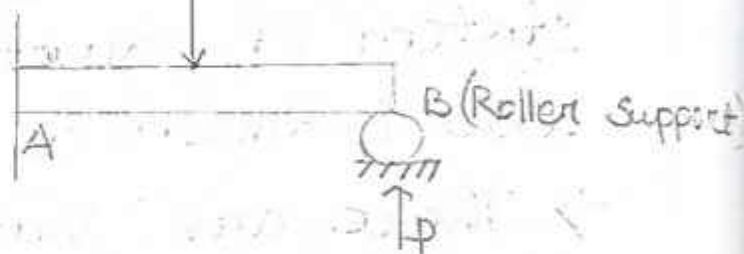
Over hanging beam

If the end portion of the beam is exerted beyond its support is known as over hanging beam.



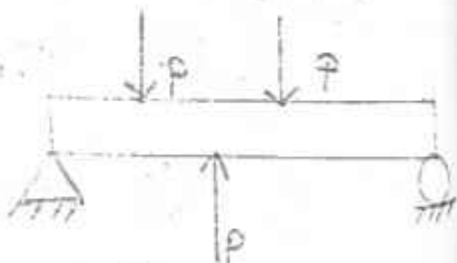
Rigidity Fixed beam

If a cantilever beam is supported at any point by a vertical pillar and other end contain rolling support.



Continuous Beam

A beam provided more than two support is known as continuous beam.



LOAD

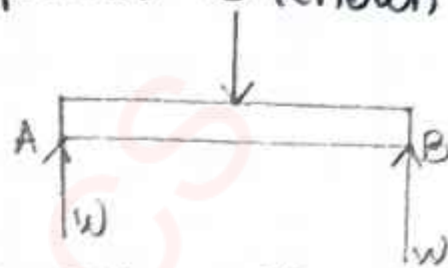
A load acting on a beam by the body force and surface force.

→ There are various types of LOAD

- ① Concentrated or point load
- ② uniformly distributed load
- ③ uniformly varying load

Concentrated or point load

A Force or load acting at a point is known as concentrated or point load.

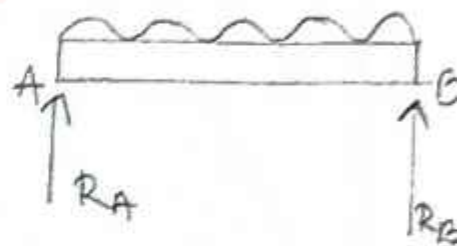


Uniform Distributed load

If the beam is uniform cross-section the load on a unit length at any point of the beam of the axis is known as Uniform Distributed load.

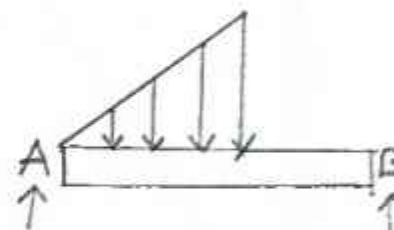
Uniform Varying Load

The load uniformly varying from one end to other end this load is known as



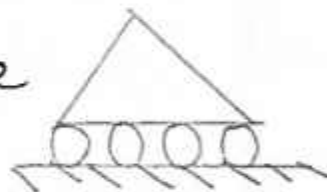
Support

- ① Roller Support
- ② Fixed Support
- ③ Simple Support
- ④ Hinge Support



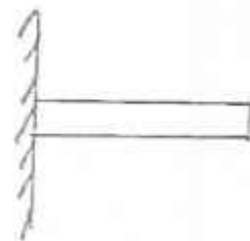
Roller Support

It can resist vertical force towards the Beam.



Fixed Support

It can ~~resist~~ rotational force towards the beam.



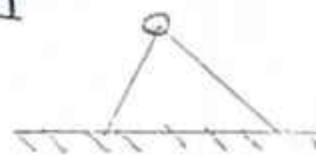
Simple Support

It can ~~resist~~ vertical force towards the beam.



Hinge Support

It resists in any direction of inclined reaction force towards the beam.



- ① An electric light fixture of weight $Q = 178\text{N}$ is supported as shown in the figure determine the tension force in the wires BA and BC if their angle is inclination.

Grivendata

$$W = 178\text{N}$$

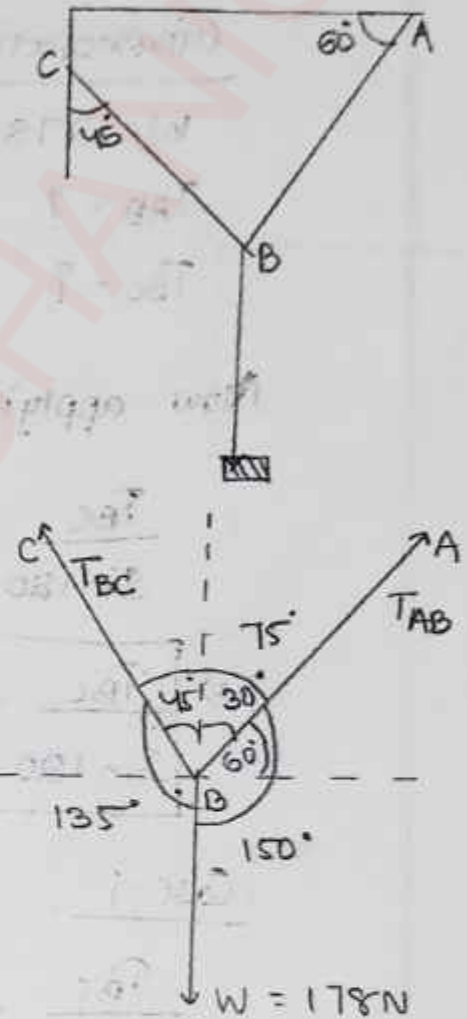
$$T_{AB} = ?$$

$$T_{BC} = ?$$

Now applying Lami's Theorem

$$\Rightarrow \frac{T_{BC}}{\sin 150} = \frac{T_{AB}}{\sin 135} = \frac{178 (W)}{\sin 75}$$

$$\Rightarrow \frac{T_{BC}}{\sin 150} = \frac{T_{AB}}{\sin 135} = \frac{178}{\sin 75}$$



Case-I

$$\frac{T_{BC}}{\sin 150} = \frac{178}{\sin 75}$$

$$\Rightarrow T_{BC} = \frac{178 \times \sin 150}{\sin 75}$$

$$\Rightarrow \boxed{T_{BC} = 92.13\text{N}}$$

Case-II

$$\frac{T_{AB}}{\sin 135} = \frac{178}{\sin 75}$$

$$\Rightarrow T_{AB} = \frac{178 \times \sin 135}{\sin 75}$$

$$\Rightarrow \boxed{T_{AB} = 130.3\text{N}} \quad (\text{ans})$$

- ② An electric light fixture of weight $W = 278\text{ N}$ is supported as shown in Figure. determine the tension force in the wires BA and BC if their angle of inclination.

Given data

$$W = 278\text{ N}$$

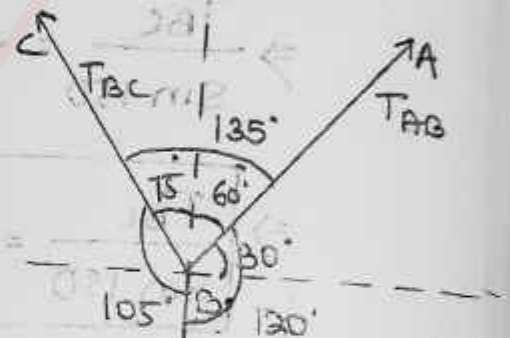
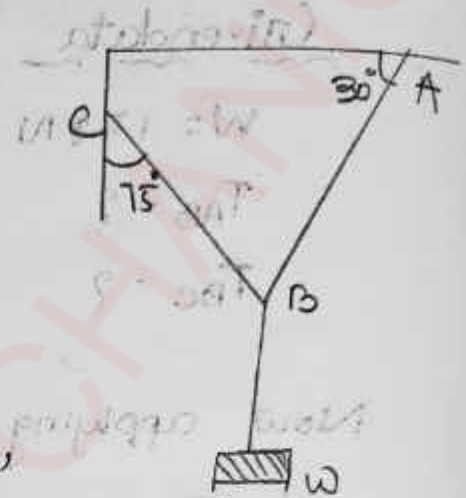
$$T_{AB} = ?$$

$$T_{BC} = ?$$

Now applying Lami's Theorem,

$$\frac{T_{BC}}{\sin 120^\circ} = \frac{T_{AB}}{\sin 105^\circ} = \frac{W}{\sin 135^\circ}$$

$$\Rightarrow \frac{T_{BC}}{\sin 120^\circ} = \frac{T_{AB}}{\sin 105^\circ} = \frac{278}{\sin 135^\circ}$$



Case-I

$$\frac{T_{BC}}{\sin 120^\circ} = \frac{278}{\sin 135^\circ}$$

$$\Rightarrow T_{BC} = \frac{278 \times \sin 120^\circ}{\sin 135^\circ}$$

$$\Rightarrow T_{BC} = 340.47\text{ N}$$

Case-II

$$\frac{T_{AB}}{\sin 105^\circ} = \frac{278}{\sin 135^\circ}$$

$$\Rightarrow T_{AB} = \frac{278 \times \sin 105^\circ}{\sin 135^\circ}$$

$$\Rightarrow T_{AB} = 379.75\text{ N}$$

③ The weight of 200N is tied to the end of a stream of a length as shown in the figure find the magnitude of force P to pull the weight at an angle of 30° , find the tension of stream.

Given data

$$W = 200\text{N}$$

$$P = ?$$

$$T = ?$$

Now applying Lami's Theorem

$$\frac{T}{\sin 90^\circ} = \frac{P}{\sin 120^\circ} = \frac{W}{\sin 150^\circ}$$

$$\Rightarrow \frac{T}{\sin 90^\circ} = \frac{P}{\sin 120^\circ} = \frac{200}{\sin 150^\circ}$$

Case-I

$$\frac{T}{\sin 90^\circ} = \frac{200}{\sin 150^\circ}$$

$$\Rightarrow T = \frac{200 \sin 90^\circ}{\sin 150^\circ}$$

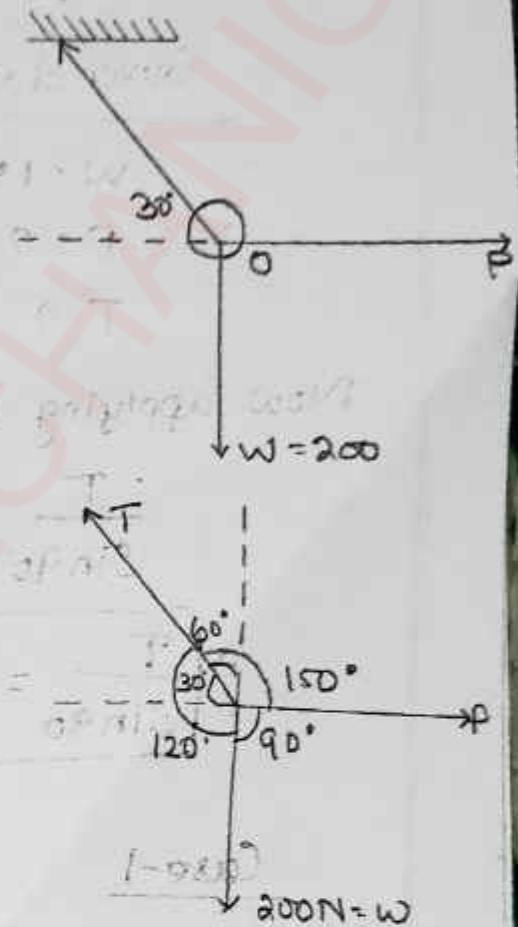
$$\Rightarrow T = 400\text{N}$$

Case-II

$$\frac{P}{\sin 120^\circ} = \frac{200}{\sin 150^\circ}$$

$$\Rightarrow P = \frac{200 \times \sin 150^\circ}{\sin 120^\circ}$$

$$\Rightarrow P = 346.41\text{N}$$



- ④ The weight of 150N is tied to the end of a stream of a length as shown in figure find the magnitude of force P to pull the weight at an angle of 60° find the tension stream.

Given data

$$W = 150\text{ N}$$

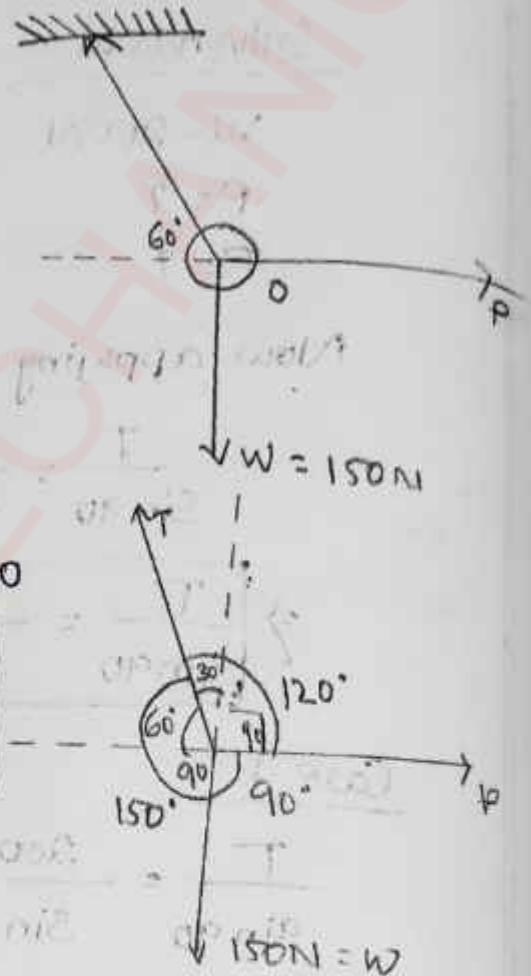
$$P = ?$$

$$T = ?$$

Now applying Lami's Theorem,

$$\frac{T}{\sin 90^\circ} = \frac{P}{\sin 150^\circ} = \frac{W}{\sin 120^\circ}$$

$$\Rightarrow \frac{T}{\sin 90^\circ} = \frac{P}{\sin 150^\circ} = \frac{150}{\sin 120^\circ}$$



Case-1

$$\frac{T}{\sin 90^\circ} = \frac{150}{\sin 120^\circ}$$

$$\Rightarrow T = \frac{150 \times \sin 90^\circ}{\sin 120^\circ}$$

$$\Rightarrow T = 173.20\text{ N}$$

Case-2

$$\frac{P}{\sin 150^\circ} = \frac{150}{\sin 120^\circ}$$

$$\Rightarrow P = \frac{150 \times \sin 150^\circ}{\sin 120^\circ}$$

$$\Rightarrow P = 86.60\text{ N}$$

⑤ The system shown in the figure is in equilibrium. If $Q = 10\text{ N}$ Find the magnitude of weight P .

Grivendata

$$Q = 10\text{ N}$$

$$T_1 = T_2 = 10\text{ N}$$

$$P = ?$$

Now apply Lami's Theorem,

$$\frac{P}{\sin 105^\circ} = \frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 135^\circ}$$

$$\Rightarrow \frac{P}{\sin 105^\circ} = \frac{10}{\sin 120^\circ} = \frac{10}{\sin 135^\circ}$$

Now,

$$\frac{P}{\sin 105^\circ} = \frac{10}{\sin 120^\circ}$$

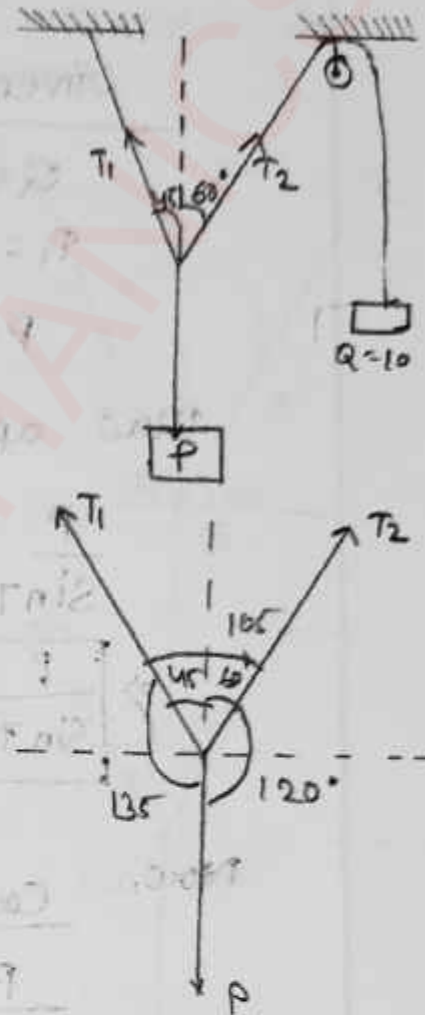
$$\Rightarrow P = \frac{10 \times \sin 105^\circ}{\sin 120^\circ}$$

$$\Rightarrow \boxed{P = 11.15\text{ N}}$$

$$\frac{P}{\sin 105^\circ} = \frac{10}{\sin 135^\circ}$$

$$\Rightarrow P = \frac{10 \times \sin 105^\circ}{\sin 135^\circ}$$

$$\Rightarrow \boxed{P = 13.66\text{ N}}$$



- ⑥ The system show in the Figure is in equilibrium if $Q = 20\text{N}$ Find the magnitude of weight P .

Given data

$$Q = 20\text{N}$$

$$T_1 = T_2 = 20\text{N}$$

$$P = ?$$

Now applying Lami's Theorem,

$$\frac{P}{\sin 70^\circ} = \frac{T_1}{\sin 130^\circ} = \frac{T_2}{\sin 150^\circ}$$

$$\Rightarrow \frac{P}{\sin 70^\circ} = \frac{20}{\sin 130^\circ} = \frac{20}{\sin 150^\circ}$$

Now,

Case-1

$$\frac{P}{\sin 70^\circ} = \frac{20}{\sin 150^\circ}$$

$$\Rightarrow P = \frac{20 \sin 70^\circ}{\sin 150^\circ}$$

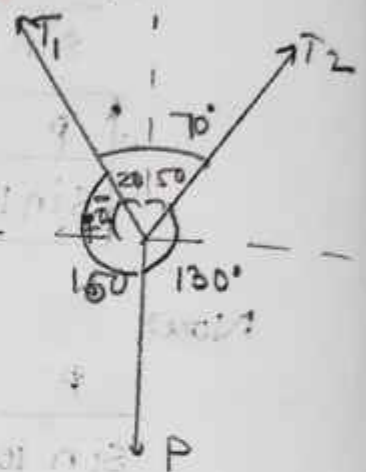
$$\Rightarrow \boxed{P = 54.94\text{N}}$$

Case-2

$$\frac{P}{\sin 70^\circ} = \frac{20}{\sin 130^\circ}$$

$$\Rightarrow P = \frac{20 \times \sin 70^\circ}{\sin 130^\circ}$$

$$\Rightarrow \boxed{P = 24.53\text{N}}$$



⑦ A ball of weight $Q = 53.4 \text{ N}$ rests in a right angle as shown in the figure. determine the force exerted of the side of D and E if own surfaces are perfectly smooth.

Given data

$$Q = 53.4 \text{ N}$$

$$R_e = ?$$

$$R_d = ?$$

Now applying Lami's Theorem

$$\frac{R_e}{\sin 150} = \frac{R_d}{\sin 120} = \frac{W}{\sin 90}$$

$$\Rightarrow \frac{R_e}{\sin 150} = \frac{R_d}{\sin 120} = \frac{53.4}{\sin 90}$$

Case-1

$$\frac{R_e}{\sin 150} = \frac{53.4}{\sin 90}$$

$$\Rightarrow R_e = \frac{53.4 \times \sin 150}{\sin 90}$$

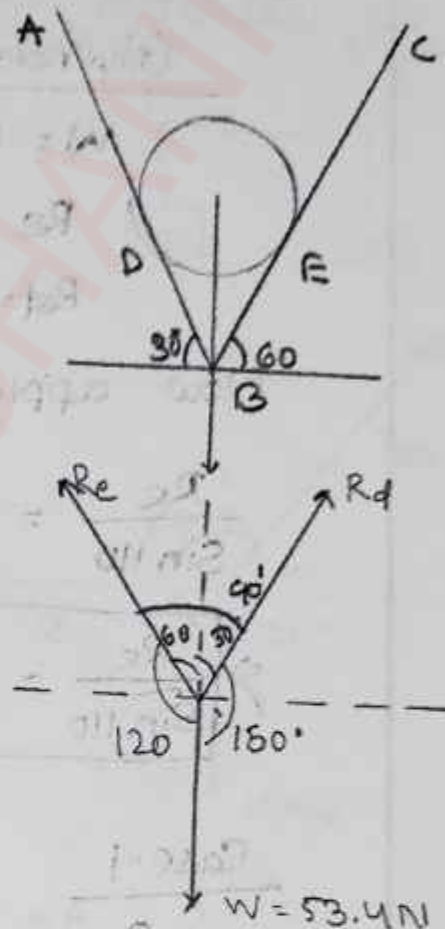
$$\Rightarrow \boxed{R_e = 26.7 \text{ N}}$$

Case-II

$$\frac{R_d}{\sin 120} = \frac{53.4}{\sin 90}$$

$$\Rightarrow R_d = \frac{53.4 \times \sin 120}{\sin 90}$$

$$\Rightarrow \boxed{R_d = 46.24 \text{ N}}$$



- ⑧ A ball of weight $Q = 64.7 \text{ N}$ rest in a right angle as shown in the figure determine the force exerted of the side of D and E if own surface are perfectly smooth.

Grivendata

$$W = 64.7 \text{ N}$$

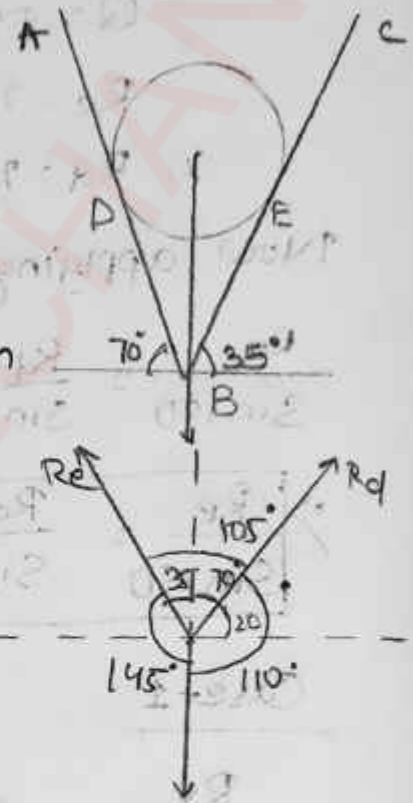
$$R_e = ?$$

$$R_d = ?$$

Now applying Lami's Theorem

$$\frac{R_e}{\sin 110} = \frac{R_d}{\sin 145} = \frac{W}{\sin 105}$$

$$\Rightarrow \frac{R_e}{\sin 110} = \frac{R_d}{\sin 145} = \frac{64.7}{\sin 105}$$



Case-I

$$\frac{R_e}{\sin 110} = \frac{64.7}{\sin 105}$$

$$\Rightarrow R_e = \frac{64.7 \times \sin 110}{\sin 105}$$

$$\Rightarrow R_e = 62.94 \text{ N}$$

Case-II

$$\frac{R_d}{\sin 145} = \frac{64.7}{\sin 105}$$

$$\Rightarrow R_d = \frac{64.7 \times \sin 145}{\sin 105}$$

$$\Rightarrow R_d = 38.41 \text{ N}$$

- Q) A cable supported at A and B carrying a load of 10 kN at D and a load of W at C. point as shown in the figure. Find the value of W so that CD remain horizontal.

Given data

$$W = ?$$

FBD-I

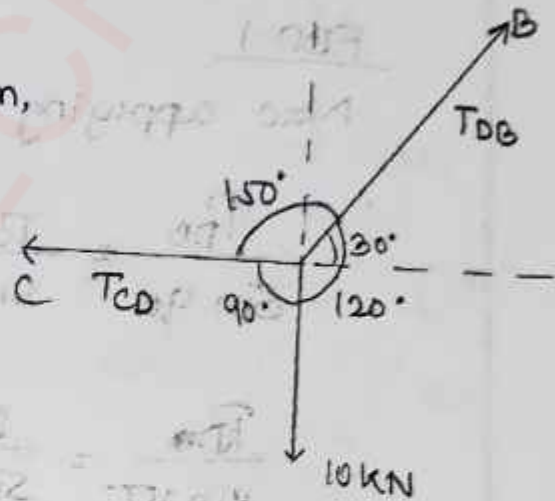
Now applying Lami's Theorem,

$$\frac{T_{DB}}{\sin 90^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{10}{\sin 150^\circ}$$

$$\frac{T_{CD}}{\sin 120^\circ} = \frac{10}{\sin 150^\circ}$$

$$\Rightarrow T_{CD} = \frac{10 \times \sin 120^\circ}{\sin 150^\circ}$$

$$\Rightarrow T_{CD} = 17.32 \text{ kN}$$



FBD-II

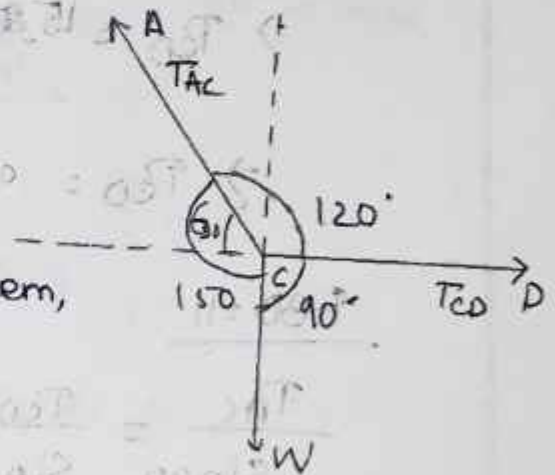
Now applying Lami's Theorem,

$$\frac{T_{AC}}{\sin 90^\circ} = \frac{T_{CD}}{\sin 150^\circ} = \frac{W}{\sin 120^\circ}$$

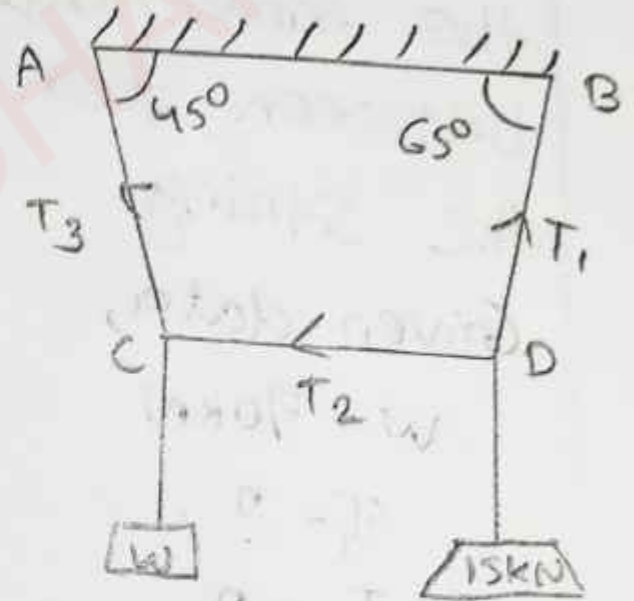
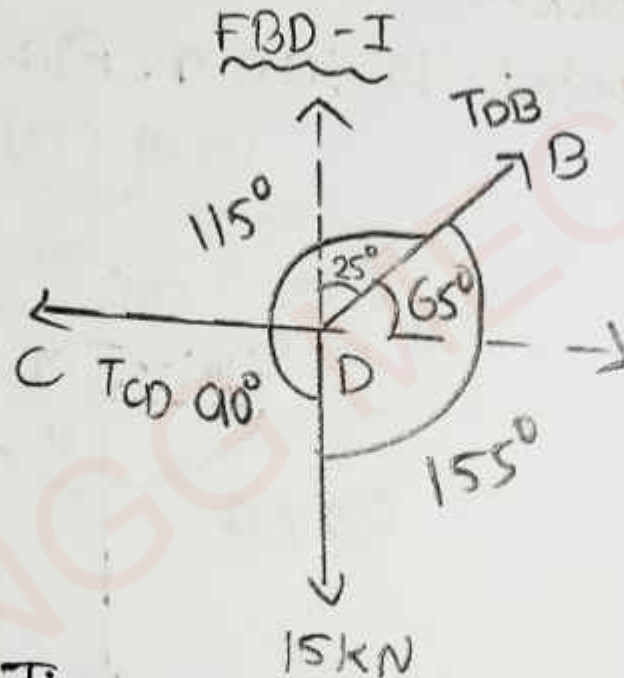
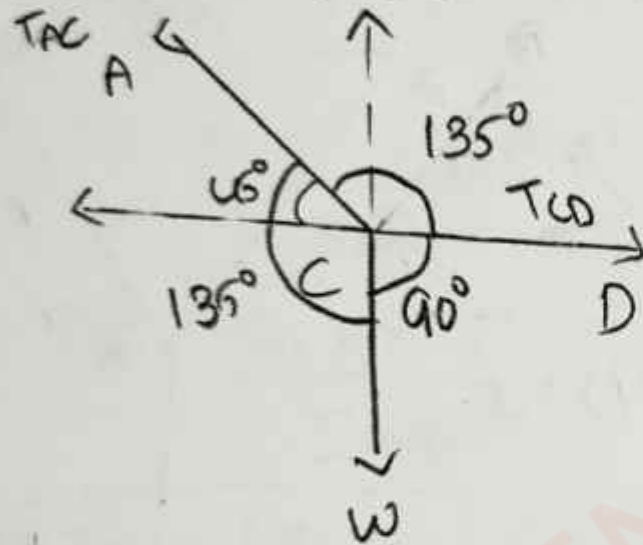
$$\frac{T_{CD}}{\sin 150^\circ} = \frac{W}{\sin 120^\circ}$$

$$\Rightarrow \frac{17.32}{\sin 150^\circ} = \frac{W}{\sin 120^\circ}$$

$$\Rightarrow W = 29.99 \text{ kN}$$



Q10 A cord supported at A & B carry a load of 15 kN at D and a load of w at C point as shown in the figure. Find the value of w , so that CD remain Horizontal. FBD - II



Now applying Lami's Theorem FBD - I

$$\frac{T_{DB}}{\sin 90} = \frac{T_{CD}}{\sin 155^\circ} = \frac{15}{\sin 115^\circ}$$

$$\frac{T_{CD}}{\sin 155^\circ} = \frac{15}{\sin 115^\circ}$$

$$\Rightarrow T_{CD} = \frac{15 \times \sin 155^\circ}{\sin 115^\circ}$$

$$\Rightarrow \boxed{T_{CD} = 6.99 \text{ N}} \text{ (Ans)}$$

Now applying Lami's Theorem FBD - II

$$\frac{T_{AC}}{\sin 90} = \frac{T_{CD}}{\sin 135} = \frac{W}{\sin 135}$$

$$\Rightarrow \frac{T_{CD}}{\sin 135} = \frac{W}{\sin 135}$$

$$\Rightarrow \frac{6.99}{\sin 135} = \frac{W}{\sin 135}$$

$$\Rightarrow W = \frac{6.99 \times \sin 135}{\sin 135}$$

$$\Rightarrow \boxed{W = 6.99 \text{ N}}$$

- 11) A body of weight 70 kN suspended by two strings whose lengths are 6 cm and 8 cm from two points in the same horizontal level. The horizontal distance between two points is 10 cm. Find the tension of the strings.

Given data

$$W = 70 \text{ kN}$$

$$T_1 = ?$$

$$T_2 = ?$$

$$AB = 10 \text{ cm}$$

$$AO = 6 \text{ cm}$$

$$BO = 8 \text{ cm}$$

Now applying Lami's theorem

$$\frac{T_1}{\sin(90^\circ + \alpha)} = \frac{T_2}{\sin(90^\circ + \beta)} = \frac{W}{\sin 90^\circ}$$

$$\Rightarrow \frac{T_1}{\sin(90^\circ + \alpha)} = \frac{T_2}{\sin(90^\circ + \beta)} = \frac{70}{\sin 90^\circ} \quad \text{--- (1)}$$

Now at $\triangle ABO$

$$\cos \alpha = \frac{b}{h} = \frac{8}{10} = 0.8$$

$$\cos \beta = \frac{b}{h} = \frac{6}{10} = 0.6$$

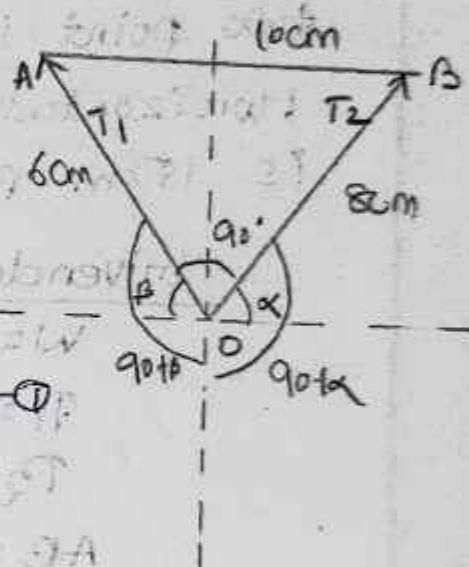
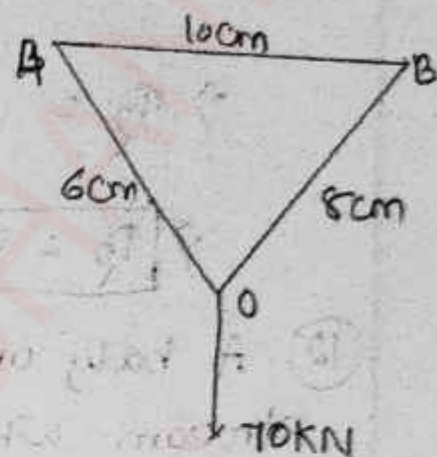
Case-1

$$\frac{T_1}{\cos \alpha} = \frac{70}{\sin 90^\circ}$$

$$\Rightarrow T_1 = \frac{70 \cos \alpha}{\sin 90^\circ}$$

$$\Rightarrow T_1 = \frac{70 \times 0.8}{\sin 90^\circ}$$

$$\Rightarrow T_1 = 56 \text{ kN}$$



Case-11

$$\frac{T_2}{\cos \beta} = \frac{70}{\sin 90}$$

$$\Rightarrow T_2 = \frac{70 \cos \beta}{\sin 90}$$

$$\Rightarrow T_2 = \frac{70 \times 0.6}{\sin 90}$$

$$\Rightarrow \boxed{T_2 = 42 \text{ kN}}$$

- (12) A body of weight 80 kN suspended by two strings whose lengths are 7m and 10m from two points in the same horizontal level. The horizontal distance between two points is 15m. Find the tension of the strings.

Given data

$$W = 80 \text{ kN}$$

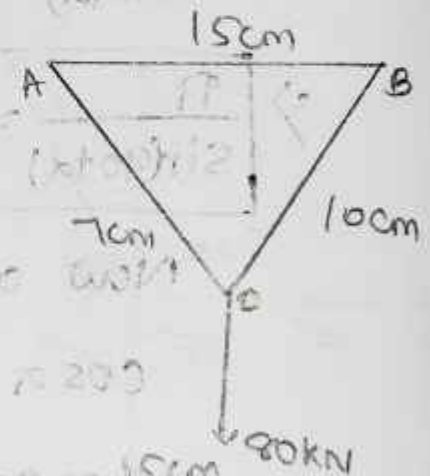
$$T_1 = ?$$

$$T_2 = ?$$

$$AB = 15 \text{ m}$$

$$AO = 7 \text{ m}$$

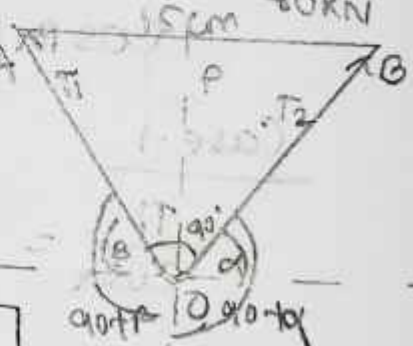
$$BO = 10 \text{ m}$$



Now applying Lami's Theorem

$$\frac{T_1}{\sin(90 + \alpha)} = \frac{T_2}{\sin(90 + \beta)} = \frac{W}{\sin 90}$$

$$\Rightarrow \boxed{\frac{T_1}{\cos \alpha} = \frac{T_2}{\cos \beta} = \frac{80}{\sin 90}} \quad \text{--- (1)}$$



Now, at ΔABO

$$\cos \alpha = \frac{b}{h} = \frac{10}{15} = 0.66$$

$$\cos \beta = \frac{b}{h} = \frac{7}{15} = 0.46$$

Case-I

$$\frac{T_1}{\cos \alpha} = \frac{80}{\sin 90}$$

$$\Rightarrow \frac{T_1}{0.66} = \frac{80}{\sin 90}$$

$$\Rightarrow T_1 = \frac{80 \times 0.66}{\sin 90}$$

$$\Rightarrow \boxed{T_1 = 52.8 \text{ kN}}$$

Case-II

$$\frac{T_2}{\cos \beta} = \frac{80}{\sin 90}$$

$$\Rightarrow \frac{T_2}{0.46} = \frac{80}{\sin 90}$$

$$\Rightarrow T_2 = \frac{80 \times 0.46}{\sin 90}$$

$$\Rightarrow \boxed{T_2 = 36.8 \text{ kN}}$$

A string of length 24cm with attached to a point on a smooth vertical wall to a point on the surface of a sphere of radius 12cm the sphere whose weight is 100kN hanging in equilibrium against the wall. Find the tension of string and reaction of the wall.

Given data

$$AB = 24\text{cm}$$

$$CO = BO = 12\text{cm}$$

$$W = 100\text{kN}$$

$$T = ?$$

$$R_c = ?$$

Now applying Lami's Theorem

$$\frac{T_i}{\sin 90} = \frac{R_c}{\sin(90+\theta)} = \frac{W}{\sin(180-\theta)}$$

$$\Rightarrow \frac{T_i}{\sin 90} = \frac{R_c}{\cos \theta} = \frac{100}{\sin \theta} \quad \text{--- (1)}$$

Now ΔAOC

$$P = CO = 12\text{cm}$$

$$AC = b$$

$$h = AO = AB + BO = 36\text{cm}$$

$$\sin \theta = \frac{P}{h} = \frac{12}{36} = 0.3$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow (0.3)^2 + (\cos^2 \theta) = 1$$

$$\Rightarrow 0.09 + \cos^2 \theta = 1$$

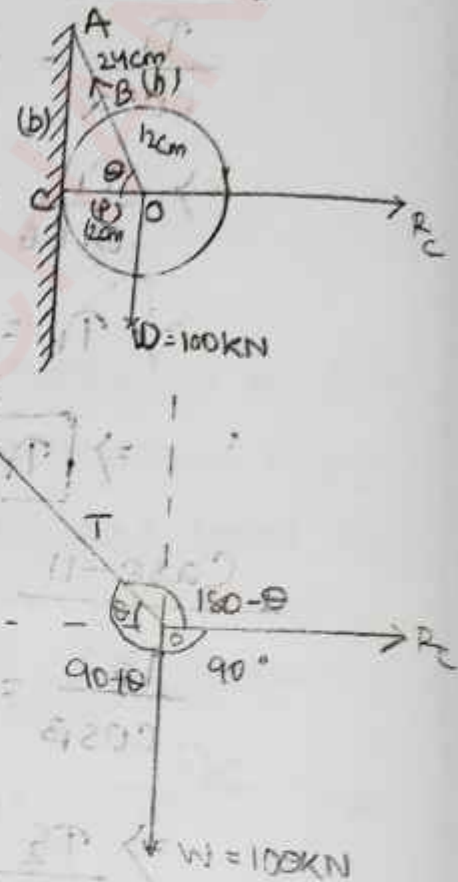
$$\Rightarrow \cos^2 \theta = 1 - 0.09$$

$$\Rightarrow \cos \theta = \sqrt{1 - 0.09}$$

$$\Rightarrow \boxed{\cos \theta = 0.95}$$

Case-1

Now eqn (1) we get



$$\frac{T}{\sin 90} = \frac{100}{\sin \theta}$$

$$\Rightarrow T = \frac{100 \times \sin 90}{0.3}$$

$$\Rightarrow \boxed{T = 333.3 \text{ kN}}$$

Case-II

$$\frac{R_c}{\cos \theta} = \frac{100}{\sin \theta}$$

$$\Rightarrow R_c = \frac{100 \times \cos \theta}{0.3}$$

$$\Rightarrow \boxed{R_c = 376.6 \text{ kN (ans)}}$$

A string of length 30cm, which attached to a point on a smooth vertical wall to a point on the surface of a sphere of radius 15cm the sphere whose weight is 200kN hanging in equilibrium against the wall. Find the tension of the string and reaction of the wall.

Given data

$$AB = 30 \text{ cm}$$

$$CO = BO = 15 \text{ cm}$$

$$W = 200 \text{ kN}$$

$$T = ?$$

$$R_c = ?$$

Now applying Lami's Theorem

$$\frac{T_1}{\sin 90} = \frac{R_c}{\sin(90+\theta)} = \frac{W}{\sin(180-\theta)}$$

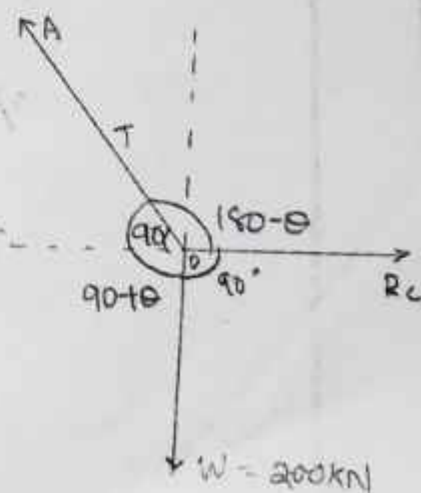
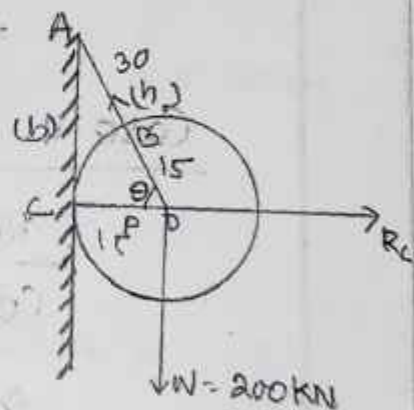
$$\Rightarrow \boxed{\frac{T_1}{\sin 90} = \frac{R_c}{\cos \theta} = \frac{200}{\sin \theta}} \quad \text{--- (1)}$$

Now $\triangle AOC$

$$P = CO = 15 \text{ cm}$$

$$AC = 16$$

$$h = AO = AB + BO = 30 \text{ cm} + 15 \text{ cm} = 45 \text{ cm}$$



$$h = 15 = 3$$

$$\sin \theta = \frac{P}{h} = \frac{15}{45} = 0.3$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow (0.3)^2 + \cos^2 \theta = 1$$

$$\Rightarrow 0.09 + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - 0.09$$

$$\Rightarrow \cos \theta = \sqrt{1 - 0.09} = 0.95$$

Case-I

Now eqⁿ ① we get

$$\frac{T}{\sin 90^\circ} = \frac{200}{\sin \theta}$$

$$\Rightarrow T = \frac{200 \times \sin 90^\circ}{\sin \theta}$$

$$\Rightarrow T = \frac{200 \times 0.3 \sin 90^\circ}{\sin \theta (0.3)}$$

$$\Rightarrow \boxed{T = 666.6 \text{ kN}}$$

Case-II

$$\frac{R_c}{\cos \theta} = \frac{200}{\sin \theta}$$

$$\Rightarrow R_c = \frac{200 \times \cos \theta}{\sin \theta}$$

$$\Rightarrow R_c = \frac{200 \times 0.95}{0.3}$$

$$\Rightarrow \boxed{R_c = 633.3 \text{ kN}}$$

A string of length ^{45cm} attached to a point on a smooth vertical wall to a point on the surface of a sphere of radius 13cm the sphere whose weight 300kN hanging in equilibrium against the wall. Find the tension stream and reaction of the wall.

Given data

$$AB = 45 \text{ cm}$$

$$CO = BO = 13 \text{ cm}$$

$$W = 300 \text{ kN}$$

$$T = ?$$

$$R_c = ?$$

Now applying Lami's Theorem,

$$\frac{T}{\sin 90^\circ} = \frac{R_c}{\sin(90^\circ + \theta)} = \frac{W}{\sin(180^\circ - \theta)}$$

$$\Rightarrow \frac{T}{\sin 90^\circ} = \frac{R_c}{\cos \theta} = \frac{300}{\sin \theta} \quad \text{--- (1)}$$

Now ΔAOC

$$P = CO = 13 \text{ cm}$$

$$AC = h$$

$$AO = (h) = AB + BO = 45 + 13 = 58 \text{ cm}$$

$$\sin \theta = \frac{P}{h} = \frac{13}{58} = 0.22$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

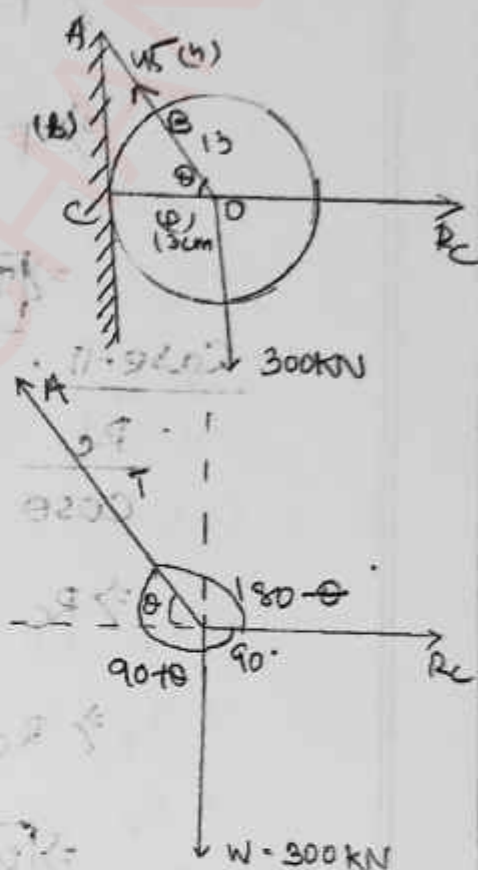
$$\Rightarrow (0.22)^2 + \cos^2 \theta = 1$$

$$\Rightarrow 0.04 + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - 0.04$$

$$\Rightarrow \cos \theta = \sqrt{1 - 0.04}$$

$$\Rightarrow \cos \theta = 0.97$$



Case-1

Now eqⁿ ① we get

$$\frac{T}{\sin 90} = \frac{300}{\sin \theta}$$

$$\Rightarrow T = \frac{300 \times \sin 90}{\sin \theta}$$

$$\Rightarrow T = \frac{300 \times \sin 90}{0.97}$$

$$\Rightarrow T = 1500 \text{ KN}$$

Case-II

$$\frac{R_c}{\cos \theta} = \frac{300}{\sin \theta}$$

$$\Rightarrow R_c = \frac{300 \times \cos \theta}{\sin \theta} = \frac{300}{0.2}$$

$$\Rightarrow R_c = \frac{300 \times 0.97}{0.2}$$

$$\Rightarrow R_c = 1455 \text{ KN}$$

A simple supported beam AB is 2.5 m carrying a two point load as shown in the figure. Find the support left reaction and right reaction of the beam.

Given data

$$AB = 2.5 \text{ m}$$

$$AC = DB = 1 \text{ m}$$

$$CD = 0.5 \text{ m}$$

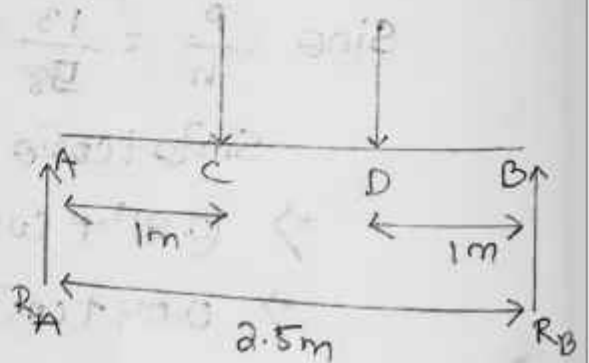
$$AD = 1.5 \text{ m}$$

$$W_1 = 2 \text{ kN}$$

$$W_2 = 4 \text{ kN}$$

$$R_A = ?$$

$$R_B = ?$$



Case-I (Moment)

→ Now taking at point A

$$\boxed{\sum M_A = 0}$$

$$\Rightarrow (-R_B \times 2.5) + (4 \times 1.5) + (2 \times 1) = 0$$

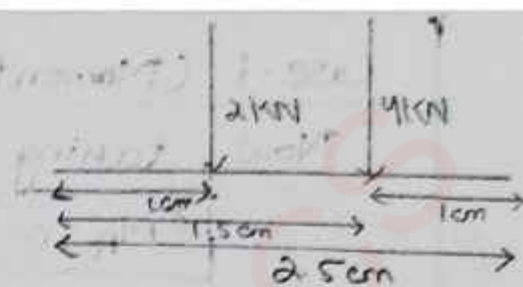
$$\Rightarrow -2.5R_B + 6 + 2 = 0$$

$$\Rightarrow -2.5R_B + 8 = 0$$

$$\Rightarrow 2.5R_B = 8$$

$$\Rightarrow R_B = \frac{8}{2.5}$$

$$\Rightarrow \boxed{R_B = 3.2 \text{ kN}}$$



Case-II (Vertical Force)

→ R

$$\boxed{\sum V = 0}$$

$$\Rightarrow R_B + R_A - 2 - 4 = 0$$

$$\Rightarrow R_A + 3.2 - 6 = 0$$

$$\Rightarrow R_A - 2.8 = 0$$

$$\Rightarrow \boxed{R_A = 2.8 \text{ kN}} \text{ (ans)}$$

A simple supported beam AB is 5.5m carry a two point load as shown in the figure find the supported left reaction & right reaction of the beam.

Given data

$$AB = 5.5 \text{ m}$$

$$CD = DB = 2 \text{ m}$$

$$AC = 1.5 \text{ m}$$

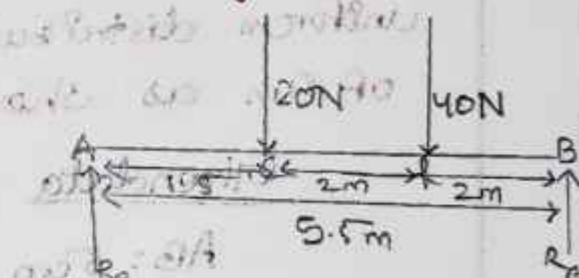
$$AD = 3.5 \text{ m}$$

$$W_1 = 20 \text{ N}$$

$$W_2 = 40 \text{ N}$$

$$R_A = ?$$

$$R_B = ?$$



Case-1 (Moment)

Now taking at point (A)

$$\boxed{\sum M_A = 0}$$

$$\Rightarrow (-R_B \times 5.5) + (40 \times 2.5) + (20 \times 1.5) = 0$$

$$\Rightarrow -5.5 R_B + 140 + 30 = 0$$

$$\Rightarrow -5.5 R_B + 170 = 0$$

$$\Rightarrow -5.5 R_B = -170$$

$$\Rightarrow R_B = \frac{170}{5.5}$$

$$\Rightarrow \boxed{R_B = 30.90 \text{ N}}$$

Case-II (Vertical force)

$$\Rightarrow R_A + R_B - 20 - 40 = 0$$

$$\Rightarrow R_A + 30.90 - 60 = 0$$

$$\Rightarrow R_A - 29.1 = 0$$

$$\Rightarrow \boxed{R_A = 29.1 \text{ N}}$$

Determine the reaction at point A & B of the simple supported beam 5m long is loaded with uniform distributed load of 10 kN/m over a length of 2m as shown in the figure.

Given data

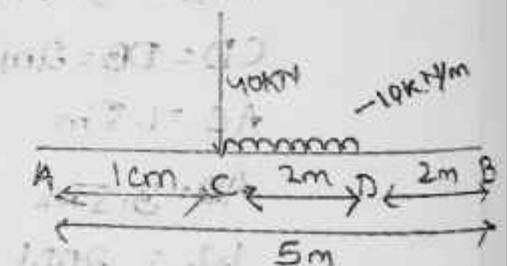
$$AB = 5 \text{ m}$$

$$AC = 1 \text{ m}$$

$$CD = DB = 2 \text{ m}$$

$$W = 40 \text{ kN}$$

$$\text{UDL} = 10 \text{ kN/m}$$



Force (UDL)

$$F = 10 \times 2 = 20 \text{ kN}$$

Length (UDL)

$$L = 1 + 1 = 2 \text{ m}$$

$$R_A = ? \quad R_B = ?$$

Case-I (Moment)

Now taking moment at point **A**

$$\sum M_A = 0$$

$$\Rightarrow (-R_B \times 5) + (20 \times 2) + (40 \times 1) = 0$$

$$\Rightarrow -5R_B + 40 + 40 = 0$$

$$\Rightarrow -5R_B + 80 = 0$$

$$\Rightarrow 5R_B = 80$$

$$\Rightarrow R_B = \frac{80}{5}$$

$$\Rightarrow R_B = 16 \text{ kN}$$

Case-II (Vertical)

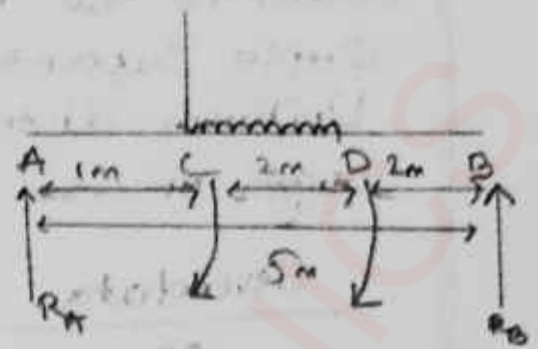
$$\sum V = 0$$

$$\Rightarrow R_A + R_B - 40 - 20 = 0$$

$$\Rightarrow R_B + 16 - 60 = 0$$

$$\Rightarrow R_B = 60 - 16$$

$$\Rightarrow R_B = 44 \text{ kN}$$



Determine the reaction at points A & B of the simple supported beam is 6m long is load with Uniform distributed load of 20kN over a length of 3m as shown in the figure.

Given data

$$AB = 6m$$

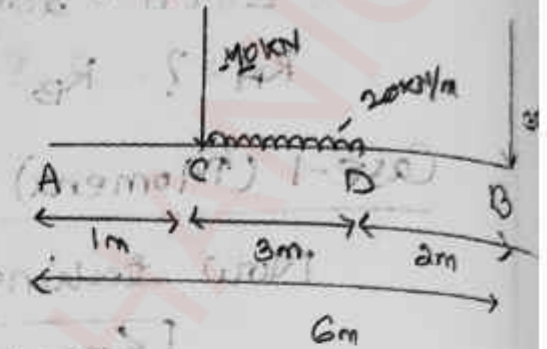
$$AC = 1m$$

$$CD = 3m$$

$$DB = 2m$$

$$W = 10kN$$

$$UDL = 20kN/m$$



Force (UDL)

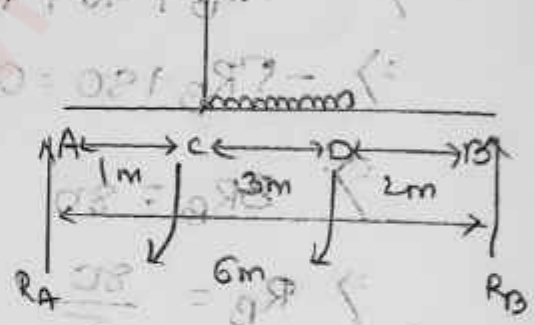
$$F = 20 \times 3 = 60kN$$

Length (UDL)

$$L = 1 + 1.5 = 2.5$$

$$R_A = ?$$

$$R_B = ?$$



Case-1 (Moment)

Now taking moment at point A

$$\sum M_A = 0$$

$$\Rightarrow (-R_B \times 6) + (60 \times 2.5) + (10 \times 1) + (20 \times 6) = 0$$

$$\Rightarrow -6R_B + 150 + 10 + 120 = 0$$

$$\Rightarrow -6R_B + 280 = 0$$

$$\Rightarrow 6R_B = 280$$

$$\Rightarrow R_B = \frac{280}{6}$$

$$\Rightarrow R_B = 46.66 \text{ kN}$$

Case-II (vertical)

$$\boxed{\sum V = 0}$$

$$\Rightarrow R_A + R_B - 10 - 30 - 60 = 0$$

$$\Rightarrow R_A + 56.66 - 90 - 10 = 0$$

$$\Rightarrow R_A - 43.34 = 0$$

$$\Rightarrow \boxed{R_A = 43.34 \text{ kN}}$$

GOVERNMENT POLYTECHNIC, BHUBANESWAR

ସରକାରୀ ପଲିଟେକ୍ନିକ ଭୁବନେଶ୍ୱର



ENGINEERING MECHANICS NOTE

CHAPTER-3 [FRICTION]

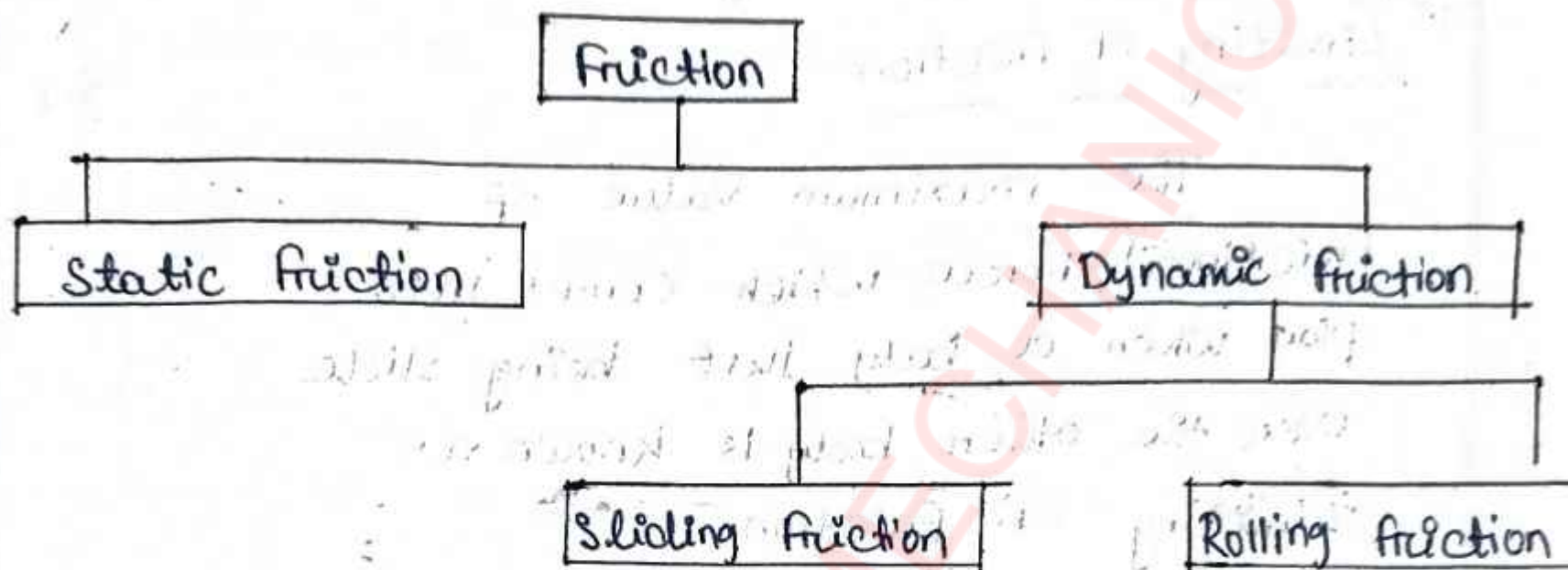
PREPARED BY

ER. RAJKUMAR PRADHAN [RAJ SIR]

Friction

The force acting on a body in the position opposite direction of body is known as friction.

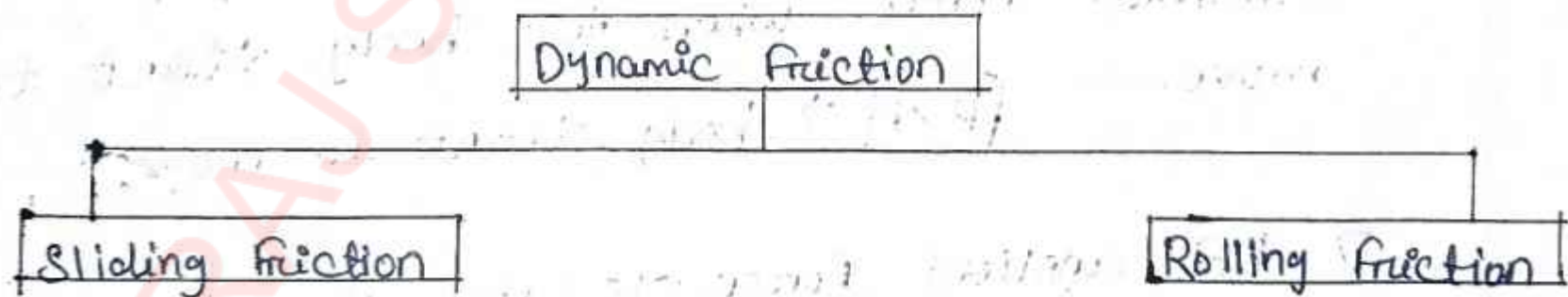
→ There are two types of friction.

Static friction

The friction experienced by a body when body is at rest; this force is known as static friction.

Dynamic friction

The friction experienced by a body when body is in motion; this force is known as dynamic friction.

Sliding friction

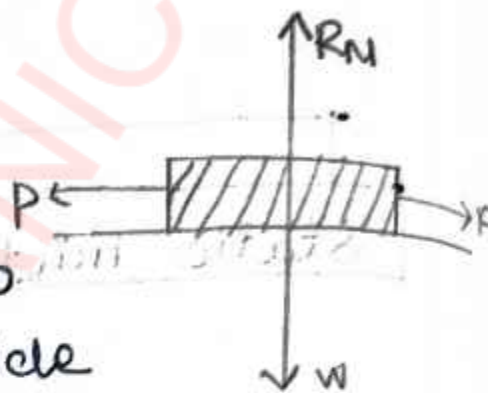
It's the force of friction exerted between two contact surfaces when one body slides over the other; this is known as sliding friction.

Rolling friction

It's the 'force' of friction exerted between a contact surface when body rolls over other body known as Rolling friction.

Limiting of friction

The maximum value of frictional force which comes into play when a body just being slide over the other body is known as Limiting of friction.



Where P - applied force

F - frictional force

R_N - Normal reaction

W - weight of body

→ If applied force is equal to frictional force then the body at rest. $P = F \rightarrow$ Rest body

→ If applied force is greater than to frictional force when the body start to move. $P > F \rightarrow$ body start to move

→ If applied force is less than to frictional force when the body is not to move. $P < F$ body, not to move.

Coefficient of Friction (μ)

The ratio between frictional force and normal reaction is known as coefficient of friction.

→ It denoted as (μ)

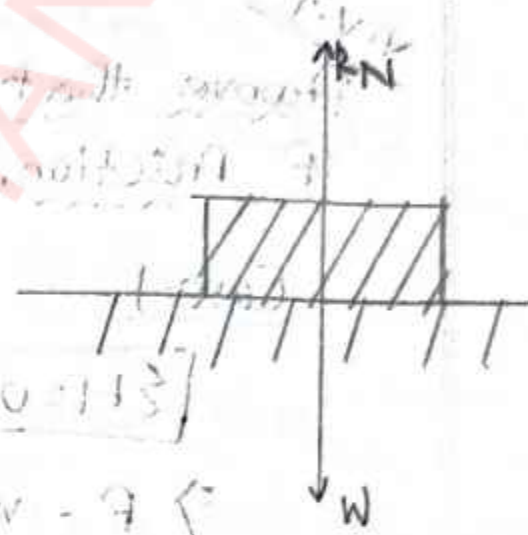
→ No unit

mathematically,

$$F \propto R_N$$

$$\Rightarrow F = \mu R_N$$

$$\Rightarrow \mu = \frac{F}{R_N}$$



Where F = Frictional force
 R_N = Normal reaction

V.V.P

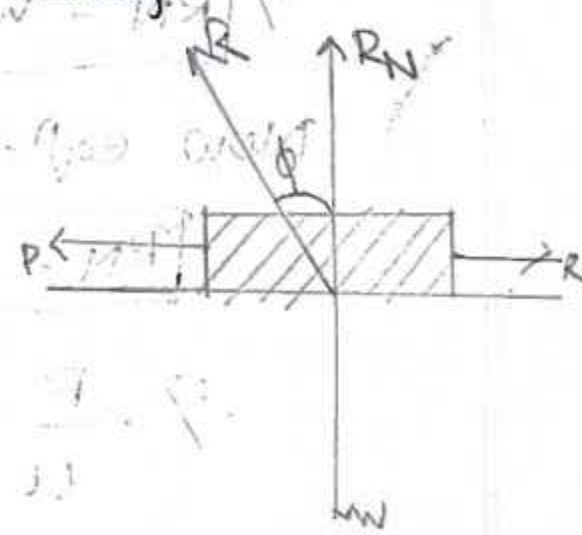
Angle of Friction (ϕ)

The angle between R_N and the resultant of friction is known as Angle of Friction.

→ It denoted as ϕ mathematically.

$$\phi = \tan^{-1} \mu$$

$$\phi = \tan^{-1} \left(\frac{F}{R_N} \right)$$



Where

F = Frictional force

R_N = Normal reaction

Angle of Repose (α)

The angle between inclination plane to the horizontal plane is known as Angle of Repose.

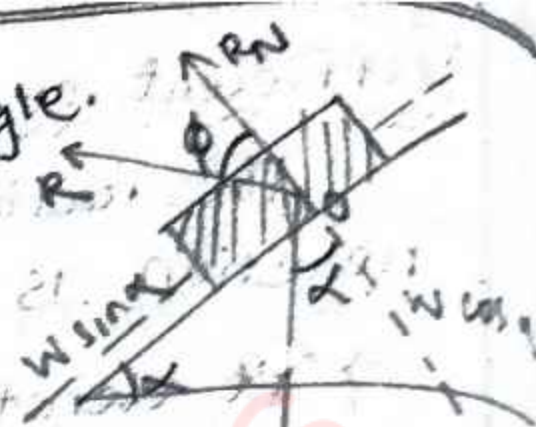
→ It also called as Inclined angle.

Mathematically,

$$\alpha = \phi$$

$$\Rightarrow \alpha = \tan^{-1} \mu$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{F}{R_N} \right)$$



V.V.D

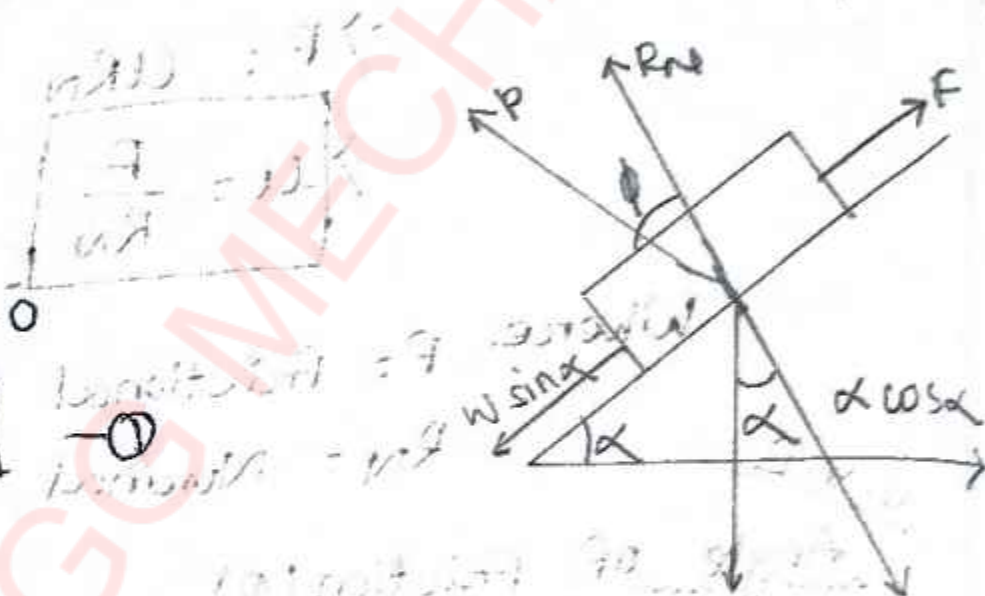
Prove that angle of repose is equal to angle of friction.

Case-I

$$\Sigma H = 0$$

$$\Rightarrow F - W \sin \alpha = 0$$

$$\Rightarrow F = W \sin \alpha \quad \text{--- (I)}$$



Case-II

$$\Sigma V = 0$$

$$\Rightarrow R_N - W \cos \alpha = 0$$

$$\Rightarrow R_N = W \cos \alpha \quad \text{--- (II)}$$

Now eqn - (II)

$$R_N = W \cos \alpha$$

$$\Rightarrow \frac{F}{R_N} = \frac{W \sin \alpha}{W \cos \alpha}$$

$$F = \mu W \cos \alpha$$

$$\Rightarrow W \sin \alpha = \mu W \cos \alpha$$

$$\Rightarrow \mu = \frac{\sin \alpha}{\cos \alpha}$$

$$\Rightarrow \tan \phi = \tan \alpha$$

$$\Rightarrow \phi = \alpha \quad \left[\begin{array}{l} \phi = \tan^{-1} \mu \\ \mu = \tan \phi \end{array} \right]$$

N.V.P. Law of Friction

→ Force is acting in opposite direction of the body
IF 'Applied' force is equal to frictional force
then body is at rest.

$$F = P \rightarrow \text{body rest}$$

→ IF the applied force is greater than the
frictional force then the body is start to
move.

$$P > F \rightarrow \text{body start to move}$$

→ IF the applied force is less than the
frictional force then the body is not to move.

$$P < F \rightarrow \text{body not to move}$$

→ Angle of repose (α) is equal to angle of
friction (ϕ).

$$\alpha = \phi$$

→ Frictional force is always directly
proportional to the normal reaction.

$$F \propto R_N$$

→ In a friction with contains constant
called coefficient of friction.

$$\mu = \frac{F}{R_N}$$

Important formula for frictional problem

$$\mu = \frac{F}{RN}$$

$$RN = \frac{F}{\mu}$$

$$F = \mu RN$$

Angle of Repose

$$\alpha = \phi$$

$$\alpha = \tan^{-1} \mu$$

$$\alpha = \tan^{-1} \left(\frac{F}{RN} \right)$$

Angle of friction

$$\phi = \tan^{-1} \mu$$

$$\Rightarrow \phi = \tan^{-1} \frac{F}{RN}$$

$$\Rightarrow \mu = \tan \phi$$

① Inclined plane

Maximum force

$$F_{\max} = W \sin \alpha$$

minimum force

$$F_{\min} = W \cos \alpha$$

② Applied horizontal

Max^m force

$$P_{\max} = W \tan (\alpha + \phi)$$

min^m force

$$P_{\min} = W \tan (\alpha - \phi)$$

(iii) Applied Parallel Force

Maximum force

$$P_{\max} = \frac{W \sin(\alpha + \phi)}{\cos \phi}$$

minimum force

$$P_{\min} = \frac{W \sin(\alpha - \phi)}{\cos \phi}$$

where

F = frictional force

R_N = Normal Reaction

W = Weight

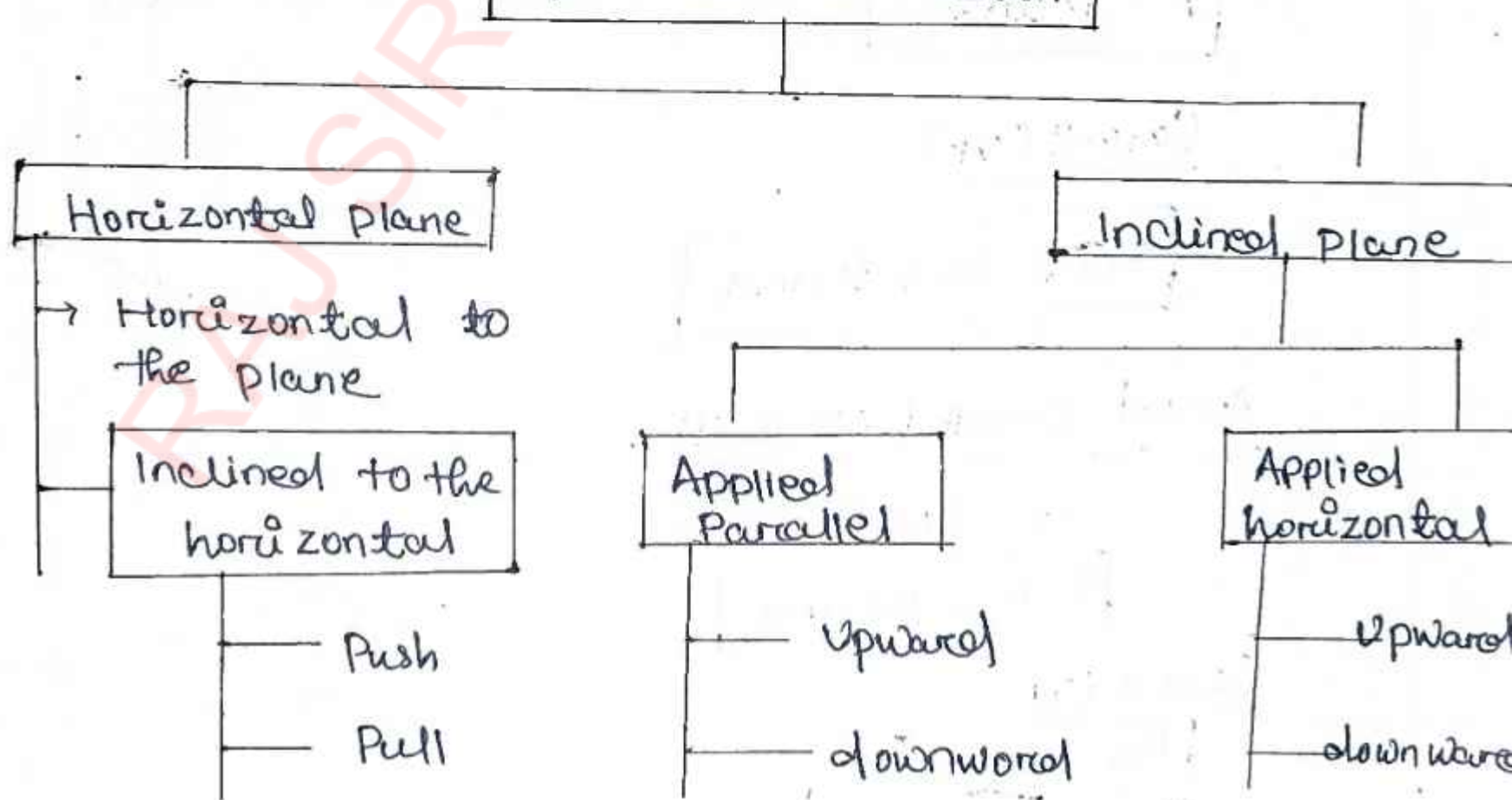
α = Inclined angle / Angle of Repose

ϕ = Angle of friction

μ = coefficient of friction.

Equilibrium of friction

Equilibrium of friction



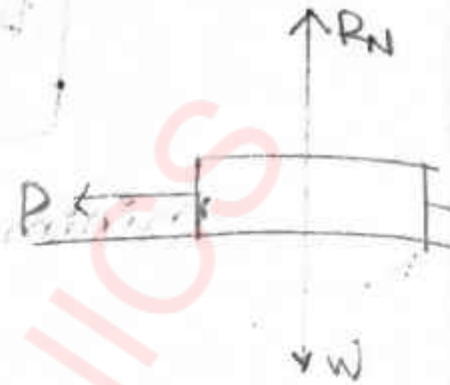
Horizontal to the plane method

Case-I (Horizontal)

$$P = F$$

Case-II (Vertical)

$$R_N = W$$



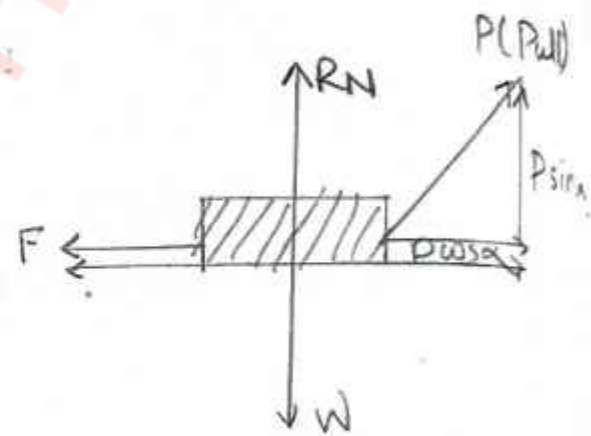
Inclined to the horizontal

Case-I (HF)

$$F = P \cos \alpha$$

Case-II (VF)

$$R_N + P \sin \alpha = W$$



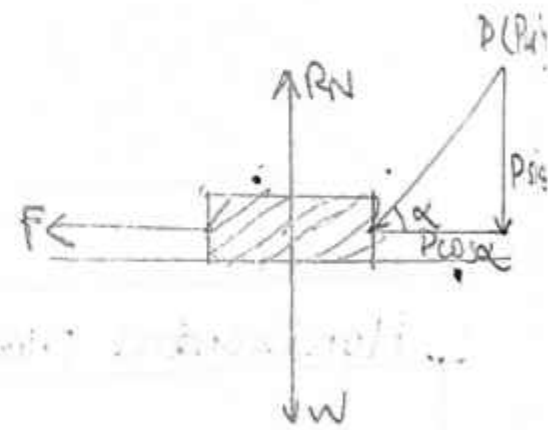
Push method

Case-I (HF)

$$F = P \cos \alpha$$

Case-II (VF)

$$R_N = W + P \sin \alpha$$



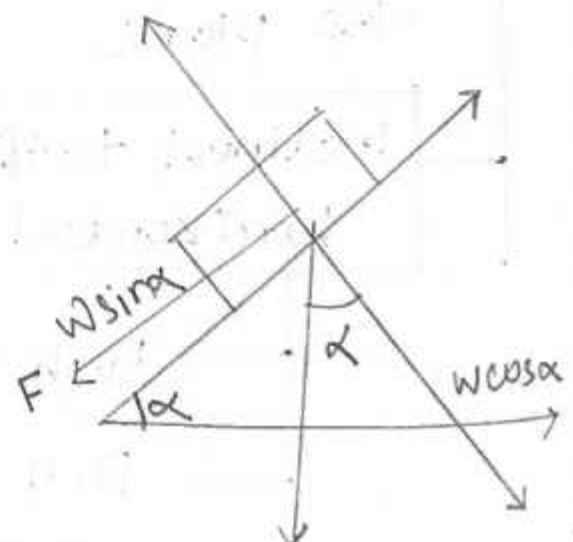
Applied parallel (Method)

Case-I (HF)

$$P + F = W \sin \alpha$$

Case-II (VF)

$$R_N = W \cos \alpha$$



- ① If the coefficient of the friction is 0.2 Find the angle of friction and angle of repose.

Given data

$$\mu = 0.2$$

$$\alpha = ?$$

$$\phi = ?$$

Angle of friction

$$\phi = \tan^{-1} \mu$$

$$\phi = \tan^{-1} (0.2)$$

$$\boxed{\phi = 11.3^\circ}$$

Angle of repose

$$\alpha = \phi$$

$$= \tan^{-1} \mu$$

$$= \tan^{-1} (0.2)$$

$$\boxed{\alpha = 11.3^\circ}$$

- ② If the coefficient of the friction is 0.3 Find the angle of friction and angle of repose.

Given data

$$\mu = 0.3$$

$$\alpha = ?$$

$$\phi = ?$$

Angle of friction

$$\phi = \tan^{-1} \mu$$

$$= \tan^{-1} (0.3)$$

$$\boxed{\phi = 16.69^\circ}$$

Angle of repose

$$\begin{aligned}\alpha &= \phi \\ &= \tan^{-1} \mu \\ &= \tan^{-1} (0.3)\end{aligned}$$

$$\boxed{\alpha = 16.69^\circ}$$

- ③ A block of weight 200N rests on an inclined plane. If the coefficient of friction is 0.35. Find the angle of repose and greatest and smallest force of the friction.

Given data

$$W = 200\text{N}$$

$$\mu = 0.35$$

$$\alpha = ?$$

$$F_{\text{max}} (\text{greatest}) = ?$$

$$F_{\text{min}} (\text{smallest}) = ?$$

Angle of Repose

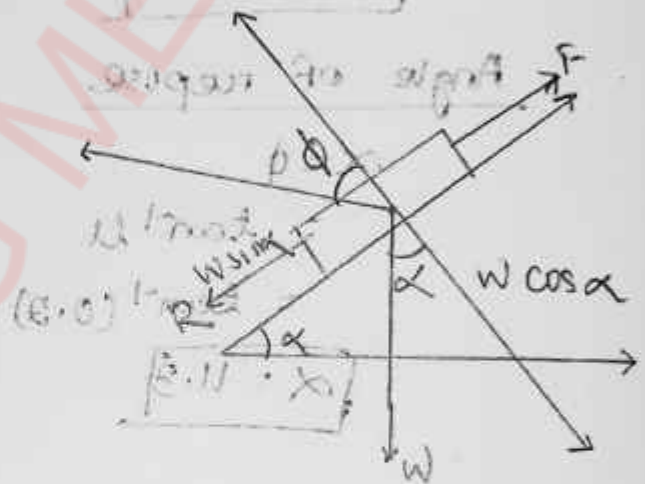
$$\begin{aligned}\alpha &= \tan^{-1} \mu \\ &= \tan^{-1} \mu \\ &= \tan^{-1} (0.35)\end{aligned}$$

$$\boxed{\alpha = 19.29^\circ}$$

Maximum force

$$\begin{aligned}F_{\text{max}} &= W \sin \alpha \\ &= 200 \sin (19.29^\circ)\end{aligned}$$

$$\boxed{F_{\text{max}} = 66.06\text{N}}$$



Minimum Force

$$F_{\min} = W \cos \alpha$$
$$= 200 \cos (19.29)$$

$$F_{\min} = 188.77 \text{ N}$$

④

A block of weight 300N rest on incline. If the coefficient of friction 0.65 find the angle of repose and greatest and smallest force of the friction.

Given data

$$W = 300 \text{ N}$$

$$\mu = 0.65$$

$$\alpha = ?$$

$$F_{\max} \text{ (greatest)} = ?$$

$$F_{\min} \text{ (Smallest)} = ?$$

$$\text{Angle of repose} = ?$$

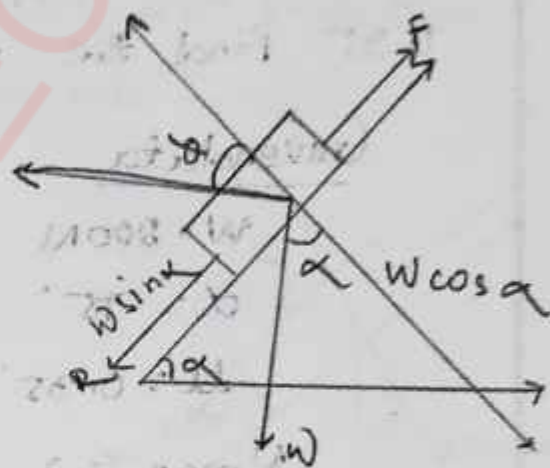
$$\alpha = \phi$$
$$= \tan^{-1} \mu$$
$$= \tan^{-1} (0.65)$$

$$\alpha = 33.02^\circ$$

Maximum Force

$$F_{\max} = W \sin \alpha$$
$$= 300 \sin (33.02)$$

$$F_{\max} = 163.47 \text{ N}$$



Minimum force

$$F_{\min} = W \cos \alpha$$

$$= 300 \cos (33.02)$$

$$F_{\min} = 251.54 \text{ N}$$

5

A. object of weight 200N rest on a inclined plane make angle 75° with the horizontal. The force is applied horizontal to the object applied coefficient of friction 0.35 find the maximum & minimum value of force.

Given data

$$W = 200 \text{ N}$$

$$\alpha = 75^\circ$$

$$\mu = 0.35$$

$$P_{\max} = ?$$

$$P_{\min} = ?$$

[Applied horizontal]

Angle of friction

$$\phi = \tan^{-1} \mu$$

$$= \tan^{-1} (0.35)$$

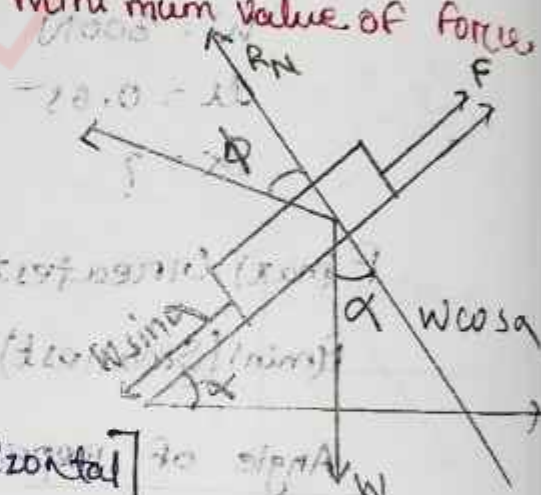
$$\phi = 19.29^\circ$$

Maximum force

$$P_{\max} = W \tan (\alpha + \phi)$$

$$200 \tan (75 + 19.29)$$

$$P_{\max} = 2666.73 \text{ N}$$



Minimum force

$$P_{\min} = W \tan(\alpha - \phi)$$
$$= 200 \tan(75 - 19.29)$$

$$P_{\min} = 243.29 \text{ N}$$

⑥

A object of weight 500N rest on a inclined plane make angle 30° with the horizontal plane the force is applied horizontal to the object applied coefficient of friction 0.37. Find the max^m and min^m value of the force.

Given data

$$W = 500 \text{ N}$$

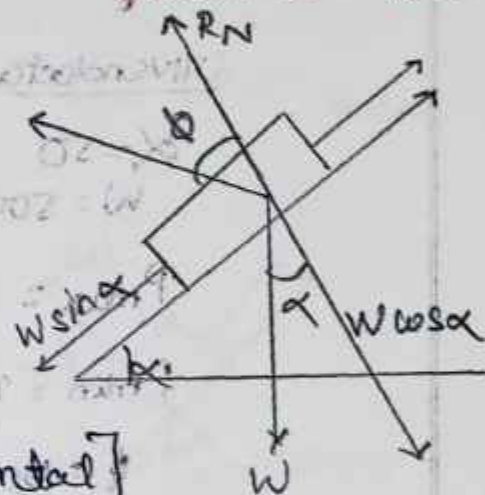
$$\alpha = 30^\circ$$

$$\mu = 0.37$$

$$P_{\max} = ?$$

$$P_{\min} = ?$$

[applied horizontal]



Angle of friction

$$\phi = \tan^{-1} \mu$$
$$= \tan^{-1}(0.37)$$

$$\phi = 20.30^\circ$$

Minimum force

$$P_{\max} = W \tan(\alpha + \phi)$$

$$= 500 \tan(30 + 20.30)$$

$$P_{\max} = 602.25 \text{ N}$$

Minimum force

$$P_{\min} = W \tan(\alpha - \phi)$$
$$= 500 \tan(30^\circ - 20^\circ)$$

$$P_{\min} = 85.46 \text{ N}$$

A body of weight 500N laying on a rough inclined plane which makes an angle of 50° with the horizontal. A force is applied to the body parallel to the plane. Find the max^m and min^m value of the force if angle of friction 30° .

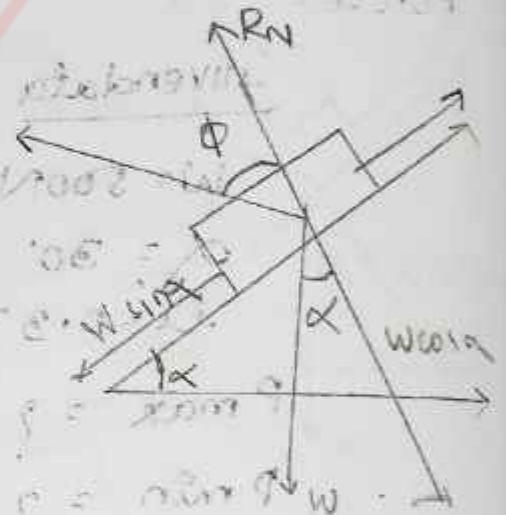
Given data

$$\alpha = 50^\circ$$
$$W = 500 \text{ N}$$

$$P_{\max} = ?$$
$$P_{\min} = ?$$

(applied force)

$$\phi = 30^\circ$$



Max^m force

$$P_{\max} = \frac{W \sin(\alpha + \phi)}{\cos \phi}$$

$$= \frac{500 \sin(50^\circ + 30^\circ)}{\cos 30^\circ}$$

$$= \frac{500 \sin 80^\circ}{\cos 30^\circ}$$

$$P_{\max} = 568.57 \text{ N}$$

$$\begin{aligned}
 P_{\min} &= \frac{W \sin(\alpha - \phi)}{\cos \phi} \\
 &= \frac{500 \sin(50 - 30)}{\cos 30} \\
 &= \frac{500 \sin 20}{\cos 30}
 \end{aligned}$$

$$P_{\min} = 197.46 \text{ N}$$

A body of weight 400N laying on a plane of inclined angle, plane which makes an angle 70° with the horizontal, force is applied to the parallel plane. Find the max^m and min^m value. If angle of friction is 25° .

Given data

$$W = 400 \text{ N}$$

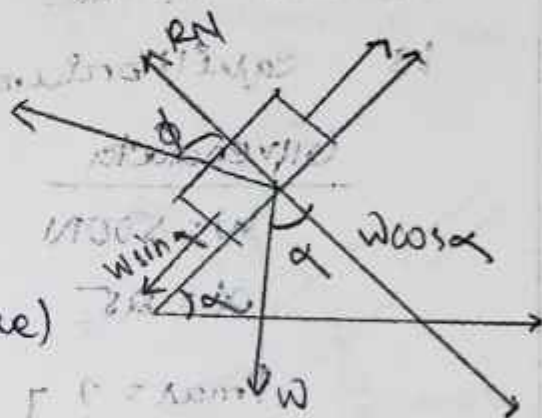
$$\alpha = 70^\circ$$

$$P_{\max} = ?$$

$$P_{\min} = ?$$

$$\phi = 25^\circ$$

(applied force)



max^m force

$$P_{\max} = \frac{W \sin(\alpha + \phi)}{\cos \phi}$$

$$= \frac{400 \sin(70 + 25)}{\cos 25}$$

$$= \frac{400 \sin 95}{\cos 25} = 439.67 \text{ N}$$

Minimum Force

$$\begin{aligned} P_{\min} &= \frac{W \sin(\alpha - \phi)}{\cos \phi} \\ &= \frac{400 \sin(70 - 25)}{\cos 25} \\ &= \frac{400 \sin 45}{\cos 25} \end{aligned}$$

$$P_{\min} = 312.08 \text{ N}$$

A body weight 500N is lying on a rough plane inclined at an angle 25° with the horizontal. It is supported by an effort parallel to the plane as shown in the figure. Determine the max^m and min^m value of P which is equilibrium if angle of friction 20° .

Given data

$$W = 500 \text{ N}$$

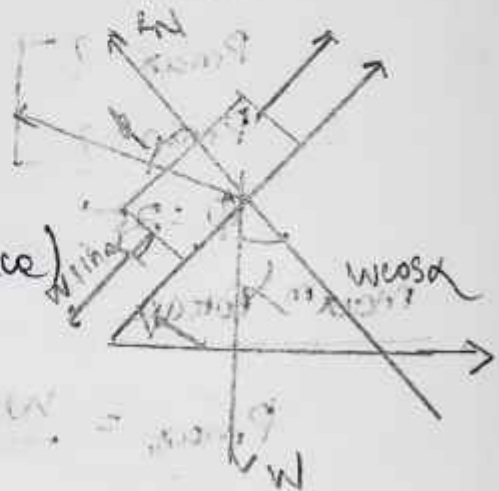
$$\alpha = 25^\circ$$

$$P_{\max} = ?$$

$$P_{\min} = ?$$

$$\phi = 20^\circ$$

(applied force)



Max^m Force

$$\begin{aligned} P_{\max} &= \frac{W \sin(\alpha + \phi)}{\cos \phi} \\ &= \frac{500 \sin(25 + 20)}{\cos 20} \\ &= \frac{500 \sin 45}{\cos 20} = 376.24 \text{ N} \end{aligned}$$

minimum force

$$P_{\min} = \frac{w \sin(\alpha - \phi)}{\cos \phi}$$
$$= \frac{500 \sin(25^\circ - 20^\circ)}{\cos 20^\circ}$$
$$= \frac{500 \sin 5^\circ}{\cos 20^\circ}$$

$$P_{\min} = 46.37 \text{ N}$$

A body weight 700 N is lying on a plane inclined at an angle 35° with the horizontal. It is supported by an effort of parallel plane as shown in the figure. determine the min^m and max^m of P which equilibrium if the angle of friction 25° .

Given data

$$W = 700 \text{ N}$$

$$\alpha = 35^\circ$$

$$P_{\max} = ?$$

$$P_{\min} = ?$$

(applied force)

$$\phi = 25^\circ$$

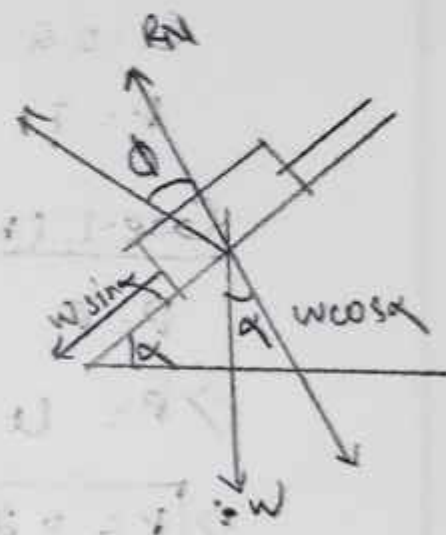
max^m force

$$P_{\max} = \frac{w \sin(\alpha + \phi)}{\cos \phi}$$

$$= \frac{700 \sin(35^\circ + 25^\circ)}{\cos 25^\circ}$$

$$= \frac{700 \sin 60^\circ}{\cos 25^\circ}$$

$$= 680.88 \text{ N}$$



$$P_{\min} = \frac{W \sin(\alpha - \phi)}{\cos \phi}$$

$$= \frac{100 \sin(35 - 25)}{\cos 25}$$

$$= \frac{100 \sin 10}{\cos 25}$$

$$P_{\min} = 134.11 \text{ N}$$

What horizontal force is required to pull a body of weight 200N along with the horizontal surface take the coefficient of friction is 0.2.

Given data

$$W = 200 \text{ N}$$

$$\mu = 0.2$$

$$P = ?$$

Case-I (HF)

$$P = F$$

$$\Rightarrow P = \mu R_N \quad \left[\because \mu = \frac{F}{R_N} \right]$$

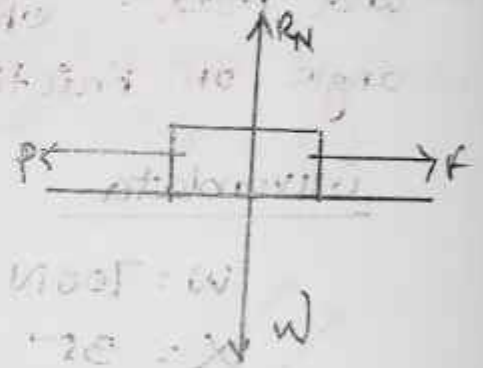
$$\Rightarrow \boxed{P = 0.2 R_N} \quad \text{--- (1)} \quad \Rightarrow F = \mu R_N$$

Case-II (VF)

$$R_N = W$$

$$R_N = 200 \text{ N}$$

Now taking (R_N) value Putting eqⁿ (1)



$$P = 0.2 RN$$

$$= 0.2 \times 200 N$$

$$\boxed{P = 40 N}$$

What horizontal force is required to a pull body of weight 300 N along with a horizontal surface take the coefficient of friction is 0.3.

Given data

$$W = 300 N$$

$$\mu = 0.3$$

$$P = ?$$

Case-I (HF)

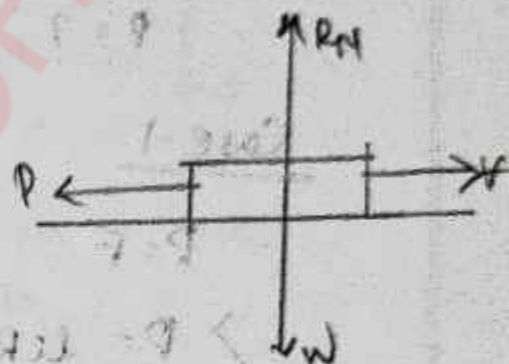
$$P = F$$

$$\Rightarrow P = \mu RN$$

$$\Rightarrow \boxed{P = 0.3 RN} \quad \text{--- (1)}$$

$$\mu = \frac{F}{RN}$$

$$\Rightarrow F = \mu RN$$



Case-II (VF)

$$RN = W$$

$$\Rightarrow RN = 300 N$$

Now (RN) value putting eqn (1)

$$P = 0.3 RN$$

$$\Rightarrow P = 0.3 \times 300 N$$

$$\Rightarrow \boxed{P = 90 N}$$

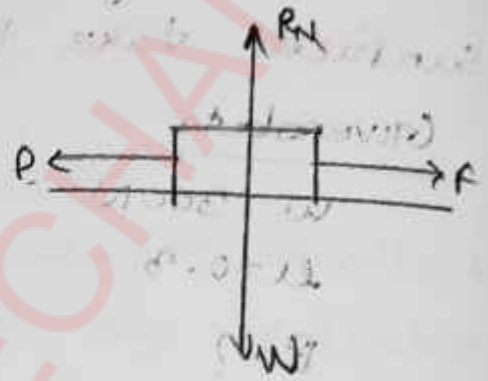
Find the horizontal effort to move the body weight is 90 N along a horizontal. The plane is gradually rise upto 15° the body will slide up.

Given data

$$W = 90$$

$$\phi = 15^\circ$$

$$P = ?$$



Case-I

$$P = F$$

$$\Rightarrow P = \mu RN$$

$$\Rightarrow P = \tan(15^\circ) RN$$

$$\boxed{P = 0.26 RN} \quad \text{--- (1)}$$

Case-II

$$\boxed{RN = W}$$

$$\Rightarrow RN = 90$$

Now (RN) value in eqn (1)

$$P = 0.26 RN$$

$$= 0.26 \times 90$$

$$\boxed{P = 23.4 \text{ N}}$$

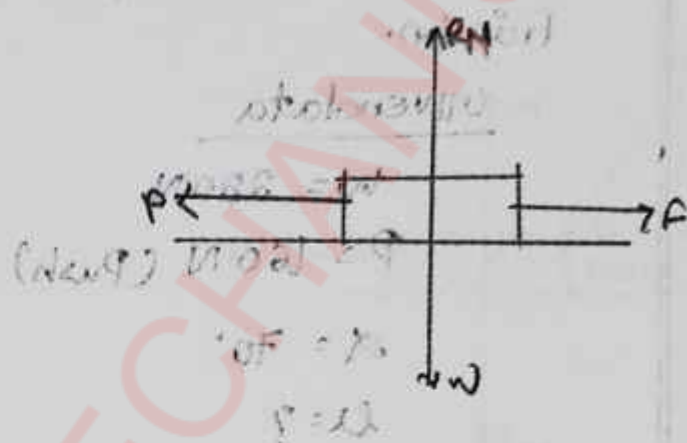
Find the horizontal effort to move the body weight, is 75N along a horizontal plane. The plane is gradually raise upto 25° . The body will slide up.

Given data

$$W = 75\text{N}$$

$$\phi = 25^\circ$$

$$P = ?$$



Case-I

$$P = F$$

$$\Rightarrow P = \mu R_N$$

$$\Rightarrow P = \tan(25^\circ) R_N$$

$$\Rightarrow \boxed{P = 0.46 R_N} \quad \text{--- (1)}$$

$$\phi = \tan^{-1} \mu$$

$$\Rightarrow \phi = \tan^{-1} \mu$$

Case-II

$$\boxed{R_N = W}$$

$$\Rightarrow R_N = 75\text{N}$$

Now R_N value putting in eqn (1)

$$P = 0.46 \times R_N$$

$$= 0.46 \times 75$$

$$\boxed{P = 34.5\text{N}}$$

A body weight 220N is placed on a rough horizontal plane to move the body horizontal plane a push of 160N inclined at 70° to the horizontal plane. Find the coefficient of friction.

Given data

$$W = 220\text{N}$$

$$P = 160\text{N (Push)}$$

$$\alpha = 70^\circ$$

$$\mu = ?$$

Case-I (HF)

$$F = P \cos \alpha$$

$$\Rightarrow F = 160 \cos 70^\circ$$

$$\Rightarrow F = 54.72\text{N}$$

Case-II (VF)

$$R_N = W + P \sin \alpha$$

$$= 220 + 160 \sin 70^\circ$$

$$R_N = 370.35\text{N}$$

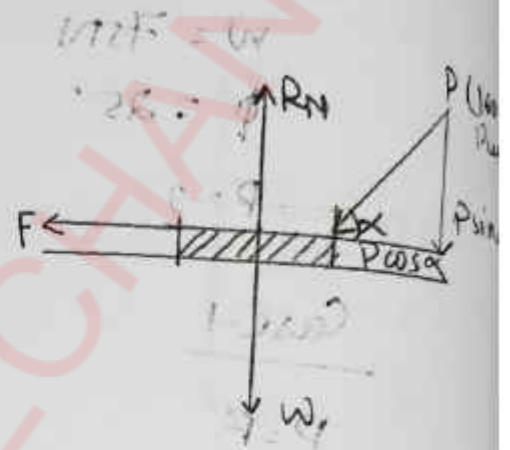
Coefficient of friction

$$\mu = \frac{F}{R_N}$$

$$= \frac{54.72}{370.35}$$

$$= 0.14$$

$$\mu = 0.14$$



A body weight 340 N is placed on a rough horizontal plane. To move the body horizontal plane a push of 130 N inclined at 55° to the horizontal plane. Find the coefficient of friction.

Given data

$$W = 340\text{ N}$$

$$P = 130\text{ N (Push)}$$

$$\alpha = 55^\circ$$

$$\mu = ?$$

Case-I (HF)

$$F = P \cos \alpha$$

$$\Rightarrow F = 130 \times \cos(55^\circ)$$

$$\Rightarrow \boxed{F = 74.56\text{ N}}$$

Case-II (VF)

$$R_N = W + P \sin \alpha$$

$$= 340 + 130 \sin(55^\circ)$$

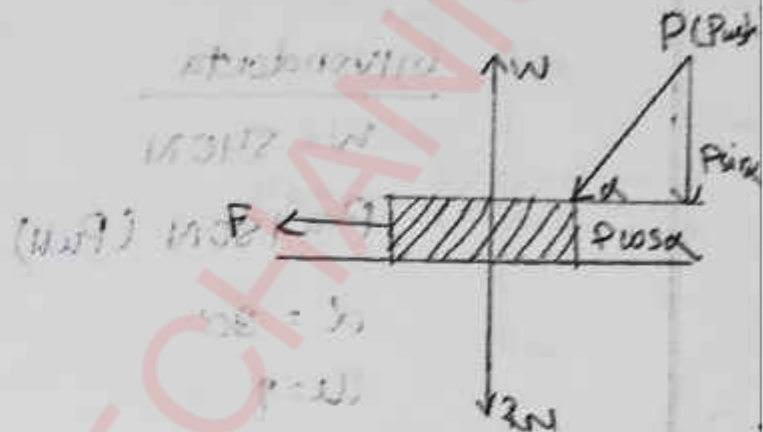
$$\boxed{R_N = 446.48\text{ N}}$$

Coefficient of force

$$\mu = \frac{F}{R_N}$$

$$= \frac{74.56}{446.48}$$

$$\boxed{\mu = 0.16}$$



A body of weight 540N is pulled along the rough of horizontal plane by a pull of 180N acting at a angle of 30° with the horizontal. Find the coefficient of friction.

Given data

$$W = 540\text{N}$$

$$P = 180\text{N (Pull)}$$

$$\alpha = 30^\circ$$

$$\mu = ?$$

Case-I (HF)

$$F = P \cos \alpha$$

$$\Rightarrow F = 180 \cos(30^\circ)$$

$$F = 155.88\text{N}$$

Case-II (VF)

$$R_N + P \sin \alpha = W$$

$$\Rightarrow R_N + 180 \sin 30 = 540$$

$$\Rightarrow R_N = 450 - 180 \sin 30$$

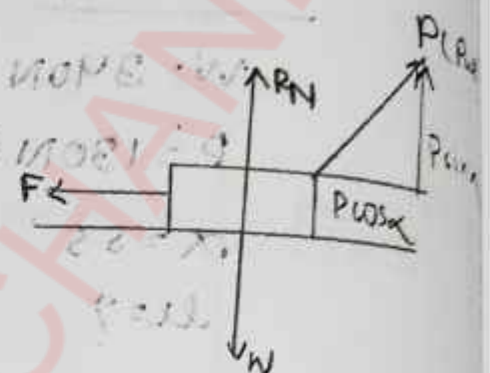
$$\Rightarrow R_N = 450$$

Coefficient of Friction

$$\mu = \frac{F}{R_N}$$

$$= \frac{155.88}{450}$$

$$\mu = 0.34$$



A body of weight 440N is pulled along the rough of horizontal plane by a pull of 175N acting at a angle of 55° with the horizontal. Find the Coefficient of force.

Given data

$$W = 440\text{N}$$

$$P = 175\text{N (Pull)}$$

$$\alpha = 55^\circ$$

$$\mu = ?$$

Case-I (HF)

$$F = P \cos \alpha$$

$$\Rightarrow F = 175 \cos 55^\circ$$

$$\Rightarrow \boxed{F = 100.37\text{N}}$$

Case-II (VF)

$$R_N + P \sin \alpha = W$$

$$\Rightarrow R_N + 175 \sin 55^\circ = 440$$

$$\Rightarrow R_N + 143.35 = 440$$

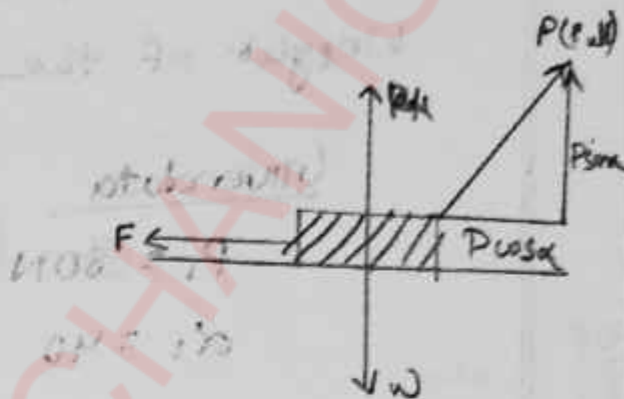
$$\Rightarrow R_N = 440 - 143.35$$

$$\Rightarrow \boxed{R_N = 296.65\text{N}}$$

Coefficient of force

$$\begin{aligned} \mu &= \frac{F}{R_N} \\ &= \frac{100.37}{296.65\text{N}} \end{aligned}$$

$$\Rightarrow \boxed{\mu = 0.33}$$



A body rest on a rough horizontal plane required a pull of 80N inclined at 40° to the plane just move it also found that push of 120N inclined at 30° to the plane find the weight of the body and coefficient of friction.

Given data

$$P_1 = 80\text{N}$$

$$\alpha_1 = 40^\circ \text{ (Pull)}$$

$$P_2 = 120\text{N}$$

$$\alpha_2 = 30^\circ \text{ (Push)}$$

$$W = ?$$

$$\mu = ?$$

Condition-1 (Pull)

$$P_1 = 80\text{N}$$

$$\alpha_1 = 40^\circ$$

Case-I (HF)

$$F = P_1 \cos \alpha_1 = 80 \cos 40^\circ$$

$$F = 61.28\text{N}$$

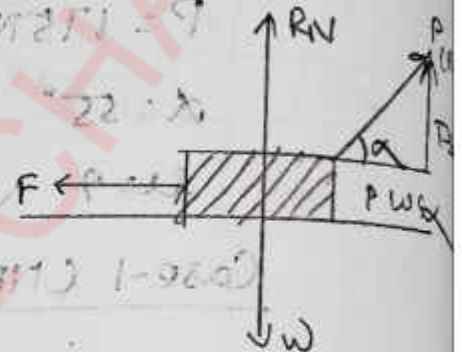
Case-II (VF)

$$R_N = P_1 \sin \alpha_1 = W$$

$$\Rightarrow R_N + 80 \sin 40^\circ = W$$

$$\Rightarrow R_N + 51.42 = W$$

$$\Rightarrow \frac{F}{\mu} + 51.42 = W$$



$$\Rightarrow \frac{61.28}{\mu} + 51.42u = w$$

$$\Rightarrow \frac{61.28 + 51.42\mu}{\mu} = w$$

$$\Rightarrow 61.28 + 51.42\mu = w\mu$$

$$\Rightarrow 61.28 = w\mu - 51.42\mu$$

$$\Rightarrow \boxed{61.28 = \mu(w - 51.42)} \quad \text{--- (I)}$$

Condition-2

$$P_2 = 120\text{N}$$

$$\alpha_2 = 30^\circ$$

Case-I (HF)

$$F = P_2 \cos \alpha_2$$

$$\Rightarrow F = 120 \cos 30^\circ$$

$$\Rightarrow \boxed{F = 103.9\text{N}}$$

Case-II (VF)

$$R_N = W + P \sin \alpha$$

$$\Rightarrow R_N = W + 120 \sin 30^\circ$$

$$\Rightarrow R_N = W + 60$$

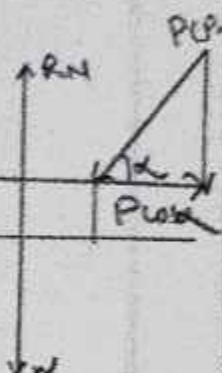
$$\Rightarrow \frac{R \cdot F}{\mu} = W + 60$$

$$\Rightarrow F = \mu W + 60\mu$$

$$\Rightarrow \boxed{103.9 = \mu(W + 60)} \quad \text{--- (II)}$$

Now eqⁿ - (I) & (II) we get

$$\frac{103.9}{61.28} = \frac{\mu(W + 60)}{(\mu W - 51.42)}$$



$$\Rightarrow 103.9 (w - 51.42) = 61.28 (w + 60)$$

$$\Rightarrow 103.9w - (103.9 \times 51.42) = 61.28w + 61.28 \times 60$$

$$\Rightarrow 103.9w - 5342.53 = 61.28w + 3676.8$$

$$\Rightarrow 103.9w - 61.28 = 5342.53 + 3676.8$$

$$\Rightarrow w(103.9 - 61.28) = 9019.33$$

$$\Rightarrow 42.62w = 9019.33$$

$$\Rightarrow w = \frac{9019.33}{42.62}$$

$$\Rightarrow \boxed{w = 211.62 \text{ N}}$$

Now (w) value putting in eqⁿ (1)

$$61.28 = \mu (w - 51.42)$$

$$\Rightarrow 61.28 = \mu (211.62 - 51.42)$$

$$\Rightarrow 61.28 = \mu (160.2)$$

$$\Rightarrow \mu = \frac{61.28}{160.2}$$

$$\Rightarrow \boxed{\mu = 0.38}$$

A body rest on a rough horizontal plane required a pull of 50N inclined at 20° to the plane just move it also found that push of 70N inclined at 20° to the plane find the weight of the body and coefficient force.

Given data

$$P_1 = 50 \text{ N (Pull)}$$

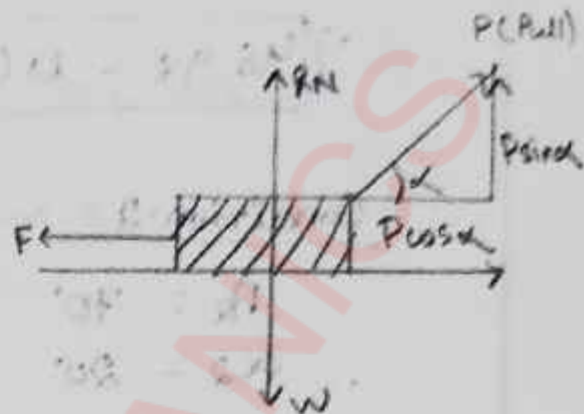
$$\alpha_1 = 20^\circ$$

$$P_2 = 70 \text{ N (Push)}$$

$$\alpha_2 = 20^\circ$$

$$W = ?$$

$$u = ?$$



Condition-1

$$P_1 = 50 \text{ N}$$

$$\alpha_1 = 20^\circ$$

Case-1 (HF)

$$F = P_1 \cos \alpha_1$$

$$= 50 \cos 20^\circ$$

$$F = 46.98 \text{ N}$$

Case-2 (VF)

$$R_N + P \sin \alpha_1 = W$$

$$\Rightarrow R_N + 50 \sin 20^\circ = W$$

$$\Rightarrow R_N + 17.10 = W$$

$$\Rightarrow \frac{F}{u} + 17.10 = W$$

$$\Rightarrow \frac{46.98}{u} + 17.10 = W$$

$$\Rightarrow \frac{46.98 + 17.10u}{u} = W$$

$$(P \cos \alpha + W) \Rightarrow 46.98 + 17.10u = uW$$

$$\Rightarrow 46.98 = w - 17.10u$$

$$\Rightarrow \boxed{46.98 = u(w - 17.10)} \quad \text{--- (i)}$$

Condition-2

$$P_2 = 70'$$

$$\alpha_2 = 20'$$

Case-I (HF)

$$F = P_2 \cos \alpha_2$$

$$\Rightarrow F = 70 \cos 20'$$

$$\Rightarrow \boxed{F = 65.77 \text{ N}}$$

Case-II (VF)

$$R_N = w + P \sin \alpha$$

$$\Rightarrow R_N = w + 70 \sin 20'$$

$$\Rightarrow R_N = w + 23.94$$

$$\Rightarrow \frac{F}{u} = w + 23.94$$

$$\Rightarrow F = u(w + 23.94)$$

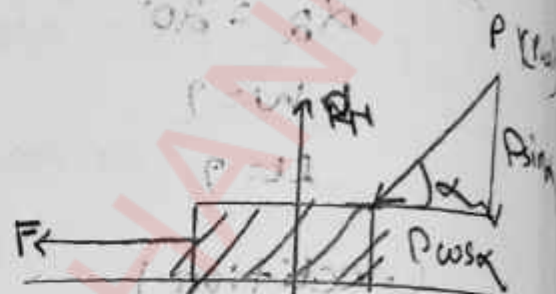
$$\Rightarrow \boxed{65.77 = u(w + 23.94)} \quad \text{--- (ii)}$$

Now eqn- (i) & (ii) we get

$$\frac{65.77}{46.98} = \frac{u(w + 23.94)}{u(w - 17.10)}$$

$$\Rightarrow 65.77 (w - 17.10) = 46.98 (w + 23.94)$$

$$\Rightarrow 65.77w - 65.77 \times 17.10 = 46.98w + 46.98 \times 23.94$$



$$\Rightarrow 65.77w - 1124.66 = 46.98w + 1124.70$$

$$\Rightarrow 65.77w - 46.98w = 1124.70 + 1124.66$$

$$\Rightarrow w(65.77 - 46.98) = 2249.36$$

$$\Rightarrow 18.79w = 2249.36$$

$$\Rightarrow w = \frac{2249.36}{18.79}$$

$$\Rightarrow w = 119.71$$

Now (w) value putting in eqn-①

$$46.98 = u(w - 17.10)$$

$$\Rightarrow 46.98 = u(119.71 - 17.10)$$

$$\Rightarrow 46.98 = u(102.61)$$

$$\Rightarrow u = \frac{46.98}{102.61}$$

$$\Rightarrow u = 0.45$$

An inclined plane as shown in the figure is used to a lead slowly a body weight 400N from a truck 1.2m high into the ground the coefficient of friction is 0.3 what min force is required to the plane for this purpose

Given data

$$W = 400\text{N}$$

$$\mu = 0.3$$

$$P = ?$$

$$AB = b = 2.4\text{m}$$

$$BC = P = 1.2\text{m}$$

In $\triangle ABC$

$$\tan \alpha = \frac{P}{b}$$

$$\Rightarrow \tan \alpha = \frac{1.2}{2.4}$$

$$\Rightarrow \tan \alpha = 0.5$$

$$\Rightarrow \alpha = \tan^{-1} 0.5$$

$$\Rightarrow \boxed{\alpha = 26.56^\circ}$$

Case-I (HP)

$$F + P = \sin \alpha$$

$$= 400 \sin 26.56$$

$$= 178.85$$

$$\Rightarrow \mu R_N + P = 178.85$$

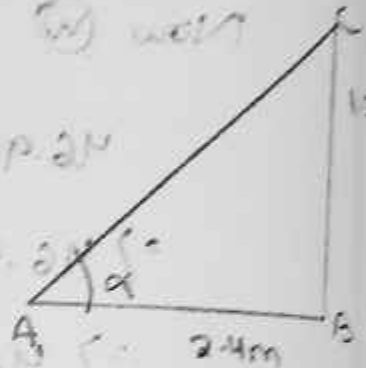
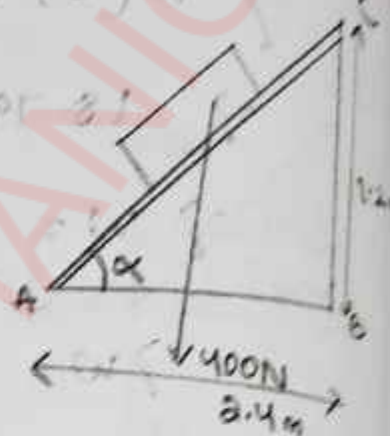
$$\Rightarrow \boxed{0.3 R_N + P = 178.85\text{N}}$$

Case-II (VF)

$$R_N = W \cos \alpha$$

$$\Rightarrow R_N = 400 \cos 26.56$$

$$\boxed{R_N = 357.78\text{N}}$$



$$\left[\mu = \frac{F}{R_N} \right]$$

$$\Rightarrow F = \mu R_N$$

An inclined plane as shown in the figure is used to a load slowly a body weight 400N from a truck 1.2m high into the ground the coefficient of friction is 0.3 what min force is required to the plane for this purpose.

Given data

$$W = 400\text{N}$$

$$\mu = 0.3$$

$$P = ?$$

$$AB = b = 2.4\text{m}$$

$$BC = p = 1.2\text{m}$$

In $\triangle ABC$

$$\tan \alpha = \frac{p}{b} \quad (0.5) = 0.5$$

$$\Rightarrow \tan \alpha = \frac{1.2}{2.4} = 0.5$$

$$\Rightarrow \tan \alpha = 0.5$$

$$\Rightarrow \alpha = \tan^{-1} 0.5 = 26.56^\circ$$

$$\Rightarrow \boxed{\alpha = 26.56^\circ}$$

Case-I (HP)

$$F + P = \sin \alpha$$

$$= 400 \sin 26.56$$

$$= 178.85$$

$$\Rightarrow \mu R_N + P = 178.85$$

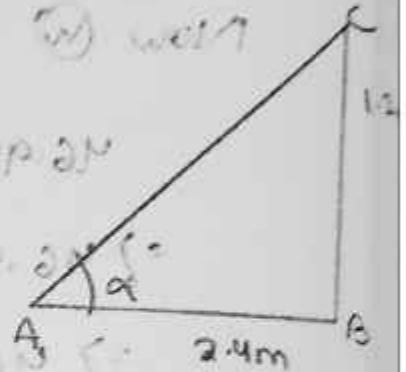
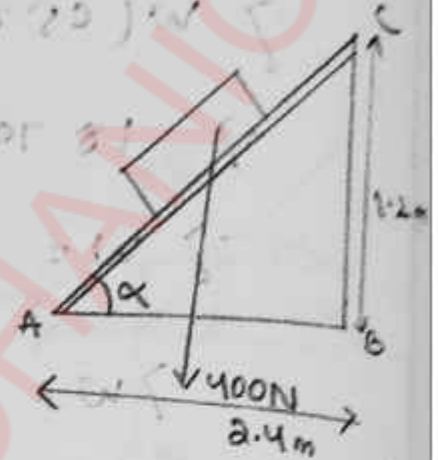
$$\Rightarrow \boxed{0.3 R_N + P = 178.85\text{N}}$$

Case-II (VF)

$$R_N = W \cos \alpha$$

$$\Rightarrow R_N = 400 \cos 26.56$$

$$\boxed{R_N = 357.78\text{N}}$$



$$\left[\because \mu = \frac{F}{R_N} \right]$$

$$\Rightarrow F = (\mu R_N)$$

Now (R_N) value putting eq-1

$$0.3 R_N + P = 178.85$$

$$\Rightarrow 0.3 \times 367.78 + P = 178.85$$

$$\Rightarrow 107.33 + P = 178.85$$

$$\Rightarrow P = 178.85 - 107.33$$

$$\Rightarrow \boxed{P = 71.52 \text{ N}}$$

An Inclined plane as shown in Figure is used to a load slowly a body weight 300N from a truck 1.4m high into the ground. The coefficient of friction is 0.2 what min^m force is required to the plane for this purpose.

Given data

$$W = 300 \text{ N}$$

$$\mu = 0.2$$

$$P = ?$$

$$AB = b = 3 \text{ m}$$

$$BC = P = 1.4 \text{ m}$$

In ΔABC

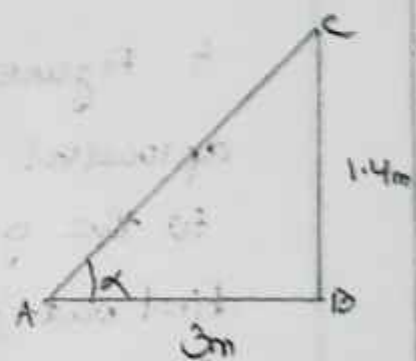
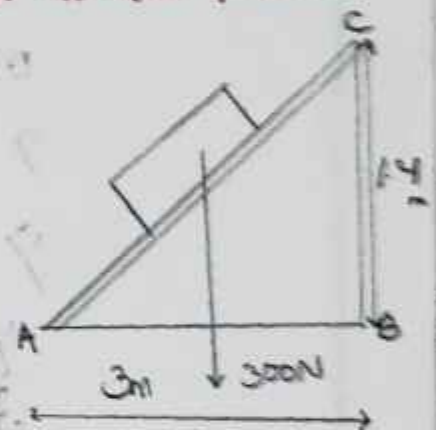
$$\tan \alpha = \frac{P}{b}$$

$$\Rightarrow \tan \alpha = \frac{1.4}{3}$$

$$\Rightarrow \tan \alpha = 0.46$$

$$\Rightarrow \alpha = \tan^{-1} 0.46$$

$$\Rightarrow \boxed{\alpha = 24.70^\circ}$$



Case-1 (HF)

$$F + P = W \sin \alpha$$

$$\Rightarrow F + P = 300 \sin 24.70$$

$$\Rightarrow R_N + P = 125.36$$

$$\Rightarrow 0.2R_N + P = 125.36$$

$$\Rightarrow R_N [P = 125.36 - 0.2R_N] \quad \text{--- (1)}$$

$$\Rightarrow P = 125.16R_N$$

Case-II (VF)

$$R_N = W \cos \alpha$$

$$R_N = 300 \cos 24.70$$

$$R_N = 272.55 \text{ N}$$

Now, R_N value putting eqⁿ (1)

$$0.2R_N + P = 125.36$$

$$\Rightarrow 0.2 \times 272.55 + P = 125.36$$

$$\Rightarrow 54.51 + P = 125.36$$

$$\Rightarrow P = 125.36 - 54.51$$

$$\Rightarrow P = 70.85 \text{ N}$$

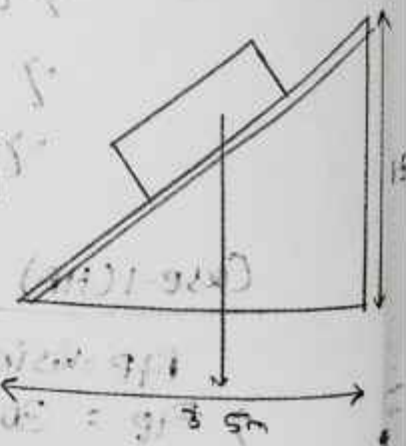
A figure as shown in Figure. Inclined plane body applied a weight 500N from a point 1.5m high to the ground the coefficient of friction is 0.25. Find out the min^m force will be apply parallel to the plane force above purpose.

Given data

$$W = 500 \text{ N}$$

$$\mu = 0.25$$

$$P = ?$$



$$AB = b = 3.5 \text{ m}$$

$$BC = p = 1.5 \text{ m}$$

In $\triangle ABC$

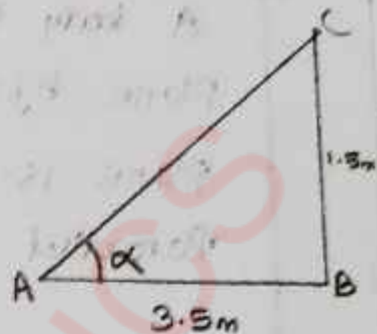
$$\tan \alpha = \frac{p}{b}$$

$$\Rightarrow \tan \alpha = \frac{1.5}{3.5}$$

$$\Rightarrow \tan \alpha = 0.42$$

$$\Rightarrow \alpha = \tan^{-1} 0.42$$

$$\Rightarrow \boxed{\alpha = 22.78^\circ}$$



Case-I (HF)

$$F + P = W \sin \alpha$$

$$\Rightarrow F + P = 500 \sin 22.78^\circ$$

$$\Rightarrow URN + P = 193.59$$

$$\Rightarrow \boxed{0.25 RN + P = 193.59 \text{ N}} \quad \text{--- (1)}$$

Case-II (VF)

$$RN = W \cos \alpha$$

$$\Rightarrow RN = 500 \cos 22.78^\circ$$

$$\Rightarrow \boxed{RN = 460.99 \text{ N}}$$

Case-II Now RN value putting eqⁿ (1)

$$0.25 RN + P = 193.59$$

$$\Rightarrow 0.25 \times 460.99 + P = 193.59$$

$$\Rightarrow 115.24 + P = 193.59$$

$$\Rightarrow P = 193.59 - 115.24$$

$$\Rightarrow \boxed{P = 78.35 \text{ N}}$$

A body of weight 450N is pulled up an inclined Plane by a force of 300N the inclination plane is 30 to the horizontal and force applied parallel to the plane.

Given data

$$W = 450\text{N}$$

$$P = 300\text{N}$$

$$\alpha = 30^\circ$$

$$\mu = ?$$

Case-I (HF)

$$P + F = W \sin \alpha$$

$$\Rightarrow 300 + F = 450 \sin 30$$

$$\Rightarrow 300 + F = 225$$

$$\Rightarrow F = 225 - 300$$

$$\Rightarrow F = -75\text{N}$$

Case-II (VF)

$$R_N = W \cos \alpha$$

$$\Rightarrow R_N = 450 \cos 30$$

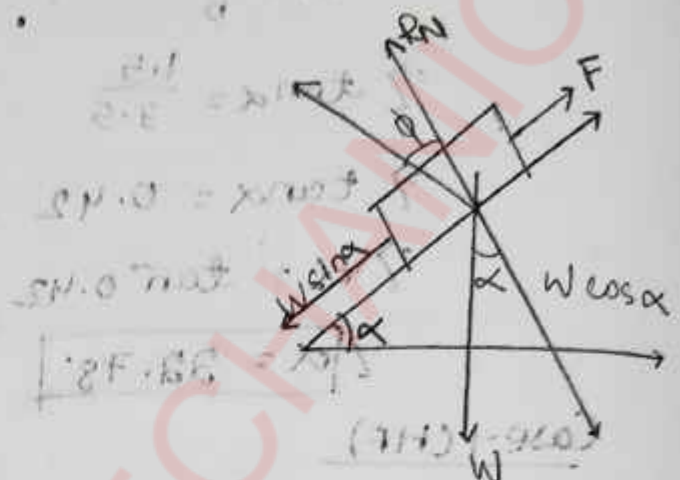
$$\Rightarrow R_N = 389.71\text{N}$$

Now coefficient of friction

$$\mu = \frac{F}{R_N}$$

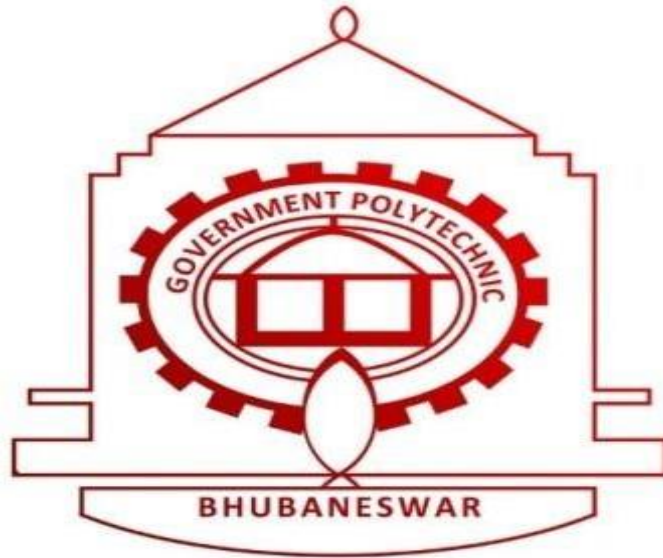
$$\Rightarrow \mu = \frac{75}{389.71}$$

$$\Rightarrow \mu = 0.19$$



GOVERNMENT POLYTECHNIC, BHUBANESWAR

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ENGINEERING MECHANICS NOTE

CHAPTER- 4
[CENTROID AND CENTRE OF
GRAVITY]

PREPARED BY

ER. RAJKUMAR PRADHAN [RAJ SIR]

Difference between Centroid and Center of Gravity.

Centroid Gravity	Center of Gravity.
→ Center of area of the plane figure like rectangular circle etc. is known as centroid of gravity. → Center point of any plane figure is known as centroid of gravity. → It deals with 2D structure. → It's the center point of any object with uniform density.	→ The point through which resultant force of gravity to the body is called center of gravity. → The point through which gravitational force applied on a body is known as center of gravity. → It deals with 3D structure. → It's the center point of any object with any density.

Position of center of gravity & Centroid.

A = total area of the body

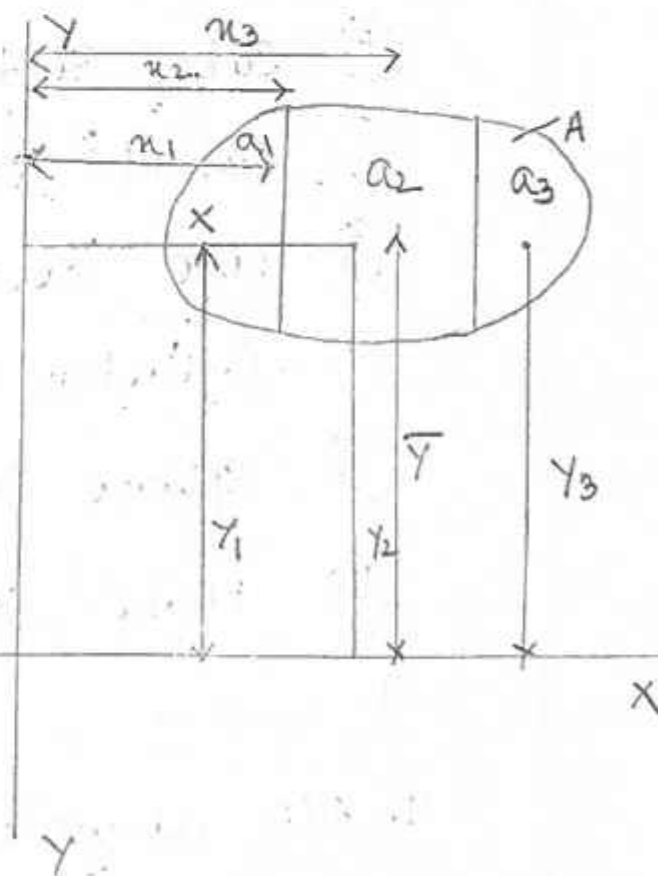
$$A = a_1 + a_2 + a_3$$

$\bar{Y}$ = Distance between $(Y-Y)$ and CG Point

$$\bar{Y} = Y_1 + Y_2 + Y_3$$

$\bar{X}$ = Distance between $(X-X)$ & CG Point

$$\bar{X} = X_1 + X_2 + X_3$$



Case-I C.G. about (X-X)

$$A\bar{Y} = a_1y_1 + a_2y_2 + a_3y_3$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

Case-II C.G. about (Y-Y)

$$A\bar{X} = a_1x_1 + a_2x_2 + a_3x_3$$

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

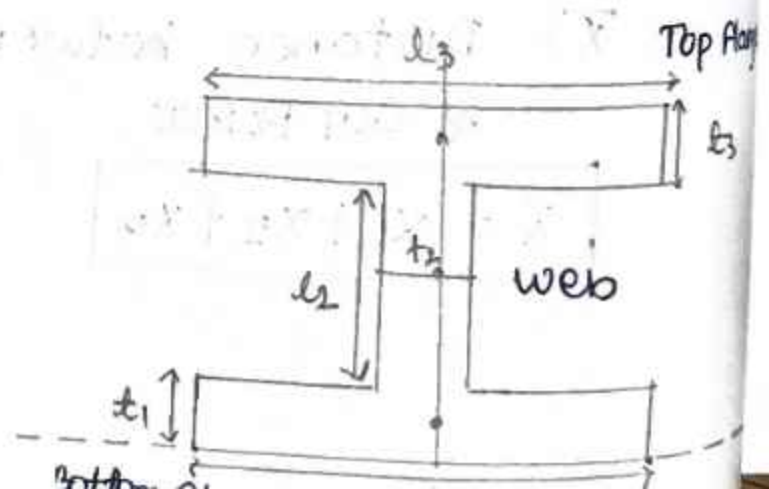
Center of gravity or Centroid Section

- ① 'I' Section
- ② 'T' Section
- ③ 'L' Section
- ④ Channel Section
- ⑤ Cube Section
- ⑥ Cuboid Section
- ⑦ cone Section
- ⑧ Cylinder Section
- ⑨ Sphere Section
- ⑩ Hemi Sphere.

T-Section

In this diagram

l_1 = length of the
bottom Flange



t_1 : thickness of the bottom Flange

l_2 : length of web

t_2 : thickness of web

l_3 : length of Top Flange

t_3 : thickness of Top Flange

① bottom Flange

$$\text{Area} = a_1 = l_1 \times t_1 \text{ m}^2$$

→ C₁ about (x-x)

$$y_1 = \frac{t_1}{2} \text{ m}$$

② web

$$\text{Area} = a_2 = l_2 \times t_2 \text{ m}^2$$

C₁ about (x-x)

$$y_2 = t_1 + \frac{l_2}{2} \text{ m}$$

③ Top Flange

$$\text{Area} = a_3 = l_3 \times t_3 \text{ m}^2$$

C₁ about (x-x)

$$y_3 = t_1 + l_2 + \frac{t_3}{2} \text{ m}$$

Now C₁ of D-section about (x-x)

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

Case-II, (C₁ about Y-Y).

$$\bar{X} = 0$$

(∵ Due to Y-Y axis passing through C₁ points)

Now C.G. of T-section

$$\bar{X}, \bar{Y} = \left(0, \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} \right)$$

T-section

Case-I [C.G. about (x-x)]

web

$$\text{Area} = [a_1 = l_1 \times t_1] \text{ m}^2$$

→ C.G. about (x-x)

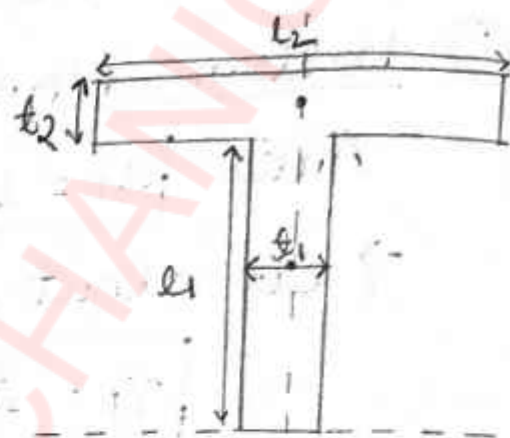
$$y_1 = \frac{l_1}{2} \text{ m}$$

Top Flange

$$\text{Area} = [a_2 = l_2 \times t_2] \text{ m}^2$$

C.G. about (x-x)

$$y_2 = l_1 + \frac{t_2}{2} \text{ m}$$



→ Now C.G. in T section about (x-x)

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} \text{ m}$$

Case-II [C.G. about (x-x)]

$$\bar{X} = 0$$

[Due to Y-Y Passing through the C.G. Point]

→ Now C.G. of T-section

$$(\bar{X}, \bar{Y}) = \left(0, \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} \right)$$

L-section [Method-1]

Case-I [Cr about (x-x)]

Rectangle-1

$$\rightarrow \text{Area} = [a_1 = l_1 \times t_1] \text{ m}^2$$

$\rightarrow$ Cr about (x-x)

$$y_1 = \frac{t_1}{2} \text{ m}$$

Rectangle-2

$$\rightarrow \text{Area} = [a_2 = l_2 \times t_2] \text{ m}^2$$

$\rightarrow$ Cr about (x-x)

$$y_2 = -\frac{t_2}{2} \text{ m}$$

$\rightarrow$ Now Cr in L-section about (x-x)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

Case-II [Cr about (y-y)]

R-1

$\rightarrow$ Cr about (y-y)

$$x_1 = \frac{l_1}{2} + \frac{t_2}{2}$$

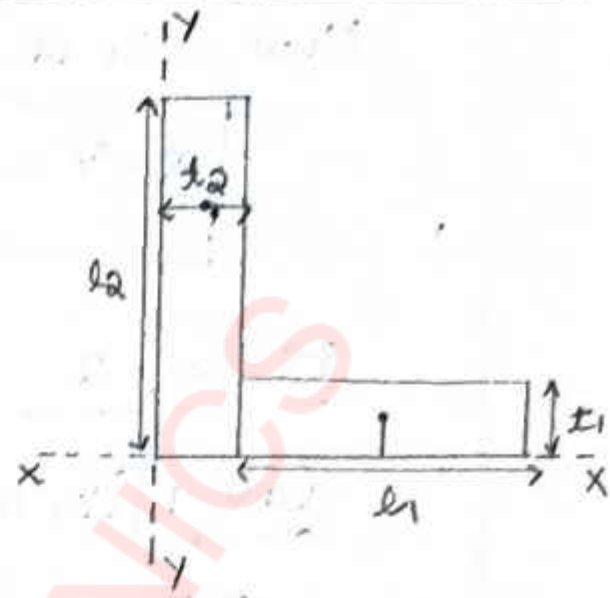
R-2

$\rightarrow$ Cr about (y-y)

$$x_2 = \frac{l_2}{2}$$

Now Cr in L section about (y-y)

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$



Now CG of L-section

$$\bar{x}, \bar{y} = \left(\frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}, \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} \right)$$

Method-2

Case-1 [CG about (x-x)]

R-1

Area = $a_1 = l_1 \times t_1$ m²

→ CG about (x-x)

$$y_1 = \frac{t_1}{2} \text{ m}$$

R-2

Area = $a_2 = l_2 \times t_2$ m²

→ CG about (x-x)

$$y_2 = t_1 + \frac{l_2}{2} \text{ m}$$

Now CG in L section about (x-x)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

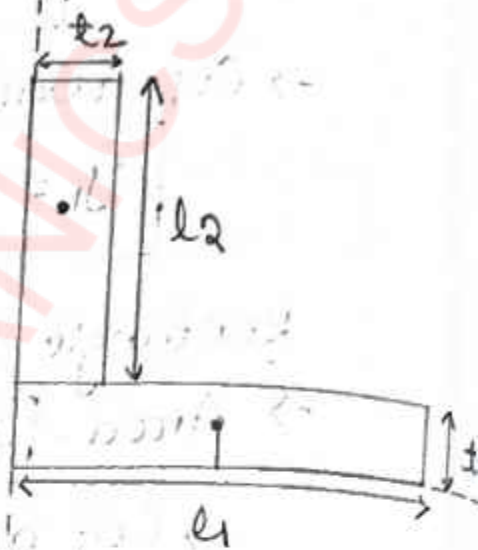
Case-II [CG about (y-y)]

R-1

Area = $a_1 = l_1 \times t_1$ m²

CG about (y-y)

$$x_1 = \frac{l_1}{2} \text{ m}$$



R-2

CG about (Y-Y)

$$\rightarrow y \left[u_2 = \frac{t_2}{2} \right] \text{ m}$$

Now CG in L section about (Y-Y)

$$\left[\bar{X} = \frac{a_1 u_1 + a_2 u_2}{a_1 + a_2} \right]$$

Now CG of L-section

$$\left[\bar{X}, \bar{Y} = \left(\frac{a_1 u_1 + a_2 u_2}{a_1 + a_2}, \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} \right) \right] \text{ m}$$

Channel section [Method-1]

Case-I [CG about (X-X)]

$$\left[\bar{Y} = 0 \right] \text{ [Due to (X-X) Passing through CG Point]}$$

Case-II [CG about (Y-Y)]

R-1

$$\text{Area} = \left[a_1 = l_1 \times t_1 \right] \text{ m}^2$$

$\rightarrow$ CG about (Y-Y)

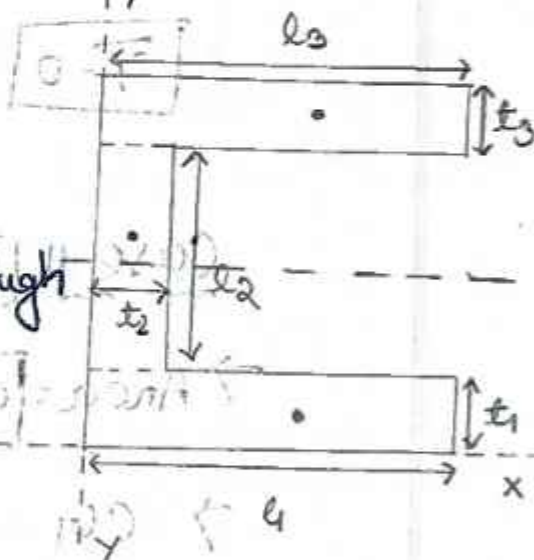
$$\left[u_1 = \frac{l_1}{2} \right] \text{ m}$$

R-2

$$\text{Area} = \left[a_2 = l_2 \times t_2 \right] \text{ m}^2$$

$\rightarrow$ CG about (Y-Y)

$$\left[u_2 = \frac{t_2}{2} \right] \text{ m}$$



R-3

$$\text{Area} = [a_3 = l_3 \times t_3] \text{ m}^2$$

→ CG about (Y-Y)

$$x_3 = \frac{l_3}{2} \text{ m}$$

→ Now CG Channel about (Y-Y)

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

Method-2

Case-I [CG about (X-X)]

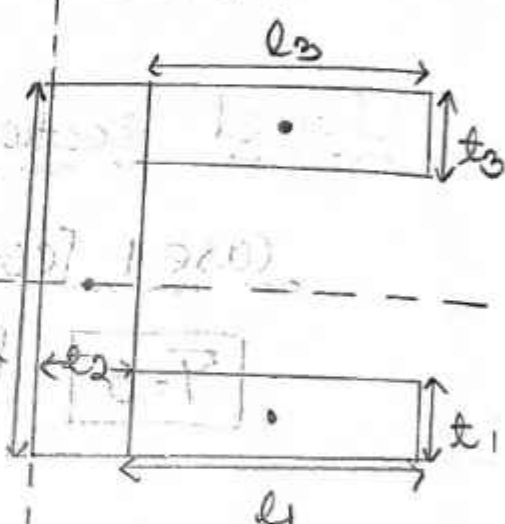
$$[\bar{y} = 0] \text{ [Due to (X-X) Passing through CG Points]}$$

Case-II [CG about (Y-Y)]

$$\rightarrow \text{Area} = [a_1 = l_1 \times t_1] \text{ m}^2$$

→ CG about (Y-Y)

$$x_1 = t_2 + \frac{l_1}{2}$$



R-2

$$\text{Area} = [a_2 = l_2 \times t_2] \text{ m}^2$$

→ CG about (Y-Y)

$$\bar{x}_2 = \frac{t_2}{2} \text{ m}$$

R-3

$$\text{Area} = a_3 = l_3 \times t_3$$

→ CG about (Y-Y)

$$u_3 = t_2 + \frac{l_3}{2}$$

Now CG in channel section about (Y-Y)

$$\bar{x} = \frac{a_1 u_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

Now CG of channel section.

$$\bar{x}, \bar{y} = \frac{a_1 u_1 + a_2 u_2 + a_3 u_3}{a_1 + a_2 + a_3}, 0$$

Square/cube



Volume

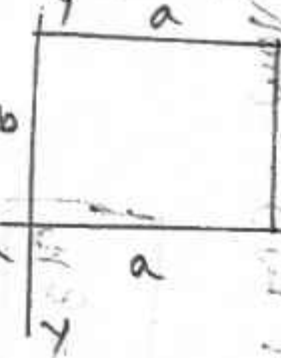
$$V = a^3$$

CG

$$\bar{y} = \frac{a}{2}$$

$$\bar{x} = \frac{b}{2}$$

Rectangle/Cuboid



Volume

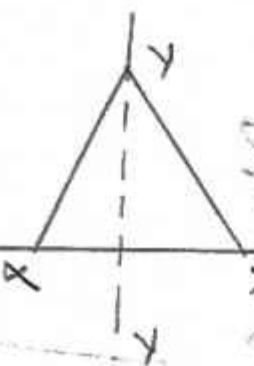
$$V = L \times B \times H$$

CG

$$\bar{y} = \frac{b}{2}$$

$$\bar{x} = \frac{a}{2}$$

Triangle

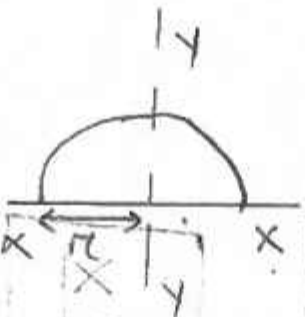


CG

$$\bar{y} = \frac{h}{3}$$

$$\bar{x} = 0$$

Semicircle



CG

$$\bar{x} = 0$$

$$\bar{y} = \frac{4r}{3\pi}$$

Circular Sector/Lamina



CG

$$\bar{x} = 0$$

$$\bar{y} = \frac{2r}{3} \frac{\sin \alpha}{\alpha}$$

Hemisphere



CG

$$\bar{y} = \frac{5r}{8}$$

$$\bar{x} = 0$$

Volume

$$V = \frac{2\pi}{3} r^3$$

Cone



Volume

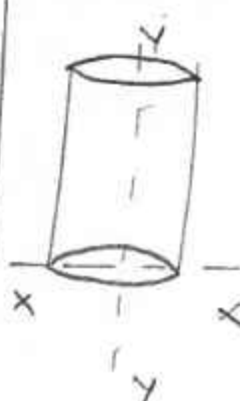
$$V = \frac{\pi}{3} r^2 h$$

CG

$$\bar{x} = 0$$

$$\bar{y} = \frac{h}{4}$$

Cylinder



Volume

$$V = \pi r^2 h$$

CG

$$\bar{x} = 0$$

$$\bar{y} = \frac{h}{2}$$



Ch-4 Find the CG of I-section as shown in the figure.

Given data

$$l_1 = 150 \text{ mm}$$

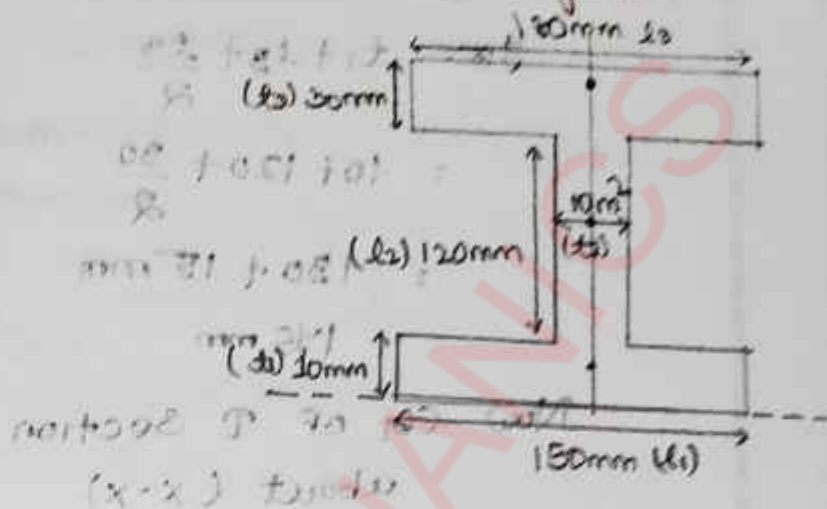
$$t_1 = 10 \text{ mm}$$

$$l_2 = 120 \text{ mm}$$

$$t_2 = 10 \text{ mm}$$

$$l_3 = 180 \text{ mm}$$

$$t_3 = 30 \text{ mm}$$



Case-1 (Bottom Flange)

$$\text{Area} = a_1 = l_1 \times t_1$$

$$= 150 \times 10 \text{ mm}^2$$

$$= 1500 \text{ mm}^2$$

CG about (x-x)

$$y_1 = \frac{t_1}{2}$$

$$= \frac{10 \text{ mm}}{2}$$

$$= 5 \text{ mm}$$

Case-1 (web)

$$\text{Area} = a_2 = l_2 \times t_2$$

$$= 120 \text{ mm} \times 10 \text{ mm}$$

$$= 1200 \text{ mm}^2$$

→ CG about (x-x)

$$y_2 = t_1 + \frac{l_2}{2}$$

$$= 10 + \frac{120 \text{ mm}}{2}$$

$$= 70 \text{ mm}$$

Top Flange

$$\text{Area} = a_3 = l_3 \times t_3$$

$$= 180 \times 30 \text{ mm}$$

$$= 5400 \text{ mm}^2$$

→ C_G about (x-x)

$$y_3 = t_1 + t_2 + \frac{t_3}{2}$$

$$= 10 + 120 + \frac{30}{2}$$

$$= 130 + 15 \text{ mm}$$

$$= 145 \text{ mm}$$

Now C_G of T Section
about (x-x)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{1500 \times 5 + 1200 \times 70 + 5400 \times 145}{1500 + 1200 + 5400}$$

$$= 107.616 \text{ mm}$$

$$\boxed{\bar{y} = 107.616 \text{ mm}}$$

Case-II C_G about (y-y)

$$\boxed{\bar{x} = 0}$$

(∵ Due to y-y axis passing through C_G Point)

Q2 Find the CG section of 'I' section of

• Bottom flange = $300\text{m} \times 20\text{m}$

• web = $100\text{m} \times 15\text{m}$

• Top flange = $350\text{m} \times 25\text{m}$

Given data,

$$l_1 = 300\text{m}$$

$$t_1 = 20\text{m}$$

$$l_2 = 100\text{m}$$

$$t_2 = 15\text{m}$$

$$l_3 = 350\text{m}$$

$$t_3 = 25\text{m}$$

Case-1 [CG about (x-x)]

① Bottom flange:-

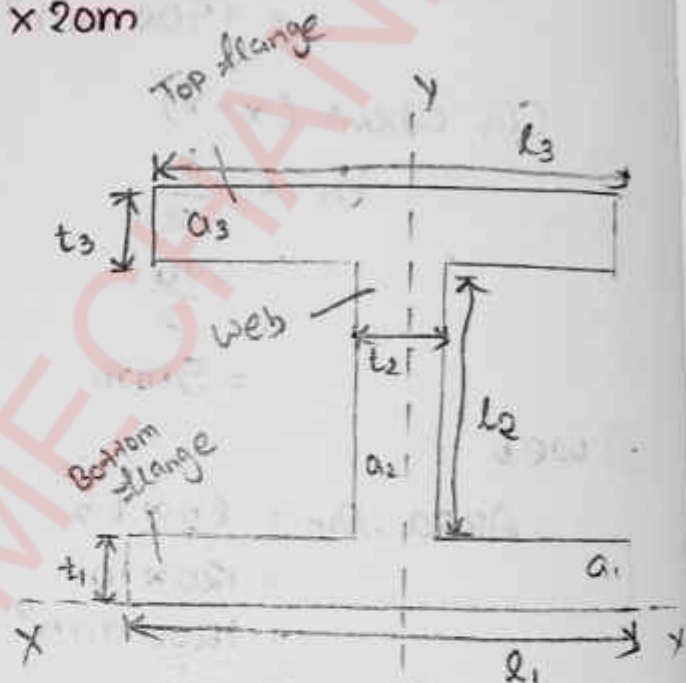
$$\begin{aligned}\text{Area } a_1 &= l_1 \times t_1 \\ &= 300 \times 20 \\ &= 6000\text{m}^2\end{aligned}$$

CG about (x-x)

$$\begin{aligned}y_1 &= \frac{t_1}{2} \\ &= \frac{20}{2} \\ &= 10\text{m}\end{aligned}$$

② web

$$\text{Area } a_2 = l_2 \times t_2$$



$$= 100 \times 15$$

$$= 1500 \text{ m}^2$$

(G₁ about (x-x))

$$y_2 = x_1 + \frac{l_2}{2}$$

$$= 20 + \frac{100}{2}$$

$$= 20 + 50$$

$$= 70 \text{ m}$$

③ Top flange

$$\text{Area } a_3 = l_3 \times t_3$$

$$= 350 \times 25$$

$$= 8750 \text{ m}^2$$

(G₁ about (x-x))

$$y_3 = t_1 + l_2 + \frac{l_3}{2}$$

$$= 20 + 100 + \frac{25}{2}$$

$$= 20 + 100 + 12.5$$

$$= 132.5 \text{ m}$$

Case-II [(G₁ about (Y-Y))]

$$\boxed{\bar{x} = 0}$$

Now CG section of (J) section about (x-x)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{6000 \times 10 + 1500 \times 70 + 8750 \times 132.5}{6000 + 1500 + 8750}$$

$$= 81.5$$

Find the C_G in T-section as shown in the Figure.

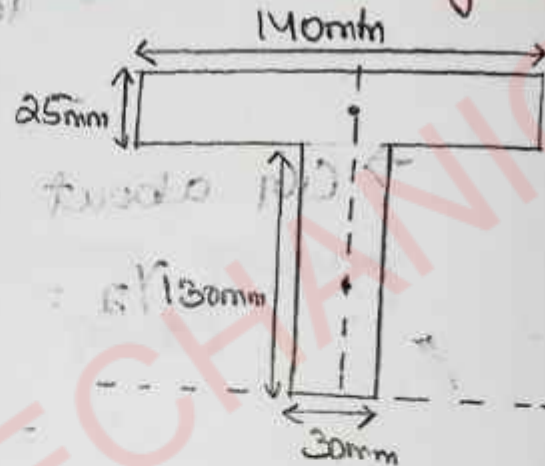
Given data

$$l_1 = 130 \text{ mm}$$

$$t_1 = 30 \text{ mm}$$

$$l_2 = 140 \text{ mm}$$

$$t_2 = 25 \text{ mm}$$



Case-1 [C_G about (x-x)]

$$\text{Area} = a_1 = l_1 \times t_1$$

$$= 130 \times 30 \text{ mm}$$

$$= 3900 \text{ mm}$$

→ C_G about (x-x)

$$y_1 = \frac{l_1}{2}$$

$$= \frac{130 \text{ mm}}{2}$$

$$= 65 \text{ mm}$$

Top Flange

$$\begin{aligned}\text{Area} = a_2 &= l_2 \times t_2 \\ &= 140 \times 25 \text{ mm}^2 \\ &= 3500 \text{ mm}^2\end{aligned}$$

→ CG about (x-x)

$$\begin{aligned}y_2 &= l_1 + \frac{t_2}{2} \\ &= 130 + \frac{25}{2} \\ &= 130 + 12.5 \\ &= 142.5 \text{ mm}\end{aligned}$$

CG in T-section about (x-x)

$$\begin{aligned}\bar{Y} &= \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} \\ &= \frac{3900 \times 65 + 3500 \times 142.5}{3900 + 3500}\end{aligned}$$

$$\boxed{\bar{Y} = 101.65 \text{ mm}}$$

Case-II [CG about (y-y)]

$$\boxed{\bar{X} = 0}$$

(Due to y-y Passing through the CG Point)

→ Now CG of T-section

$$\bar{X}, \bar{Y} = (0, 101.65)$$

Find the CG of T-section IF

Web = $180\text{ mm} \times 20\text{ mm}$

Top Flange = $130\text{ mm} \times 30\text{ mm}$

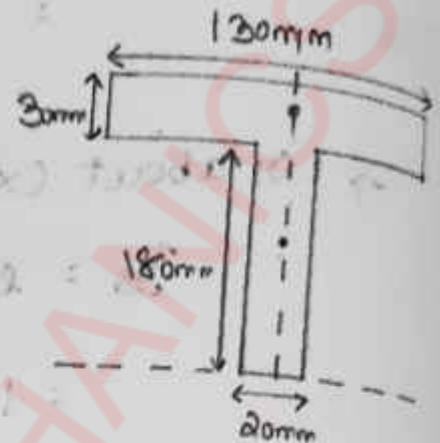
Given data

$$l_1 = 180\text{ mm}$$

$$t_1 = 20\text{ mm}$$

$$l_2 = 130\text{ mm}$$

$$t_2 = 30\text{ mm}$$



Case-1 [CG about (X-X)]

$$\text{Area: } a_1 = l_1 \times t_1$$

$$= 180 \times 20\text{ mm}^2$$

$$= 3600\text{ mm}^2$$

→ CG about (X-X)

$$y_1 = \frac{l_1}{2}$$

$$= \frac{180}{2}$$

$$= 90\text{ mm}$$

Top Flange

$$\text{Area: } a_2 = l_2 \times t_2$$

$$= 130 \times 30\text{ mm}^2$$

$$= 3900\text{ mm}^2$$

CG about (X-X)

$$y_2 = \frac{l_1 + t_2}{2}$$

$$= 180 + \frac{30}{2}\text{ mm}$$

$$= 180 + 15\text{ mm}$$

$$= 195\text{ mm}$$

Now CG in T-section about (x-x)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{3600 \times 40 + 3900 \times 195}{3600 + 3900}$$

$$\bar{y} = 144.6 \text{ mm}$$

Case-II

$$\bar{x} = 0$$

($\because$ Due to Y-Y Passing through CG Point)

$\rightarrow$ Now CG of T-section

$$(\bar{x}, \bar{y}) = (0, 144.6)$$

Find the CG in L-Section as shown in the figure.

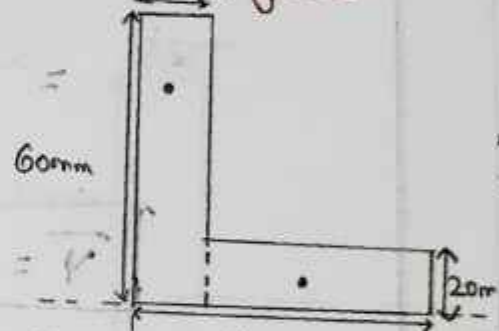
Given data

$$l_1 = 130 \text{ mm}$$

$$t_1 = 20 \text{ mm}$$

$$l_2 = 60 \text{ mm}$$

$$t_2 = 20 \text{ mm}$$



Case-I [CG about (x-x)]

R-1

$$\rightarrow \text{Area: } a_1 = l_1 \times t_1$$

$$= 130 \times 20 \text{ mm}^2$$

$$= 2600 \text{ mm}^2$$

CG about (x-x)

$$y_1 = \frac{t_1}{2}$$

$$= \frac{20}{2} \text{ mm}$$

$$= 10 \text{ mm}$$

R-2

$$\text{Area} = a_2 = l_2 \times t_2$$

$$= 60 \times 20 \text{ mm}^2$$

$$= 1200 \text{ mm}^2$$

CG about (x-x)

$$\rightarrow y_2 = \frac{l_2}{2}$$

$$= 30 \text{ mm}$$

Now CG in L-section about (x-x)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{2200 \times 10 + 1200 \times 30}{2200 + 1200}$$

$$\bar{y} = 17.05 \text{ mm}$$

Case-II [CG about (y-y)]

R-1

CG about (y-y)

$$x_1 = t_2 + \frac{l_1}{2}$$

$$= 20 + \frac{130}{2}$$

$$= 20 + 65$$

$$= 85 \text{ mm}$$

R-2

CG about (y-y)

$$x_2 = \frac{t_2}{2}$$

$$x_2 = \frac{20}{2}$$

$$= 10 \text{ mm}$$

Now C_G in L-Section about (Y-Y)

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{2200 \times 75 + 1200 \times 10}{2200 + 1200}$$

$$\boxed{\bar{x} = 52.05 \text{ mm}}$$

Now C_G of L-section

$$\boxed{\bar{x}, \bar{y} = (52.05, 17.05)}$$

Find C_G L-Section as shown in the Figure.

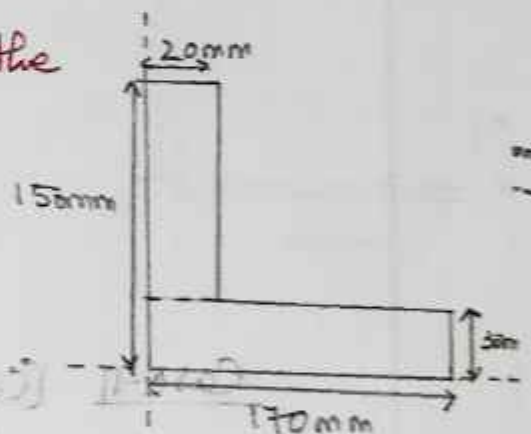
Given data

$$l_1 = 170 \text{ mm}$$

$$t_1 = 30 \text{ mm}$$

$$l_2 = 150 \text{ mm} \quad (150 - 30 = 120 \text{ mm})$$

$$t_2 = 20 \text{ mm}$$



Case-1 C_G about (x-x)

$$\rightarrow \text{Area} = a_1 = l_1 \times t_1$$

$$= 170 \times 30 \text{ mm}^2$$

$$= 5100 \text{ mm}^2$$

C_G about (Y-Y)

$$\rightarrow y_1 = \frac{t_1}{2}$$

$$= \frac{30}{2} \text{ mm}$$

$$= 15 \text{ mm}$$

R-2

$$Area = a_2 = l_2 \times t_2$$

$$(1-1) = 150 \times 20 \text{ mm}^2 \\ = 3000 \text{ mm}^2$$

CG about (x-x)

$$\bar{x}_2 = \frac{t_1 + t_2}{2}$$

$$= \frac{30 + 150}{2} \\ = 30 + 60 \\ = 90 \text{ mm}$$

Now CG L-section about (Y-Y) axis

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{5100 \times 15 + 3000 \times 90}{5100 + 3000}$$

$$\bar{y} = 39 \text{ mm}$$

Case-II [CG about (Y-Y)]

R-1

CG about (Y-Y)

$$u_1 = \frac{t_1}{2}$$

$$= \frac{170}{2} \text{ mm}$$

$$= 85 \text{ mm}$$

R-2

CG about (Y-Y)

$$u_2 = \frac{t_2}{2} = \frac{20}{2}$$

$$= 10 \text{ mm}$$

CG in L-section about (Y-Y)

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{5100 \times 85 + 2400 \times 10}{5100 + 3000}$$

$$\boxed{\bar{x} = 61 \text{ mm}}$$

Now CG of L-section

$$\boxed{\bar{x}, \bar{y} = (61 \text{ mm}, 39 \text{ mm})}$$

Find the CG in L-section in following figure

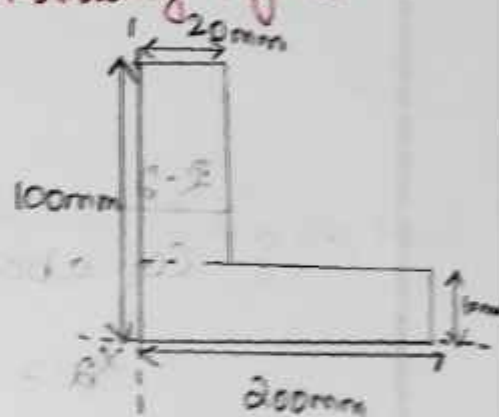
Given data

$$l_1 = 200 \text{ mm}$$

$$t_1 = 10 \text{ mm}$$

$$l_2 = 100 \text{ mm}$$

$$t_2 = 20 \text{ mm}$$



Case-1 CG about (x-x)

$$\begin{aligned} \text{Area} = a_1 &= l_1 \times t_1 \\ &= 200 \times 10 \text{ mm}^2 \\ &= 2000 \text{ mm}^2 \end{aligned}$$

CG about (x-x)

$$\begin{aligned} y_1 &= \frac{t_1}{2} \\ &= \frac{10 \text{ mm}}{2} \\ &= 5 \text{ mm} \end{aligned}$$

R-2

CG about X-X

$$\text{Area} = a_2 = l_2 \times t_2 = 100 \times 20 = 2000 \text{ mm}^2$$

CG about (x-x)

$$\bar{x}_a = \frac{x_1 + x_2}{2}$$

$$= \frac{10 + 90}{2}$$

$$= 10 + 45$$

$$= 55 \text{ mm}$$

Now CG in L section, about (y-y)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{2000 \times 5 + 1800 \times 55}{2000 + 1800}$$

$$= 28.68 \text{ m}$$

$$= 28.68 \text{ m}$$

R-2

CG about (y-y)

$$x_2 = \frac{x_2}{2}$$

$$= \frac{20}{2}$$

$$= 10 \text{ mm}$$

R-1

CG about (y-y)

$$x_1 = \frac{x_1}{2}$$

$$= \frac{200}{2}$$

$$= 100 \text{ mm}$$

Now CG about (y-y) in L-section

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{2000 \times 100 + 1800 \times 10}{2000 + 1800} = 53.10$$

Now CG of L section

$$(\bar{x}, \bar{y} = 53.10, 28.68 \text{ mm})$$

Find the CG in Channel section as shown in the figure.

Given data

$$l_1 = 300 \text{ mm}$$

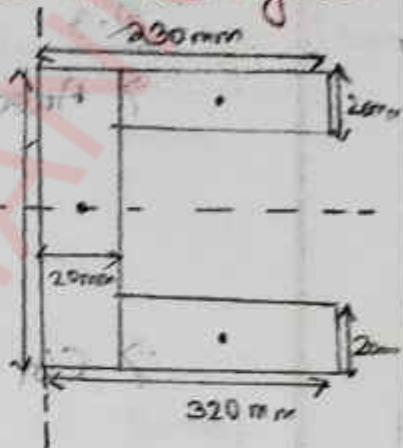
$$t_1 = 20 \text{ mm}$$

$$l_2 = 160 \text{ mm}$$

$$t_2 = 20 \text{ mm}$$

$$l_3 = 20 \text{ mm}$$

$$t_3 = 20 \text{ mm}$$



Case-I [CG about (X-X)]

$$\boxed{\bar{y} = 0}$$

[Due to (X-X) Passing through CG Point]

Case-II [CG about (Y-Y)]

R-1

$$\rightarrow \text{Area} = a_1 = l_1 \times t_1$$
$$= 300 \times 20 \text{ mm}$$

$$= 6000 \text{ mm}^2$$

CG about (Y-Y)

$$\rightarrow y_1 = t_2 + \frac{l_1}{2}$$

$$= 20 + \frac{300}{2}$$

$$= 170 \text{ mm}$$

R-2

$$\rightarrow \text{Area} = a_2 = l_2 \times t_2$$

$$= 160 \times 20 \text{ mm}$$

$$= 3200 \text{ mm}^2$$

CG about (Y-Y)

$$\rightarrow n_2 = \frac{t_2}{2}$$
$$= \frac{20}{2}$$

$$= 10 \text{ mm}$$

R-3

$$\rightarrow \text{Area} = a_3 = l_3 \times t_3$$
$$= 210 \times 20 \text{ mm}^2$$
$$= 4200 \text{ mm}^2$$

CG about (Y-Y)

$$n_3 = t_2 + \frac{l_3}{2}$$

$$= 20 + \frac{210}{2}$$

$$= 20 + 105$$

$$= 125 \text{ mm}$$

CG in Channel Section about (Y-Y)

$$\bar{X} = \frac{a_1 n_1 + a_2 n_2 + a_3 n_3}{a_1 + a_2 + a_3}$$

$$= \frac{6000 \times 170 + 3200 \times 10 + 4200 \times 125}{6000 + 3200 + 4200}$$

$$\bar{X} = 117.68 \text{ mm}$$

Now CG in Channel Section

$$\bar{X}, \bar{Y} = (117.68 \text{ mm}, 0)$$

Find the CG Channel Section as shown in the figure.

Given data

$$l_1 = 160 \text{ mm}$$

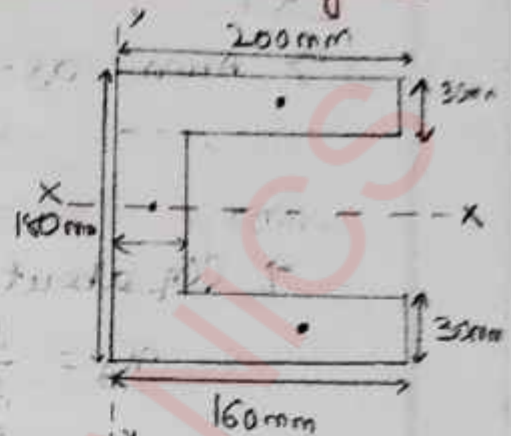
$$t_1 = 30 \text{ mm}$$

$$l_2 = 120 \text{ mm}$$

$$t_2 = 20 \text{ mm}$$

$$l_3 = 200 \text{ mm}$$

$$t_3 = 30 \text{ mm}$$



Case-I [CG about (X-X)]

$$\boxed{\bar{Y} = 0} \text{ [Due to X-X Passing through CG Point]}$$

Case-II [CG about (Y-Y)]

R-1

$$\begin{aligned} \rightarrow \text{Area} &= a_1 = l_1 \times t_1 \\ &= 160 \times 30 \text{ mm} \\ &= 4800 \text{ mm}^2 \end{aligned}$$

$\rightarrow$ CG about (Y-Y)

$$\begin{aligned} x_1 &= \frac{l_1}{2} \\ &= \frac{160 \text{ mm}}{2} = 80 \text{ mm} \end{aligned}$$

R-2

$$\begin{aligned} \rightarrow \text{Area} &= a_2 = l_2 \times t_2 \\ &= 120 \times 20 \text{ mm} \\ &= 2400 \text{ mm}^2 \end{aligned}$$

CG about (Y-Y)

$$\begin{aligned} x_2 &= \frac{t_2}{2} \\ &= \frac{20 \text{ mm}}{2} \\ &= 10 \text{ mm} \end{aligned}$$

R-3

$$\begin{aligned}\rightarrow \text{Area} &= a_3 = l_3 \times t_3 \\ &= 200 \times 20 \text{ mm} \\ &= 6000 \text{ mm}^2\end{aligned}$$

$\rightarrow$ CG about (Y-Y)

$$\begin{aligned}x_3 &= \frac{l_3}{2} \\ &= \frac{200}{2} \\ &= 100 \text{ mm}\end{aligned}$$

CG in channel section about (Y-Y)

$$\begin{aligned}\bar{X} &= \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} \\ &= \frac{4800 \times 80 + 2400 \times 10 + 6000 \times 100}{4800 + 2400 + 6000}\end{aligned}$$

$$\boxed{\bar{X} = 76.9369 \text{ mm}}$$

Now CG in channel section about (Y-Y)

$$\boxed{\bar{X}, \bar{Y} = (76.9369 \text{ mm}, 0)}$$

Find the CG in channel section as shown in the figure.

Given data

$$l_1 = 90 \text{ mm}$$

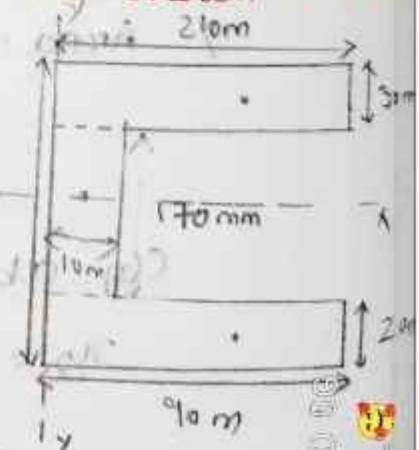
$$t_1 = 20 \text{ mm}$$

$$l_2 = 120 \text{ mm}$$

$$t_2 = 10 \text{ mm}$$

$$l_3 = 210 \text{ mm}$$

$$t_3 = 20 \text{ mm}$$



Case-1 [CG about (x-x)]

$\nabla = 0$ [∵ Due to (x-x) Passing through CG Points]

Case-11 [CG about (Y-Y)]

$$\begin{aligned}\rightarrow \text{Area} &= a_1 = l_1 \times t_1 \\ &= 90 \times 20 \text{ m}^2 \\ &= 1800 \text{ m}^2\end{aligned}$$

CG about (Y-Y)

$$\begin{aligned}u_1 &= \frac{l_1}{2} \\ &= \frac{90}{2} = 45 \text{ mm}\end{aligned}$$

R-2

$$\begin{aligned}\text{Area} &= a_2 = l_2 \times t_2 \\ &= 120 \times 10 \text{ m}^2 \\ &= 1200 \text{ m}^2\end{aligned}$$

CG about (Y-Y)

$$\begin{aligned}u_2 &= \frac{t_2}{2} \\ &= \frac{10}{2} = 5 \text{ m}\end{aligned}$$

R-3

$$\begin{aligned}\text{Area} &= a_3 = l_3 \times t_3 \\ &= 210 \times 30 \text{ m}^2 \\ &= 6300 \text{ m}^2\end{aligned}$$

CG about (Y-Y)

$$\begin{aligned}u_3 &= \frac{l_3}{2} \\ &= \frac{210}{2} \\ &= 105 \text{ m}\end{aligned}$$

CG in Channel section about (Y-Y)

$$\begin{aligned}\bar{x} &= \frac{a_1 u_1 + a_2 u_2 + a_3 u_3}{a_1 + a_2 + a_3} \\ &= \frac{1800 \times 45 + 1200 \times 5 + 6300 \times 105}{1800 + 1200 + 6300} = 80.48\end{aligned}$$

Now CG in channel section about X & Y

$$\bar{X}, \bar{Y} = (80.48, 0)$$

Calculate the center of gravity of cube of length = 20m.

Given data

$$a = 20m$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$

Case-I [CG about (x-x)]

$$\begin{aligned}\bar{Y} &= \frac{a}{2} \\ &= \frac{20}{2} = 10m\end{aligned}$$

Case-II [CG about (y-y)]

$$\begin{aligned}\bar{X} &= \frac{a}{2} \\ &= \frac{20}{2} = 10m\end{aligned}$$

Now CG of cube section

$$\bar{X}, \bar{Y} = (10m, 10m)$$

Calculate the center of gravity of cube length = 30m

Given data

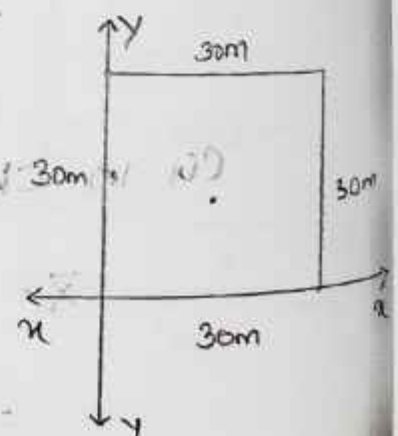
$$a = 30m$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$

Case-I [CG about (x-x)]

$$\bar{X} = \frac{a}{2} = \frac{30}{2} = 15m$$



Case-II [C.G. about (Y-Y)]

$$\bar{Y} = \frac{a}{2}$$

$$= \frac{30}{2} = 15\text{m}$$

Now C.G. in Channel section

$$\boxed{\bar{X}, \bar{Y} = (15\text{m}, 15\text{m})}$$

Calculate the center of gravity of cube of length = 30m with 8m and height 4m.

Given data

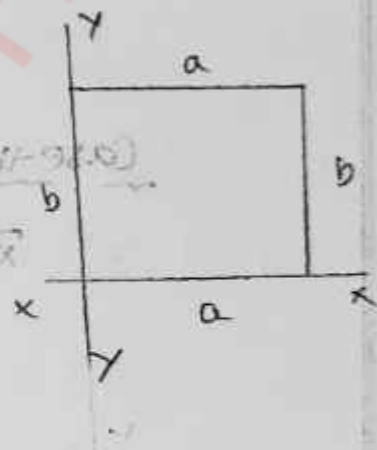
$$L = 15\text{m} = (a)$$

$$B = 8\text{m} = (b)$$

$$H = 4\text{m} = (h)$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$



Case-I [C.G. about (X-X)]

$$\bar{Y} = \frac{b}{2}$$

$$= \frac{8\text{m}}{2} = 4\text{m}$$

Case-II [C.G. about (Y-Y)]

$$\bar{X} = \frac{a}{2}$$

$$= \frac{15}{2} \text{m} = 7.5\text{m}$$

Now C.G. of cuboid

$$\boxed{\bar{X}, \bar{Y} = (7.5\text{m}, 4\text{m})}$$

Calculate the center of gravity Cuboid of length 20m with 14m & height 3m.

Given data

$$L = 20\text{m} = a$$

$$B = 14\text{m} = b$$

$$H = 3\text{m} = h$$



Case-I [CG about (x-x)]

$$\bar{y} = \frac{b}{2}$$

$$= \frac{14}{2}$$

$$= 7\text{m}$$

Case-II [CG about (y-y)]

$$\bar{x} = \frac{a}{2}$$

$$= \frac{20}{2}$$

$$= 10\text{m}$$

Now CG in cuboid

$$\boxed{\bar{x}, \bar{y} = (10\text{m}, 7\text{m})}$$

Find the center of gravity of triangle of height 6m.

Given data

$$H = 6\text{m}$$

$$\bar{x} = ?$$

$$\bar{y} = ?$$

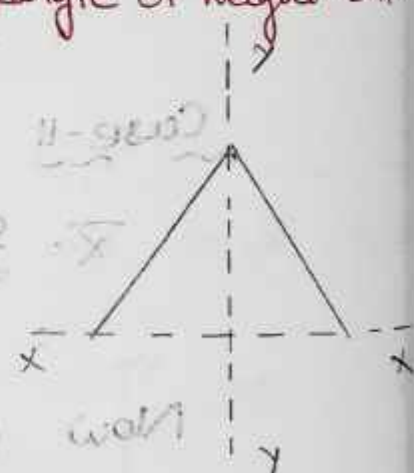
Case-I [CG about (x-x)]

Case-I CG about (y-y)

$$\boxed{\bar{x} = 0}$$

Case-II [CG about (x-x)]

$$\bar{y} = \frac{h}{3} = \frac{6}{3} = 2\text{m}$$



Now C.G. of Triangle

$$\boxed{\bar{X}, \bar{Y} = 0, 2m}$$

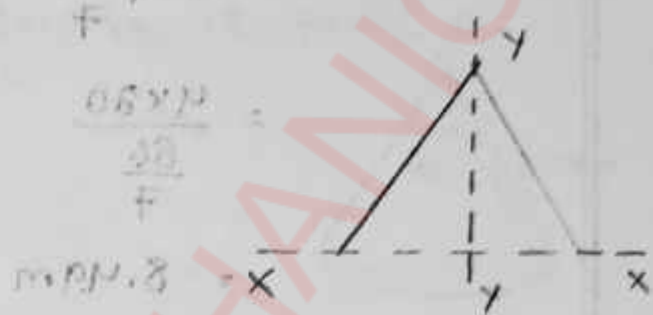
Find the center of gravity of triangle of height = 8m.

Given data

$$\text{height} = 8m$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$



Case-I [C.G. about (y-y)]

$$\boxed{\bar{X} = 0}$$

Case-II [C.G. about (x-x)]

$$\bar{Y} = \frac{h}{3}$$

$$= \frac{8}{3}$$

$$= 2.6$$

Now C.G. of triangle

$$\boxed{\bar{X}, \bar{Y} = 0, 2.6m}$$

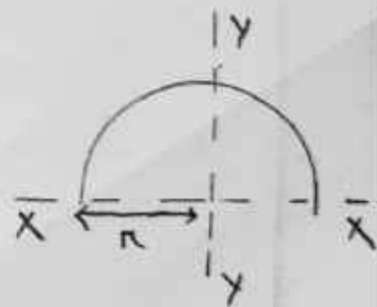
Find the C.G. of Semi circle if radius = 20m.

Given data

$$r = 20m$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$



Case-I [C.G. about (y-y)]

$$\boxed{\bar{X} = 0} \text{ (Due to Y-Y Passing through C.G. Point)}$$

Case-II [CG about (x-x)]

$$\bar{Y} = \frac{4r}{3\pi}$$

$$= \frac{4 \times 20}{3 \times \frac{22}{7}}$$

$$= \frac{4 \times 20}{66}$$

$$= 8.49 \text{ m}$$

Now CG in semicircle

$$\bar{X}, \bar{Y} = (0, 8.49)$$

Find the CG of semicircle if radius = 10 mm.

Given data

$$r = 10 \text{ mm}$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$

Case-I [CG about (y-y)]

$$\bar{X} = 0$$

[Due to (y-y) Passing through CG Point]

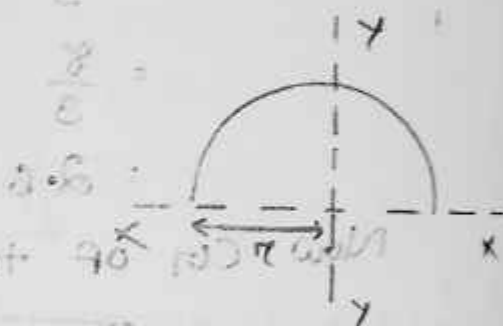
Case-II [CG about (x-x)]

$$\bar{Y} = \frac{4r}{3\pi}$$

$$= \frac{4 \times 10}{3 \times \frac{22}{7}}$$

$$= \frac{40}{66}$$

$$= \frac{40 \times 7}{66} = 4.24 \text{ m}$$



Now C.G. about semi circle

$$\bar{X}, \bar{Y} = 0, 4.24 \text{ m}$$

A plane lamina of 220mm radius as shown in the Figure. Find the C.G. of lamina at point O.

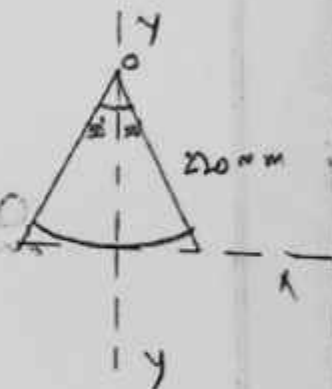
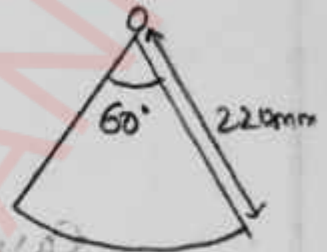
Given data

$$r = 220 \text{ mm}$$

$$\alpha = 30^\circ = \frac{\pi}{6} \text{ radian}$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$



Case-I [C.G. about (X-X)]

$$\bar{Y} = \frac{2r}{3} \left(\frac{\sin \alpha}{\alpha} \right)$$

$$= \frac{2 \times 220}{3} \times \frac{\sin 30}{\frac{\pi}{6}}$$

$$= \frac{2 \times 220 \times \sin 30}{3 \times \frac{\pi}{6}}$$

$$= \frac{220 \times 2 \times \sin 30}{3 \times 0.52}$$

$$\bar{Y} = 141.02 \text{ mm}$$

Case-II [C.G. about (Y-Y)]

$$\bar{X} = 0$$

[Due to Y-Y Passing through C.G. Point]

Now C.G. in Lamina

$$\bar{X}, \bar{Y} = 0, 141.02 \text{ mm}$$

A plane lamina of $r = 110\text{mm}$ as shown in the figure. And the C.G. of lamina at Point O.

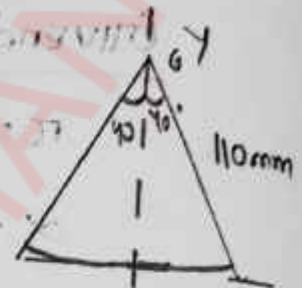
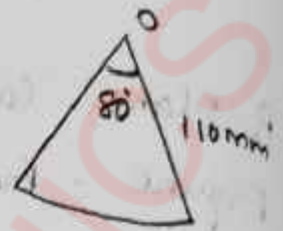
Given data

$$r = 110\text{mm}$$

$$\alpha = 40^\circ = \frac{\pi}{4.5}$$

$$\bar{x} = ?$$

$$\bar{y} = ?$$



Case-I [C.G. about (Y-Y)]
 $\boxed{\bar{x} = 0}$ [Due to (Y-Y) Passing through C.G. Point]

Case-II [C.G. about (X-X)]

$$\bar{y} = \frac{2\pi}{3} \times \frac{\sin \alpha}{\alpha}$$

$$= \frac{2 \times 110}{3} \times \frac{\sin 40^\circ}{\frac{\pi}{4.5}}$$

$$= \frac{220 \times \sin 40^\circ}{3 \times \frac{\pi}{4.5}}$$

$$= \frac{216.65}{3 \times \left(\frac{3.14}{4.5}\right)} \quad (\because \pi = 3.14)$$

$$= 67.55\text{mm}$$

Now C.G. in lamina at point O.

$$\boxed{\bar{x}, \bar{y} = 0, 67.55\text{mm}}$$

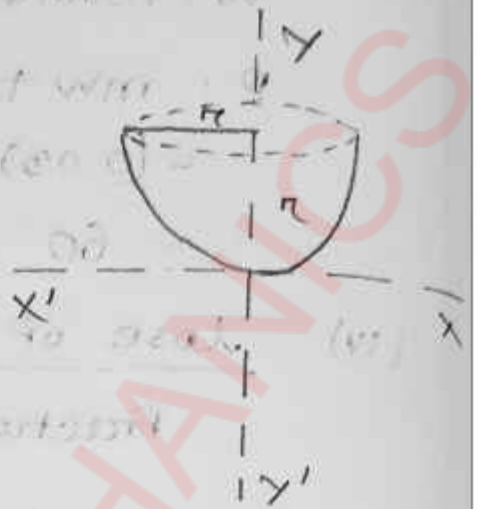
Calculate the center of Gravity of hemisphere
radius is 230mm.

Given data

$$r = 230\text{mm}$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$



Case-I [CG about (x-x)]:

$$\begin{aligned}\bar{Y} &= \frac{5r}{8} \\ &= \frac{5 \times 230}{8}\end{aligned}$$

$$\boxed{\bar{Y} = 143.75\text{mm}}$$

Case-II [CG about (y-y)]:

$$\boxed{\bar{X} = 0} \quad (\because \text{Due to (y-y) axis passing through CG Point})$$

Now CG of hemisphere

$$\boxed{\bar{X}, \bar{Y} = 0, 143.75\text{mm}}$$

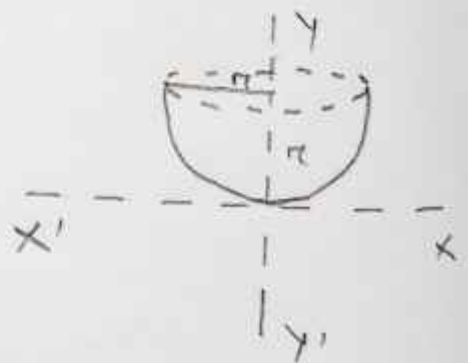
Calculate the Center of Gravity of hemisphere
radius is 15mm.

Given data

$$r = 15\text{mm}$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$



Case-I [CG about (x-x)]:

$$\bar{Y} = \frac{5r}{8}$$

$$= \frac{5 \times 15}{8}$$

$$\bar{y} = 9.375 \text{ mm}$$

Case-II [CG about (x-y)]

$\bar{x} = 0$ ($\because$ Due to (y-y) Passing through CG Point)

Now CG of hemisphere

$$\bar{x}, \bar{y} = 0, 9.375 \text{ mm}$$

Calculate the center of gravity of cone the radius of cone 20m & height 30m.

Given data

$$R = 20\text{m}$$

$$h = 30\text{m}$$

$$\bar{x} = ?$$

$$\bar{y} = ?$$

Case-I [CG about (x-x)]

$$\bar{y} = \frac{h}{4}$$

$$\begin{aligned} \bar{y} &= \frac{30}{4} \\ &= 7.5\text{m} \end{aligned}$$

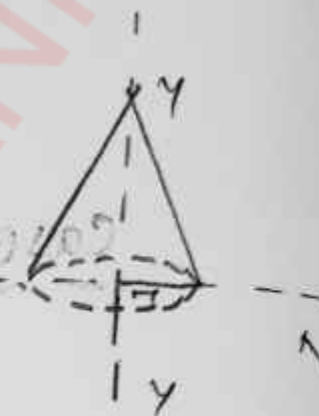
Case-II [CG about (y-y)]

$$\boxed{\bar{x} = 0}$$

[Due to y-y Passing through CG Point]

Now CG of Cone

$$\boxed{\bar{x}, \bar{y} = 0, 7.5\text{m}}$$



Calculate the center of gravity of cone if the radius of cone 10mm & height = 45mm.

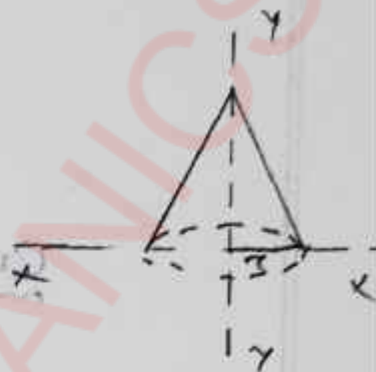
Given data

$$\bar{x} = ?$$

$$\bar{y} = ?$$

$$r = 10\text{mm}$$

$$h = 45\text{mm}$$



Case-I [CG about (x-x)]

$$\bar{y} = \frac{h}{4}$$

$$= \frac{45}{4}$$

$$= 11.25\text{mm}$$

Case-II [CG about (y-y)]

$$\bar{x} = 0$$

[Due to y-y, Passing through CG Point]

Now CG in cone

$$\bar{x}, \bar{y} = 0, 11.25\text{mm}$$

Find the center of gravity of cylinder where radius the cylinder = 25m & height = 60m.

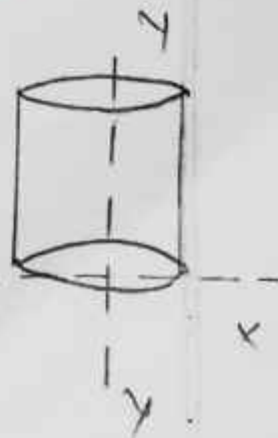
Given data

$$r = 25\text{m}$$

$$h = 60\text{m}$$

$$\bar{x} = ?$$

$$\bar{y} = ?$$



Case-I [CG about (x-x)]

$$\begin{aligned}\bar{y} &= \frac{h}{2} \\ &= \frac{60\text{m}}{2} \\ &= 30\text{m}\end{aligned}$$

Case-II [CG about (y-y)]

$\bar{x} = 0$ [Due to (y-y) Passing through CG Point]

Now CG in cylinder

$$\bar{x}, \bar{y} = 0, 30\text{m}$$

And the CG of cylinder where radius of cylinder = 26m & height = 55m.

Given data

$$r = 26\text{m}$$

$$h = 55\text{m}$$

$$\bar{x} = ?$$

$$\bar{y} = ?$$

Case-I [CG about (x-x)]

$$\bar{y} = \frac{h}{2}$$

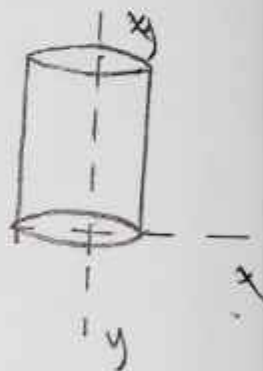
$$= \frac{55}{2}$$

$$= 27.5\text{m}$$

Case-II [CG about (y-y)]

$$\bar{x} = 0$$

[Due to (y-y) Passing through CG Point]



Now CG in Cylinder

$$\bar{X}, \bar{Y} = 0, 27.5 \text{ m} \quad (\text{ans})$$

Q. IF

Bottom Flange = $200 \text{ mm} \times 100 \text{ mm}$

Top Flange = $150 \text{ mm} \times 50 \text{ mm}$

Web = $300 \text{ mm} \times 50 \text{ mm}$

Find CG of 'I' section.

Given data

$$l_1 = 200 \text{ mm}$$

$$t_1 = 100 \text{ mm}$$

$$l_2 = 300 \text{ mm}$$

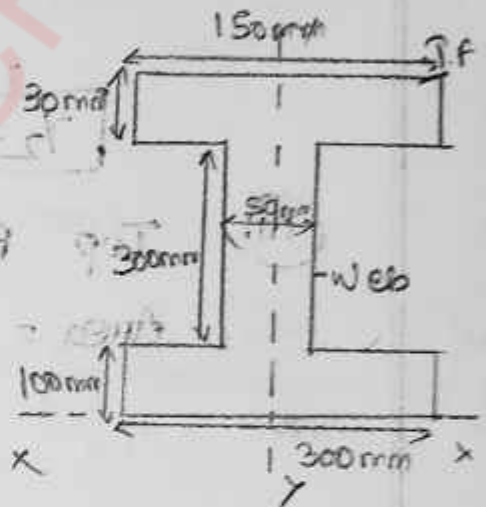
$$t_2 = 50 \text{ mm}$$

$$l_3 = 150 \text{ mm}$$

$$t_3 = 30 \text{ mm}$$

$$\bar{X} = ?$$

$$\bar{Y} = ?$$



Case-1 [about (x-x)]

→ Bottom Flange

$$\text{Area} = a_1 = l_1 \times t_1$$

$$= 200 \times 100$$

$$a_1 = 20000 \text{ mm}^2$$

CG about (x-x)

$$y_1 = \frac{t_1}{2}$$

$$= \frac{100}{2}$$

$$y_1 = 50 \text{ mm}$$

web

$$\text{Area} = a_2 = l_2 \times t_2$$
$$= 300 \times 50$$

$$a_2 = 15000 \text{ mm}^2$$

→ CG about (x-x)

$$y_2 = t_1 + \frac{l_2}{2}$$
$$= 100 + \frac{300}{2}$$
$$= 100 + 150$$

$$y_2 = 250 \text{ mm}$$

(iii) Top Flange

$$\text{Area} = a_3 = l_3 \times t_3$$
$$= 150 \times 50$$

$$a_3 = 7500 \text{ mm}^2$$

→ CG about (x-x)

$$y_3 = t_1 + l_2 + \frac{t_3}{2}$$
$$= 100 + 300 + \frac{50}{2}$$
$$= 400 + 25$$

$$y_3 = 425$$

Now, CG about (x-x)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$
$$= \frac{30000 \times 50 + 15000 \times 250 + 7500 \times 425}{30000 + 15000 + 7500}$$

$$\bar{y} = 160.7 \text{ mm}$$

Case-II [CG about (Y-Y)]

$$\boxed{\bar{X} = 0}$$

($\because$ Due to Y-Y axis passing through CG Points)

Now CG of I-Section

$$\boxed{\bar{X}, \bar{Y} = 0, 160.7 \text{ mm}}$$

Find CG of L Section (100mm x 80mm x 20mm)

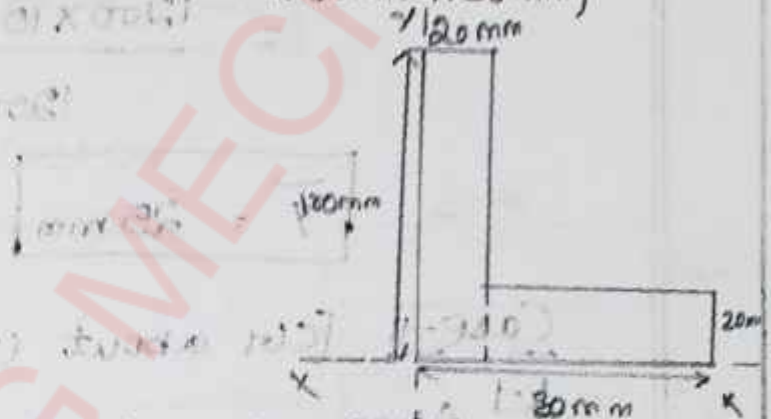
Given data

$$l_1 = 60 \text{ mm}$$

$$t_1 = 20 \text{ mm}$$

$$l_2 = 100 \text{ mm}$$

$$t_2 = 20 \text{ mm}$$



Case-1 [CG about (x-x)]

R-1

$$\begin{aligned} \text{Area} = a_1 &= l_1 \times t_1 \\ &= 60 \times 20 \text{ mm}^2 \end{aligned}$$

$$\boxed{a_1 = 1200 \text{ mm}^2}$$

CG about (x-x)

$$\begin{aligned} \rightarrow y_1 &= \frac{t_1}{2} \\ &= \frac{20}{2} = 10 \text{ mm} \end{aligned}$$

R-2

$$\begin{aligned} \text{Area} = a_2 &= l_2 \times t_2 \\ &= 100 \times 20 \text{ mm}^2 \end{aligned}$$

$$\boxed{a_2 = 2000 \text{ mm}^2}$$

(C_g about (x-x))

$$\rightarrow y_2 = \frac{t_2}{2}$$
$$= \frac{100}{2}$$

$$\boxed{y_2 = 50 \text{ mm}}$$

Now (C_g about (x-x))

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{1200 \times 10 + 2000 \times 50}{1200 + 2000}$$

$$= \frac{1200 + 2000}{1200 + 2000}$$

$$\boxed{\bar{y} = 25 \text{ mm}}$$

Case-II (C_g about (y-y))

R-1

$$\text{Area} = a_1 = l_1 \times t_1$$

$$= 60 \times 20 \text{ mm}^2$$

$$= 1200 \text{ mm}^2$$

(C_g about (y-y))

$$x_1 = t_1 + \frac{l_1}{2}$$

$$= 20 + \frac{60}{2}$$

$$= 20 + 30$$

$$\boxed{x_1 = 50 \text{ mm}}$$

R-2

$$x_2 = \frac{t_2}{2}$$

$$= \frac{20}{2}$$

$$\boxed{x_2 = 10 \text{ mm}}$$

Now CG about (Y-Y)

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{1200 \times 60 + 2000 \times 10}{1200 + 2000}$$

$$\boxed{\bar{x} = 35 \text{ mm}}$$

Now CG about L-section

$$\boxed{\bar{x}, \bar{y} = 35 \text{ mm}, 25 \text{ mm}}$$

Q-3

Find CG of Channel Section

100mm X 50mm X 15mm.

Given data

$$l_1 = 50 \text{ mm}$$

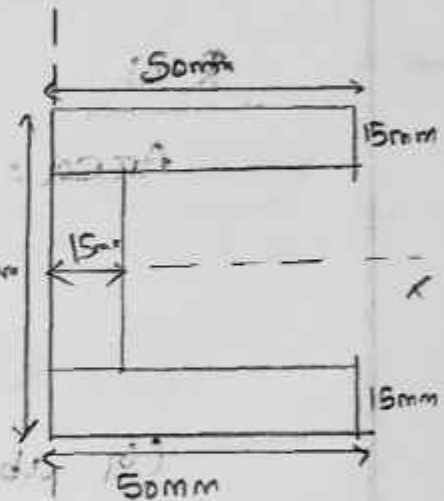
$$t_1 = 15 \text{ mm}$$

$$l_2 = 70 \text{ mm}$$

$$t_2 = 15 \text{ mm}$$

$$l_3 = 50 \text{ mm}$$

$$t_3 = 15 \text{ mm}$$



Case-I [CG about (X-X)]

$$\boxed{\bar{y} = 0}$$

(Due to X-X Passing through CG Points)

Case-II [CG about (Y-Y)]

$$\rightarrow \text{Area} = a_1 = l_1 \times t_1$$

$$= 50 \times 15 \text{ mm}$$

$$= 750 \text{ mm}^2$$

CG about (Y-Y)

$$x_1 = \frac{l_1}{2} = \frac{50}{2} = 25 \text{ mm}$$

R-II

$$\text{Area} = a_2 = l_2 \times t_2$$

$$= 70 \times 15$$

$$a_2 = 1050 \text{ mm}^2$$

(G about (Y-Y))

$$\rightarrow x_2 = \frac{t_2}{2}$$

$$= \frac{15}{2}$$

$$x_2 = 7.5 \text{ mm}$$

R-III

$$\text{Area} = a_3 = l_3 \times t_3$$

$$= 50 \times 15 \text{ mm}^2$$

$$a_3 = 750 \text{ mm}^2$$

(G about (Y-Y))

$$\rightarrow x_3 = \frac{l_3}{2}$$

$$= \frac{50}{2}$$

$$x_3 = 25 \text{ mm}$$

(G about

$$\bar{y} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{750 \times 25 + 1050 \times 75 + 750 \times 25}{750 + 1050 + 750}$$

$$= \frac{750 \times 25 + 1050 \times 75 + 750 \times 25}{750 + 1050 + 750}$$

$$\bar{y} = 17.8 \text{ mm}$$

Now, CG about Channel Section

$$\bar{X}, \bar{Y} = 0, 17.8 \text{ mm}$$

Q-4 A body consist of right circular cone of height 40mm and radius 30mm placed on a solid hemisphere of radius 30mm of same horizontal. Find the CG of the body?

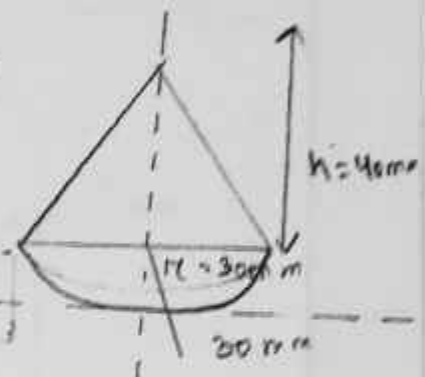
Given data

$$h = 40 \text{ mm}$$

$$r = 30 \text{ mm (cone)}$$

$$r = 30 \text{ mm (Hemisphere)}$$

$$(\bar{X}, \bar{Y}) = ?$$



Case-1 [CG of hemisphere about (x-x)]

→ Volume

$$V_1 = \frac{2\pi}{3} \times r^3$$

$$V_1 = \frac{2 \times 3.14}{3} \times (30)^3$$

$$(\because \pi = 3.14)$$

$$V_1 = 56520 \text{ mm}^3$$



CG about (x-x)

$$Y_1 = \frac{5r}{8}$$

$$= \frac{5 \times 30}{8} = \frac{150}{8} = 18.75 \text{ mm}$$

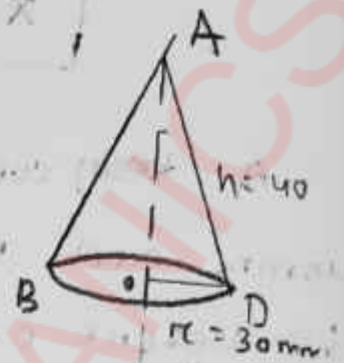
Case-II [CG about of right circular cone about (x-x)]

→ Volume

$$V_2 = \frac{\pi}{3} \times r^2 \times h$$

$$= \frac{3.14}{3} \times (30)^2 \times 40$$

$$V_2 = 37680 \text{ mm}^3$$



CG about (x-x)

$$Y_2 = 30 + \frac{40}{4}$$

$$= 30 + 10$$

$$Y_2 = 40 \text{ mm}$$

Now CG of the body about (x-x)

$$\bar{Y} = \frac{V_1 Y_1 + V_2 Y_2}{V_1 + V_2}$$

$$= \frac{56520 \times 18.75 + 37680 \times 40}{56520 + 37680}$$

$$56520 + 37680$$

$$\bar{Y} = 27.25 \text{ mm}$$

CG about the body

$$X, \bar{Y} = 0, 27.25 \text{ mm}$$

- 5 A semi circular Area is removed from a trapezium as shown in the figure, determine centroid of the remaining area.

Given data

$$AD = BC = 30 \text{ mm}$$

$$BE = 30 \text{ mm}$$

$$AB = DC = 80 \text{ mm}$$

$$\bar{x}, \bar{y} = ?$$

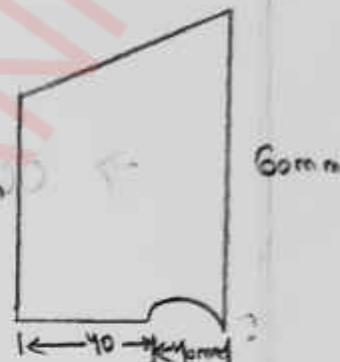
$$\text{mm}^2 = 80$$

$$(x-x) \text{ to } 30 \text{ mm}$$

$$\frac{30}{2} + 0.5 = 15.5$$

$$\text{off set}$$

$$\text{mm}^2 = 57$$



- ① Rectangle (ABCD)

$$\rightarrow \text{Area} = l \times b$$

$$= 80 \times 30$$

$$a_1 = 2400 \text{ mm}^2$$

CG about (x-x)

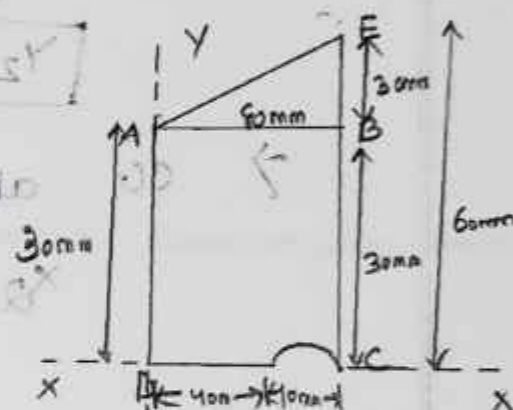
$$y_1 = \frac{30}{2}$$

$$y_1 = 15 \text{ mm}$$

$\rightarrow$ CG about (y-y)

$$x_1 = \frac{80}{2}$$

$$x_1 = 40 \text{ mm}$$



$$\text{mm}^2 = 80$$

② Triangle (ABE)

$$\rightarrow \text{Area} = a_2 = \frac{b \times h}{2}$$

$$= \frac{30 \times 80}{2}$$

$$a_2 = 1200 \text{ mm}^2$$

$\rightarrow$ CG about (X-X)

$$Y_2 = 30 + \frac{30}{3}$$

$$= 30 + 10$$

$$Y_2 = 40 \text{ mm}$$

$\rightarrow$ CG about (Y-Y)

$$X_2 = h \times \frac{2}{3}$$

$$= 80 \times \frac{2}{3}$$

$$X_2 = 53.33 \text{ mm}$$

③ Semi Circle

$$d = 40 \text{ mm}$$

$$r = 20 \text{ mm}$$

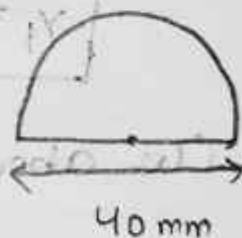
Area,

$$a_3 = \frac{\pi}{2} \times r^2$$

$$= \frac{3.14}{2} \times (20)^2$$

$$\therefore (\pi = 3.14)$$

$$a_3 = 628 \text{ mm}^2$$



CG about (x-x)

$$y_3 = \frac{4r}{3\pi}$$
$$= \frac{4 \times 20}{3 \times 3.14}$$

$$y_3 = 8.49 \text{ mm}$$

→ CG about (y-y)

$$x_3 = 40 + \frac{40}{2}$$

$$= 40 + 20$$

$$x_3 = 60 \text{ mm}$$

Now CG of trapezium remove Semicircular area

→ CG about (x-x)

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3}$$

$$= \frac{2400 \times 15 + 1200 \times 40 - 628 \times 8.49}{2400 + 1200 - 628}$$

$$\bar{y} = 26.46 \text{ m}$$

→ CG about (y-y)

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3}$$

$$= \frac{2400 \times 40 + 1200 \times 53.33 - 628 \times 60}{2400 + 1200 - 628}$$

$$\bar{x} = 41.15 \text{ mm}$$

GOVERNMENT POLYTECHNIC, BHUBANESWAR

ସରକାରୀ ପଲିଟେକ୍ନିକ ଭୁବନେଶ୍ୱର



ENGINEERING MECHANICS NOTE

CHAPTER- 5 **[SIMPLE LIFTING MACHINE]**

PREPARED BY

ER. RAJKUMAR PRADHAN [RAJ SIR]

A machine is a device which receive energy and doing a useful work.

e.g. - Steam engine, I.C engine etc.

→ They've of 2 two types machine

(1) Simple machine

(2) Compound machine

Simple machine

Its a device which do useful work at one end by receive less effort.

→ There are two types of simple machine

(1) Primary simple machine

(2) Secondary simple machine

Ex - lever,

→ Pulley

→ Differential pulley

→ Differential screw

} Primary simple machine

} Secondary S.m
Jack

Compound machine

The combination of simple machine is known as compound machine.

e.g. - screw, differential pulley etc.

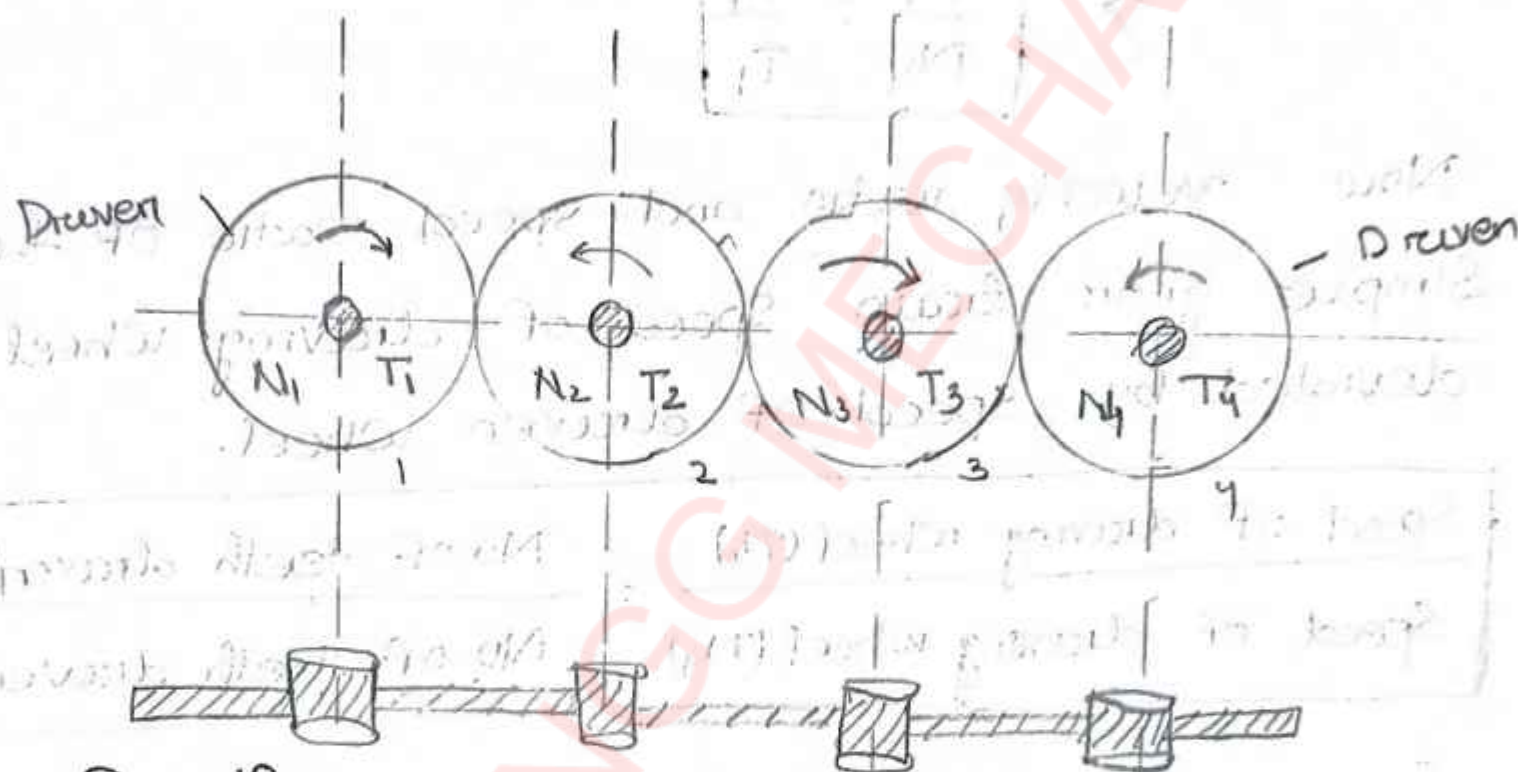
V.V.V.E Gear Train

The combination of gears to transmit power from one shaft to another shaft is known as gear train.

→ There are different types of gear train

- (1) Simple gear trains
- (2) Compound gear trains
- (3) Reverted gear trains
- (4) Epicyclic gear train

Simple gear train



→ In this diagram N_1 , N_2 , N_3 and N_4 are speed of the gear 1, 2, 3, 4.

→ Similarly T_1 , T_2 , T_3 and T_4 are no. of length of the gear 1, 2, 3, 4.

Mathematically,

Gear 1 and Gear 2 we get

$$\boxed{\frac{N_1}{N_2} = \frac{T_2}{T_1}} \quad - (1)$$

Gear 3 and Gear 4 we get

$$\boxed{\frac{N_2}{N_3} = \frac{T_3}{T_2}} \quad - - - (2)$$

Gear 3 and Gear 4 we get

$$\boxed{\frac{N_3}{N_4} = \frac{T_4}{T_3}} \quad \text{--- eqn (3)}$$

Now, Speed ratio of simple gear train

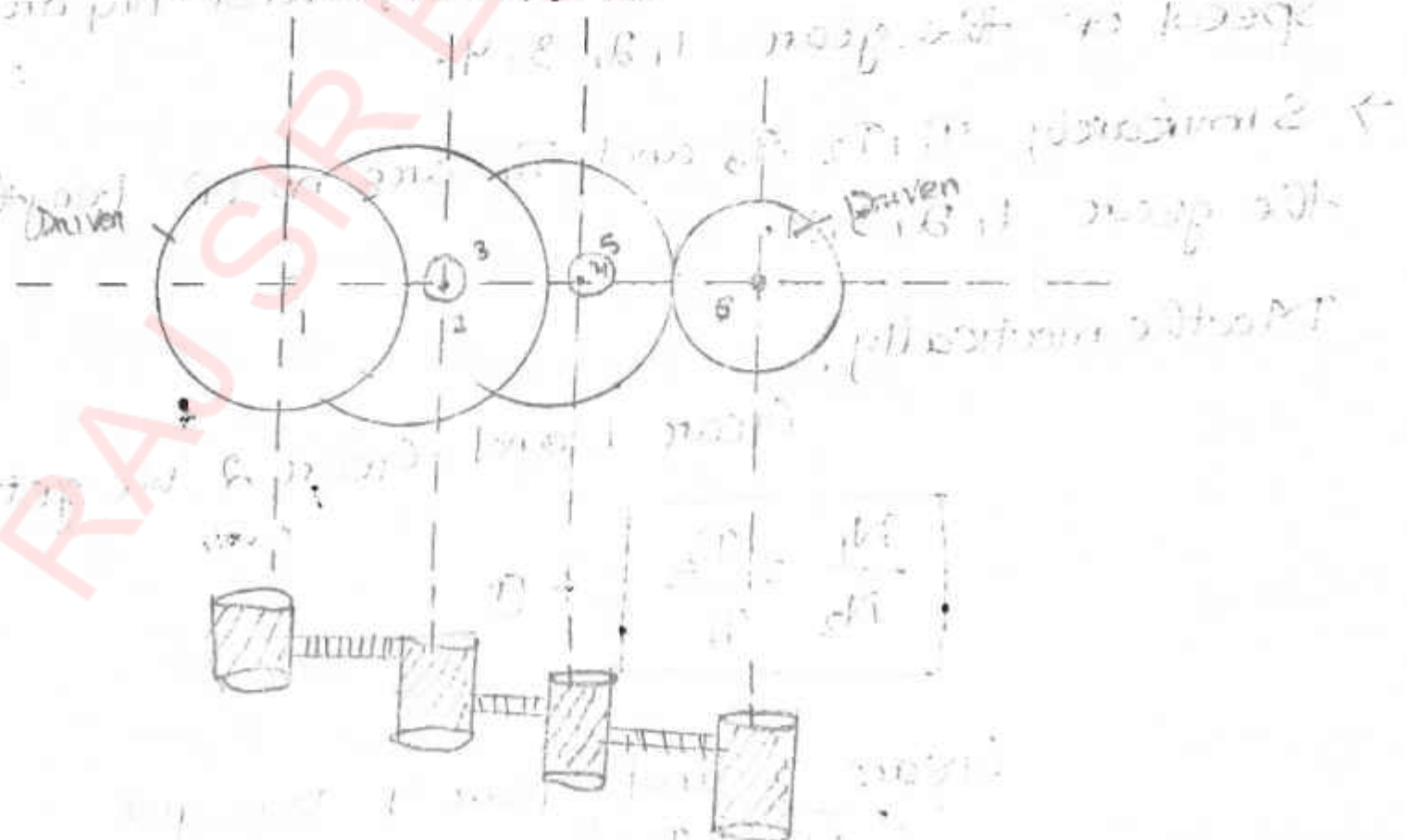
$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \times \frac{T_4}{T_3}$$

$$\Rightarrow \boxed{\frac{N_1}{N_4} = \frac{T_4}{T_1}}$$

Now velocity ratio and speed ratio of the Simple gear train Speed of driving wheel divided by Speed of driven wheel.

$$\boxed{\frac{\text{Speed of driving wheel } (N_1)}{\text{Speed of driven wheel } (N_4)} = \frac{\text{No of teeth driver } (T_1)}{\text{No of teeth driven } (T_4)}}$$

Compound Gear train



In this diagram N_1, N_2, N_3 and N_4, N_5, N_6 = Speed of gear 1, 2, 3, 4, 5, 6.

Mathematically,

→ G_1 and G_2 we get,

Velocity ratio

$$\boxed{\frac{N_1}{N_2} = \frac{T_2}{T_1}} \quad \text{--- (i)}$$

→ G_3 and G_4 we get

Velocity ratio

$$\boxed{\frac{N_3}{N_4} = \frac{T_4}{T_3}} \quad \text{--- (ii)}$$

→ G_5 and G_6 we get

Velocity ratio

$$\boxed{\frac{N_5}{N_6} = \frac{T_6}{T_5}} \quad \text{--- (iii)}$$

Now (i), (ii), (iii) we get velocity ratio of Compound gear

$$\boxed{\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}}$$

Lifting Machine

It's a device which enables to lift a heavy load (w) by applying comparatively small effort.

OR

The device which enable to lift a load by using less effort.

There are two types of lifting machine.

- ① Simple lifting machine
- ② Compound lifting machine

→ Simple lifting Machine

The device which enable to lift a load by using less effort is known as Simple lifting Machine.

Eg - : lever, Pulley, Inclined plane etc

Compound lifting Machine

A device contain combination of simple lifting machine lifts a load with less effort. This machine is known as compound lifting Machine.

Eg - Screw jacks, wheels and axels.

Mechanical Advantage -

The ratio between weight lifted load (W) & applied effort.

Mathematically,

$$MA = \frac{\text{load}}{\text{effort}}$$

$$\Rightarrow \boxed{MA = \frac{W}{P}} \quad \underline{\text{Imp}}$$

→ It has no unit.

Velocity ratio (VR)

The ratio between distance moved by effort and distance moved by load.

Mathematically,

$$VR = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$\Rightarrow VR = \frac{y}{x}$$

→ There is no unit.

Efficiency (η)

The ratio between mechanical advantage and velocity ratio is known as efficiency of simple machine.

$$\eta = \frac{MA}{VR} \quad \text{Imp}$$

The ratio between output and input of the machine known as efficiency of the machine.

→ It has no unit.

Mathematically, we know

$$\frac{MA}{VR} = \frac{\left(\frac{W}{P}\right)}{\left(\frac{y}{x}\right)}$$

$$\Rightarrow \eta = \frac{W}{P} \times \frac{x}{y}$$

$$\Rightarrow \eta = \frac{Wx}{Py} \quad \text{Imp}$$

$$\Rightarrow \eta = \frac{\text{Output}}{\text{Input}} \quad \text{Imp}$$

Where Wx = output of the machine.

Py = Input of the machine.

Ratio between MA, VR & Efficiency [Most Imp]

We know that

(Mechanical energy) $MA = \frac{\text{Load}}{\text{Effort}}$

$$\Rightarrow \boxed{MA = \frac{W}{P}} \quad \text{--- (i)}$$

Then we know that

(Velocity ratio) $VR = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$

$$\boxed{VR = \frac{y}{x}} \quad \text{--- (ii)}$$

lastly we know that

Efficiency : $\boxed{\eta = \frac{MA}{VR}}$

$$\Rightarrow \frac{MA}{VR} = \frac{\left(\frac{W}{P}\right)}{\left(\frac{y}{x}\right)}$$

$$\Rightarrow \eta = \frac{\text{Output}}{\text{Input}}$$

$$\Rightarrow \eta = \frac{W \times x}{P \times y}$$

$$\Rightarrow \eta \cdot (P \times y) = W \times x$$

$$\Rightarrow \eta = \frac{W}{P} \cdot \frac{x}{y}$$

$$\Rightarrow \boxed{\eta = \frac{MA}{VR}}$$

Law of Machine (V.V.I)

outline relationship between applied effort and lifted load.

Mathematically,

$$P = mW + C$$

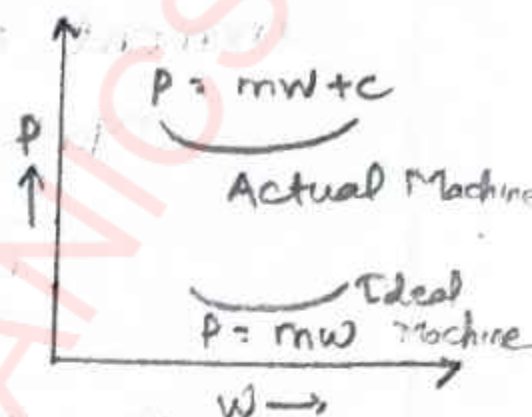
Where

P = effort

W = load

C = Machine Friction.

m = slope of line.



Maximum MA:-

$$\text{Max MA} = \frac{1}{m}$$

Minimum η :-

$$\eta_{\text{min}} = \frac{1}{m \times VR}$$

Lost of Friction:- (Effort)

$$P_{\text{friction}} = P - \frac{W}{VR}$$

Ideal Machine:-

- (i) A machine work input is equal to the output.
- (ii) In an ideal machine, efficiency is 100% .
- (iii) In an ideal machine, frictional force is zero.

So,

Mathematically,

$$P = W$$

Where,

P = effort

W = load

Reversible Machine (River)

The machine which is capable of doing some work in the reversible direction of the moment of effort.

Mathematically,

$$\boxed{\text{Output of machine} > \text{Machine friction}}$$

$$\Rightarrow \text{output} > \text{input} - \text{output}$$

$$\Rightarrow (W \times u) > (P \times y) - (W \times u)$$

$$\Rightarrow (W \times u) + (W \times u) > (P \times y)$$

$$\Rightarrow 2(W \times u) > (P \times y)$$

$$\Rightarrow \frac{W \times u}{P \times y} > \frac{1}{2}$$

$$\Rightarrow \frac{W}{P} > \frac{y}{2u}$$

$$\Rightarrow \frac{MA}{VR} > 50\%$$

$$\Rightarrow \boxed{\eta > 50\%}$$

→ Irreversible Machine (Self locking)

The machine which is not capable of doing same work in the reversible direction after removal of effort.

Mathematically,

$$\boxed{\text{Output of machine} < \text{input of machine}}$$

$$\Rightarrow \text{output} < \text{input} - \text{output}$$

$$\Rightarrow (W \times r) < (P \times R) - (W \times r)$$

$$\Rightarrow (W \times r) + (W \times r) < (P \times R)$$

$$\Rightarrow 2(W \times r) < (P \times R)$$

$$\Rightarrow \frac{W \times r}{P \times R} < \frac{1}{2}$$

$$\Rightarrow \frac{\left(\frac{W}{P}\right)}{\left(\frac{R}{r}\right)} < \frac{1}{2}$$

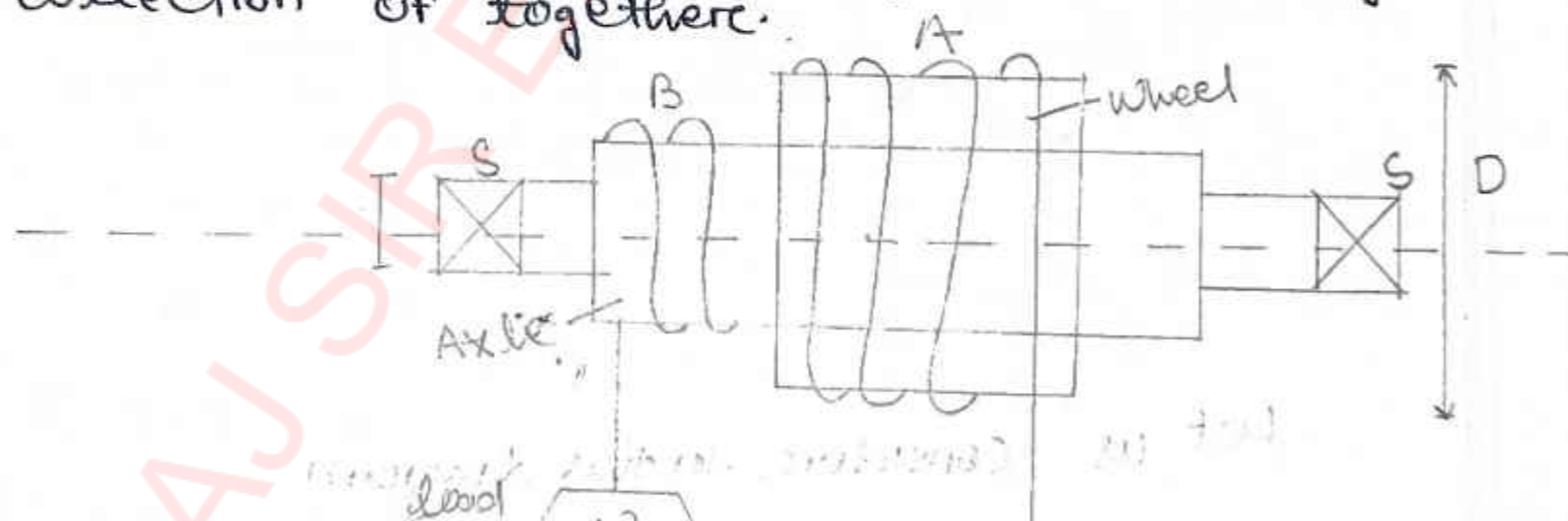
$$\Rightarrow \frac{MA}{VR} < 50\%$$

$$\Rightarrow \boxed{\eta < 50\%}$$



Simple Wheel and Axle:

It's a simple machine at which wheel and axle turn together acting like rotating collection of together.



In this diagram

B = axle

A = wheel

D = Diameter of the wheel

d = diameter of the axle

W = load lifted at axle

P = effort applied at wheel

Mathematically, we know,

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

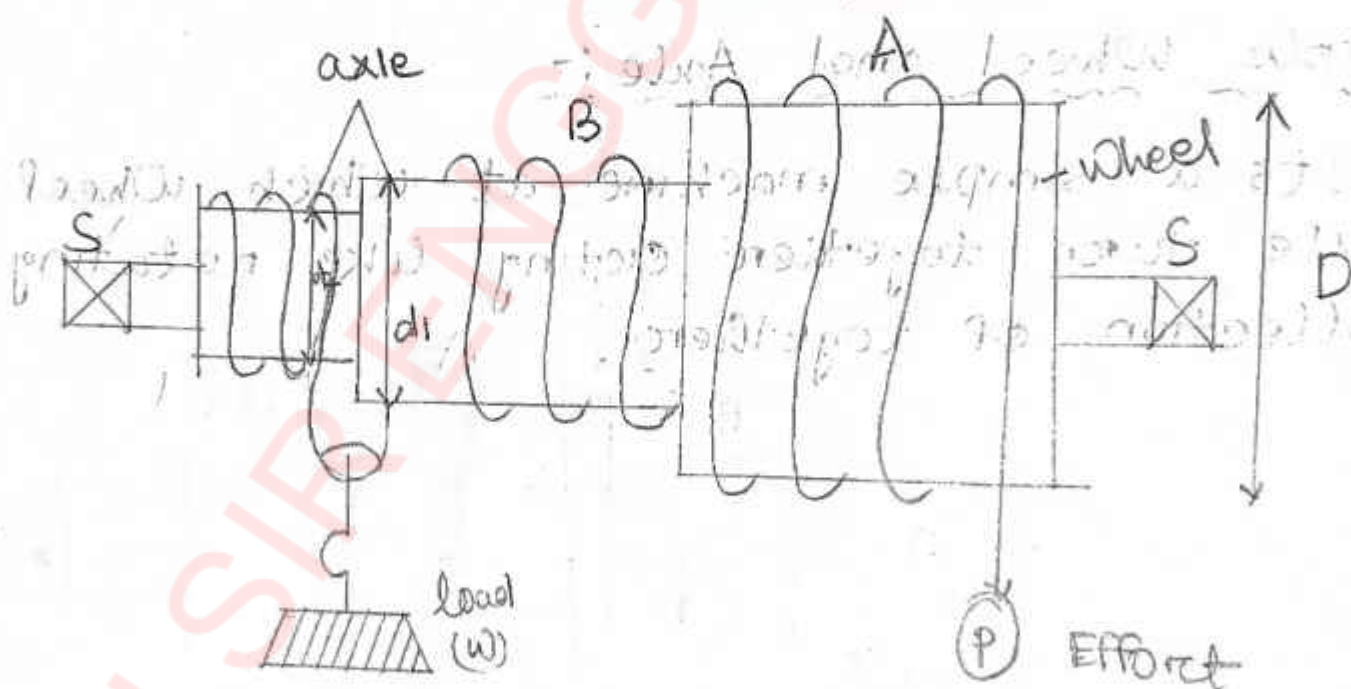
$$= \frac{\pi D}{\pi d}$$

$$VR = \frac{D}{d}$$

$$MA = \frac{W}{P}$$

$$\rightarrow \text{Efficiency} = \frac{MA}{VR}$$

→ Differential wheel and axle :-



Let us consider, in this diagram

A = Wheel, wheel effort, P is applied.

B and C are two single axle where load W is lifted.

S = Supporting spindle

D = Diameter of the wheel

d = diameter of axle 'B'

d_2 = diameter of axle &
mathematically velocity ratio

$$VR = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{Y}{x}$$

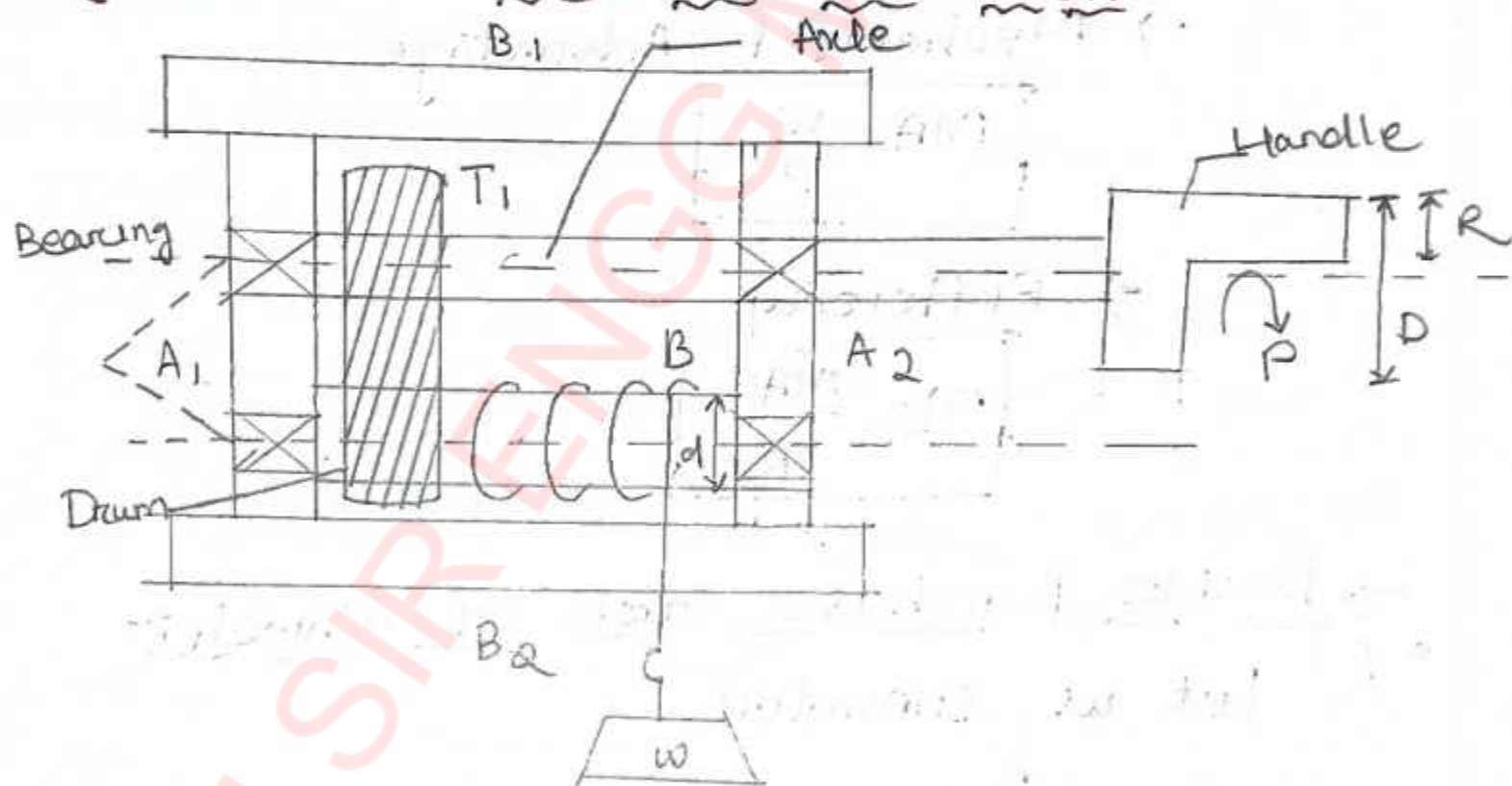
$$VR = \frac{2D}{d_1 - d_2}$$

Imp

$$\rightarrow MA = \frac{W}{P}$$

$$\rightarrow \text{Efficiency } (\eta) = \frac{MA}{VR}$$

Single purchase crab and winch.



Let us consider

B = drum

A = Axle

T_1 = No. of teeth on pinion gear

T_2 = No. of teeth Spur gear

A_1 & A_2 = Vertical bar

D = Diameter of handle

R = radius / length of the handle
 d = diameter of drum

Now mathematically,

$$\text{Velocity ratio} = VR = \frac{Y}{x}$$

but according to the simple purchase or wheel,

$$VR = \frac{Y}{x}$$

$$\therefore VR = \frac{DT_2}{dT_1}$$

→ Mechanical Advantage

$$MA = \frac{W}{P}$$

→ Efficiency

$$\eta = \frac{MA}{VR}$$

→ Double purchase carb on which:-

Let us consider,

D = diameter of handle crutching

R = Radius / length of handle

d = diameter of drum

T_1 and T_3 = no. of teeth on pinion gear

T_2 and T_4 = no. of teeth on spur gear

B = drum

A & C = Axle

A_1 & A_2 = Vertical bar

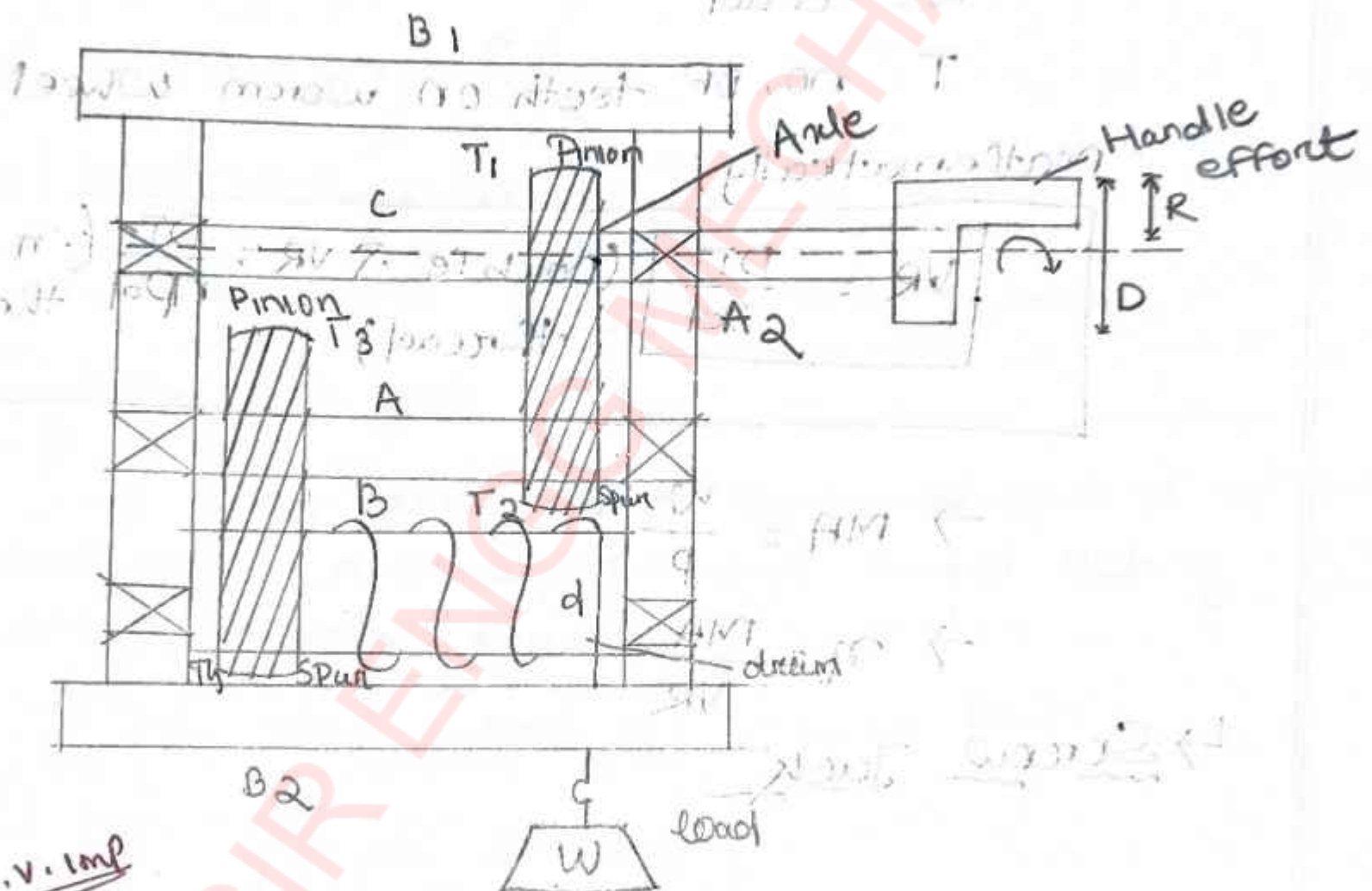
B_1 & B_2 = Top and bottom bar

Mathematically,

$$VR = \frac{DT_2 T_4}{dT_1 T_3}$$

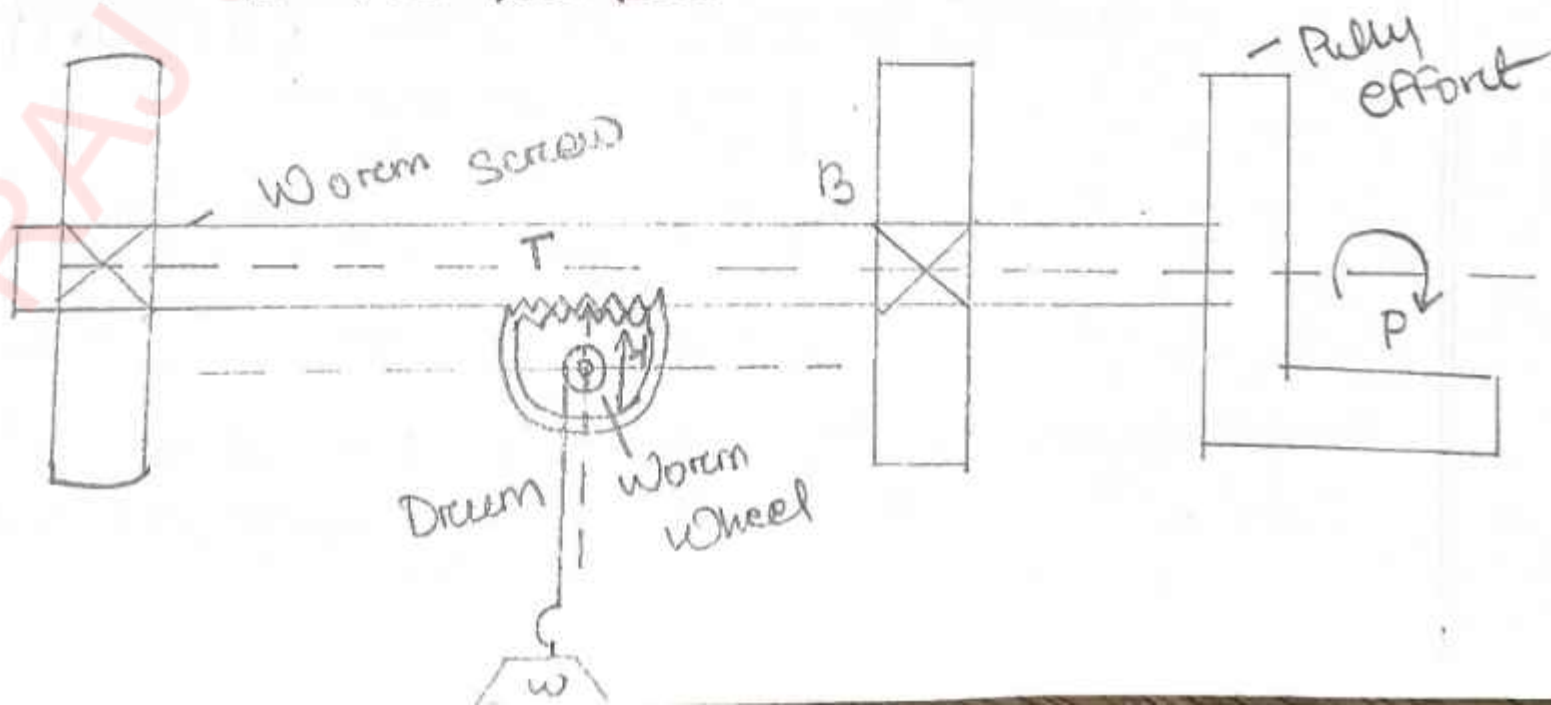
→ Mechanical Advantage, $MA = \frac{W}{P}$

→ Efficiency, $\eta = \frac{MA}{VR}$



V.V. Imp

→ Worm or wheel (worm)



Worm device is a gear arrangement device where in the form of screw

In this diagram

R = length of handle

D = diameter of handle rotation

d = diameter of pulley

P = Effort

W = load

T = no. of teeth on worm wheel

Mathematically,

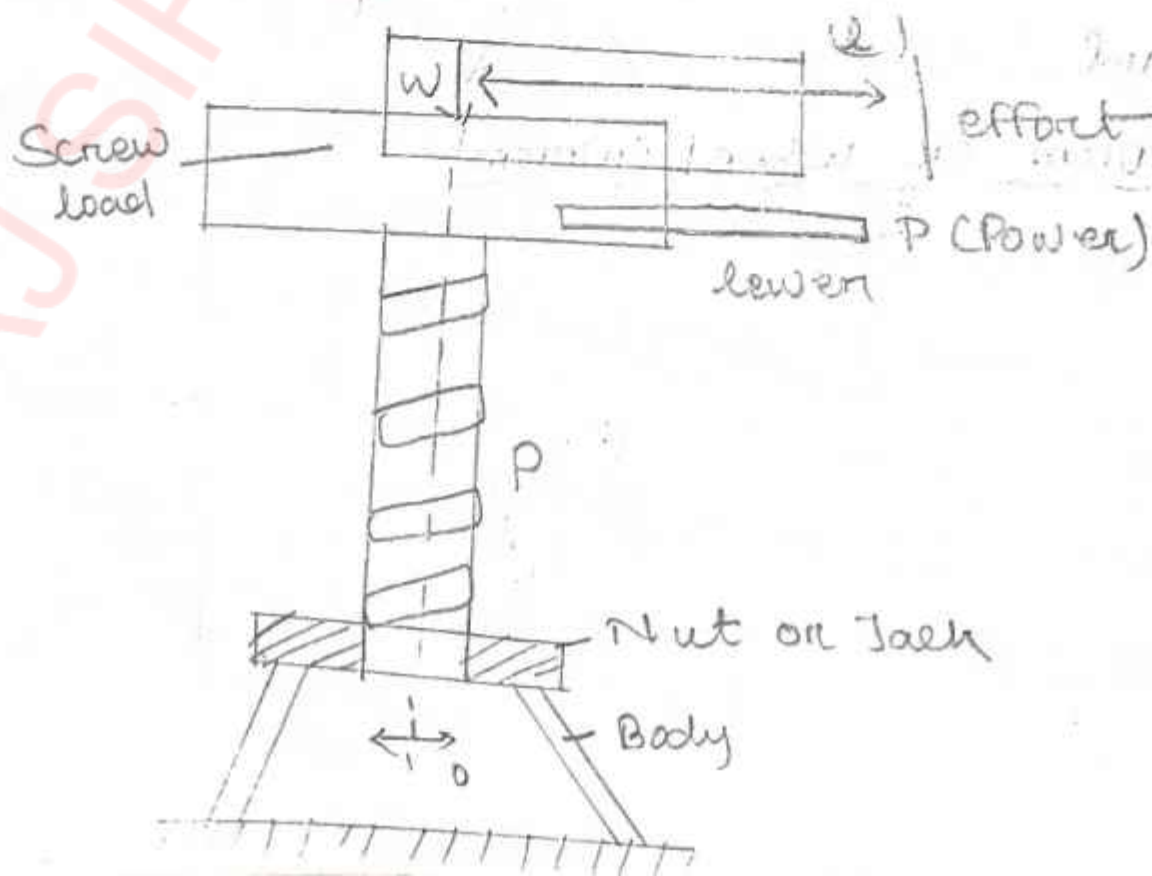
$$VR = \frac{DT}{2d}$$

(Double & $VR = \frac{DT}{\pi d}$ $\because n = \pi d$ thread)

$$\rightarrow MA = \frac{W}{P}$$

$$\rightarrow \eta = \frac{MA}{VR}$$

→ Screw Jack



mathematically,

l = length of the effort arm

~~V.V.Imp~~
 P = Pitch of screw

W = load

P = effort

$$VR = \frac{2\pi l}{P} \quad \dots \dots \quad (\because P \text{ is small})$$

$$MA = \frac{W}{P}$$

$$\eta = \frac{MA}{VR}$$

→ Newton's Law of Machine :-

There are 3 types newton's law.

① Newton's 1st law :-

A body continues in state of rest or uniform motion in a straight line until acting upon by any external force.

→ It also called as inertia law.

② Newton's 2nd law :-

Rate of change of momentum is directly proportional to the applied force.

$$F = \frac{dp}{dt}$$

$$F = kma$$

$$F = ma$$

Newton's 3rd law :-

To every action there is an equal and opposite reaction.

Chapter - 5 (Simple machine)

The velocity ratio of simple machine is 10, the effort applied is 150N. Determine efficiency of load 1200N?

Given data

$$VR = 10$$

$$P = 150 \text{ N}$$

$$\eta = ?$$

$$W = 1200 \text{ N}$$

① Mechanical Advantage

$$MA = \frac{W}{P}$$
$$= \frac{1200}{150}$$

$$\boxed{MA = 8}$$

②

Efficiency

$$\eta = \frac{MA}{VR}$$
$$= \frac{8}{10}$$

$$\eta = 0.8$$

$$\boxed{\eta = 80\%}$$

Q2

A simple lifting machine effort of 500N rise a load 12.5kN. If the efficiency of the machine is 65%. What is the mechanical Advantage & Velocity ratio of the machine?

Given data

$$P = 500 \text{ N}$$

$$W = 12.5 \text{ kN}$$

$$\eta = 65\% = 0.65$$

$$MA = ?$$

$$VR = ?$$

① Mechanical Advantage

$$MA = \frac{W}{P}$$

$$= \frac{12,500}{500}$$

$$\boxed{MA = 25}$$

② Efficiency

$$\eta = \frac{MA}{VR}$$

$$\Rightarrow 0.65 = \frac{25}{VR}$$

$$\Rightarrow 0.65 VR = 25$$

$$\Rightarrow VR = \frac{25}{0.65}$$

$$\Rightarrow \boxed{VR = 38.46}$$

- ③ The velocity ratio of simple machine is 20. The effort applied is 130N. Determine efficiency if load 1300N.

Given data

$$VR = 20$$

$$P = 130$$

$$W = 1300$$

$$\eta = ?$$

①

Mechanical Advantage

$$MA = \frac{W}{P}$$

$$= \frac{1300}{130}$$

$$\boxed{MA = 10}$$

⑧ Efficiency

$$\eta = \frac{MA}{VR}$$

$$= \frac{10}{20}$$

$$= 0.5$$

$$\boxed{\eta = 50\%}$$

⑨ A simple lifting machine effort of 400 N lifts a load 14.2 kN. If the efficiency of the machine is 55%. What is the Mechanical Advantage and velocity ratio?

Given data

$$P = 400 \text{ N}$$

$$W = 14,200 \text{ N}$$

$$\eta = 55\%$$

$$MA = ?$$

$$VR = ?$$

⑩ Mechanical Advantage

$$MA = \frac{W}{P}$$

$$= \frac{14,200}{400}$$

$$\boxed{MA = 35.5}$$

⑪ Efficiency (η)

$$\eta = \frac{MA}{VR}$$

$$0.55 = \frac{35.5}{VR}$$

$$\Rightarrow 0.55 VR = 35.5$$

$$\Rightarrow VR = \frac{35.5}{0.55}$$

$$\Rightarrow \boxed{VR = 64.54}$$

- ⑤ A simple machine effort of 100N, by moving through a distance of 1m lift a load of 1000N through a distance of 0.1m. Find the velocity ratio, Mechanical Advantage and efficiency of the machine.

Given data

$$P = 100N$$

$$y = 1m$$

$$W = 1000N$$

$$x = 0.1m \text{ (load)}$$

$$MA = ?$$

$$VR = ?$$

$$\eta = ?$$

① Mechanical Advantage

$$MA = \frac{W}{P}$$

$$= \frac{1000}{100}$$

$$\boxed{MA = 10}$$

② Velocity ratio

$$VR = \frac{y}{x} = \frac{1m}{0.1} = 10m$$

⑤ Efficiency

$$\eta = \frac{MA}{VR}$$
$$= \frac{10}{10}$$

$$\Rightarrow \boxed{\eta = 1} \text{ (Ideal machine)}$$

A simple machine effort of 200N by using through a distance of 2m lift a load of 500N through a distance of 0.2m. Find the velocity ratio, mechanical advantage and efficiency of the machine.

Given data

$$P = 200 \text{ N}$$

$$W = 500 \text{ N}$$

$$y = 2 \text{ m}$$

$$x = 0.2 \text{ m}$$

$$VR = ?$$

$$MA = ?$$

$$\eta = ?$$

① Mechanical Advantage

$$MA = \frac{W}{P}$$
$$= \frac{500}{200}$$

$$\Rightarrow \boxed{MA = 2.5}$$

② Velocity ratio

$$VR = \frac{y}{x} = \frac{2 \text{ m}}{0.2 \text{ m}} = 10 \text{ m}$$

③ Efficiency

$$\eta = \frac{MA}{VR}$$

$$= \frac{2.5}{10}$$

$$\boxed{\eta = 25}$$

- ④ M.T
An effort of 200N is required to lift 6000N load and an effort of 400N is required to lift 14500N on a simple machine established the law of machine & find the load lifted by effort of 300N.

Given data

$$P_1 = 200N$$

$$W_1 = 6000N$$

$$P_2 = 400N$$

$$W_2 = 14500N$$

$$P_3 = 300N$$

$$W_3 = ?$$

Case-I

$$P_1 = 200N$$

$$W_1 = 6000N$$

$$P = W_m + C$$

$$\boxed{200 = 6000m + C} \quad \text{--- ①}$$

Case-II

$$P_2 = 400N$$

$$W_2 = 14500N$$

$$P_2 = W_2 m + c$$

$$\Rightarrow \boxed{400 = 14500 m + c} \quad \text{--- (1)}$$

Now eqn. (2) & (1) we get

$$400 = 14500 m + c$$

$$200 = 6000 m + c$$

$$200 = 8500 m + c$$

$$\Rightarrow m = \frac{200}{8500}$$

$$\Rightarrow \boxed{m = 0.02}$$

m value Putting in eqn. (1)

$$200 = 6000 m + c$$

$$\Rightarrow 200 = 6000 \times 0.02 + c$$

$$\Rightarrow 200 = 120 + c$$

$$\Rightarrow c = 200 - 120$$

$$\Rightarrow \boxed{c = 80}$$

Establish law of machine

$$P = mW + c$$

$$\Rightarrow \boxed{P = 0.02 W + 80} \quad \text{--- (3)}$$

Case-II (load)

$$P_3 = 300 \text{ N}$$

$$W_3 = ?$$

$\rightarrow$ Now new to establish law of machine

$$P = W_m + C$$

$$\Rightarrow 300 = 0.02 W + 80$$

$$\Rightarrow 300 - 80 = 0.02 W_3$$

$$\Rightarrow 220 = 0.02 W_3$$

$$\Rightarrow W_3 = \frac{220 \times 1000}{0.02} = 11000$$

$$\Rightarrow \boxed{W_3 = 11000 \text{ N}}$$

An effort of 400N is required to lift 500N load and effort of 500N is required to lift 12.500N on a simple machine. Established law of machine & find the load lifted by effort 14.50N.

Given data

$$P_1 = 400 \text{ N}$$

$$W_1 = 8000 \text{ N}$$

$$P_2 = 500 \text{ N}$$

$$W_2 = 12500 \text{ N}$$

$$P_3 = 500 \text{ N}$$

$$W_3 = ?$$

Case-1

$$P_1 = W_{m1} + C$$

$$\Rightarrow \boxed{400 = 8000m + C} \quad \text{--- (1)}$$

Case-I

$$P_2 = W_2 m + c$$

$$\Rightarrow \boxed{700 = 17,500m + c} \quad \text{--- (1)}$$

Now eqn. (2) and (1) we get,

$$700 = 17,500m + c$$

$$400 = 80,000m + c$$

$$300 = 95,000m$$

$$\Rightarrow m = \frac{300}{95,000}$$

$$\Rightarrow \boxed{m = 0.03}$$

m value putting in eqn. (2)

$$700 = 17500m + c$$

$$\Rightarrow 700 = 17500 \times 0.03 + c$$

$$\Rightarrow 700 = 525 + c$$

$$\Rightarrow c = 700 - 525$$

$$\Rightarrow \boxed{c = 175}$$

Establish law of machine

$$P = Wm + c$$

$$\Rightarrow \boxed{500 = W \cdot 0.03 + c} \quad \text{--- (3)}$$

Case-II (load)

Now new establish law of machine

$$P = W_3 + c$$

$$\Rightarrow 500 = W_3 + 175$$

$$\Rightarrow 500 = 0.03 W_3 + 175$$

$$\Rightarrow 500 - 175 = 0.03 W_3$$

$$\Rightarrow 325 = 0.03 W_3$$

$$\Rightarrow W_3 = \frac{325}{0.03}$$

$$\Rightarrow W_3 = 10833.33 \text{ N}$$

(9)

In a lifting machine effort of 98 N lift a load of 2450 N and an efforts of 127.4 N. lift a load of 3920 N.

- (i) Establish law of Machine.
- (ii) Calculate the effort required to lift a load of 5880 N.
- (iii) Find the load of effort of 196 N.
- (iv) What is the max^m. efficiency of VR of a machine 75.
- (v) Find the max^m. Mechanical Advantage.

Given data

$$P_1 = 98 \text{ N}$$

$$W_1 = 2450 \text{ N}$$

$$P_2 = 127.4 \text{ N}$$

$$W_2 = 3920 \text{ N}$$

① Law of Machine

$$P = mW + C$$

Case-1

$$P_1 = 98 \text{ N}$$

$$W_1 = 2450 \text{ N}$$

$$P_1 = W_1 m + c$$

$$\Rightarrow 98 = 2450m + c \quad \text{--- (i)}$$

Then

$$P_2 = W_2 m + c$$

$$\Rightarrow 127.4 = 3920m + c \quad \text{--- (ii)}$$

From eqⁿ (i) & (ii) we get

$$127.4 = 3920m + c$$

$$98 = 2450m + c$$

$$29.4 = 1470m$$

$$\Rightarrow m = \frac{29.4}{1470}$$

$$\Rightarrow m = 0.02$$

Now value putting eqⁿ (i)

$$98 = 2450m + c$$

$$\Rightarrow 98 = 2450 \times 0.02 + c$$

$$\Rightarrow 98 = 49 + c$$

$$\Rightarrow c = 98 - 49$$

$$\Rightarrow c = 49$$

Now Establish law of machine

$$P = Wm + c$$

$$P = 0.02m + 49 \quad \text{--- (iii)}$$

$$W_4 = 5880 \text{ N}$$

Now eqⁿ (iii)

$$P = 0.02W_3 + 49$$

$$\Rightarrow P = 0.02 \times 5880 + 49$$

$$\Rightarrow P = 117.6 + 49$$

$$\Rightarrow \boxed{P = 166.6 \text{ N}}$$

(iv) $P_4 = 196 \text{ N}$

$$W_4 = ?$$

Now eq (3)

$$P = 0.02W_4 + 49$$

$$\Rightarrow P_4 = 0.02 \times W_4 + 49$$

$$\Rightarrow 196 - 49 = 0.02W_4$$

$$\Rightarrow 147 = 0.02W_4$$

$$\Rightarrow W_4 = \frac{147}{0.02}$$

$$\Rightarrow \boxed{W_4 = 7350 \text{ N}}$$

(v)

$$VR = 75$$

$$\eta_{\max} = \frac{1}{m \times VR}$$

$$\Rightarrow \eta_{\max} = \frac{1}{0.02 \times 75}$$

$$\Rightarrow \boxed{\eta_{\max} = 0.66 \text{ or } 66\%}$$

(v)

$$\text{Max } m \cdot MA = ?$$

$$MA_{\max} = \frac{1}{m}$$

$$\Rightarrow MA_{\max} = \frac{1}{0.02}$$

$$\Rightarrow \boxed{MA_{\max} = 50}$$

10 An effort of machine effort of 88 N lift a load of 2250 N and an effort of 125.4 N lift load of 4920 N.

- (1) Establish law of machine
- (2) Calculate the effort require to lift a load of 6880 N.
- (3) Find the load of effort of 186 N.
- (4) What is the max^m efficiency if VR of a machine is 65.
- (5) Find Max^m MA.

Given data

$$P_1 = 88 \text{ N}$$

$$W_1 = 2250 \text{ N}$$

$$P_2 = 125.4 \text{ N}$$

$$W_2 = 4920 \text{ N}$$

① Law of machine

$$P = mW + C$$

Case-I

$$P_1 = 88 \text{ N}$$

$$W_1 = 2250 \text{ N}$$

$$P_1 = W_1 m + C$$

$$\Rightarrow 88 = 2250m + C \quad \text{--- (I)}$$

Case-II

$$P_2 = W_2 m + C$$

$$\Rightarrow 125.4 = 4920m + C \quad \text{--- (II)}$$

From eqn-② & ① we get

$$125.4 = 4920m + C$$

$$88 = 2250m + C$$

$$\Rightarrow 37.4 = 2670m$$

$$\Rightarrow m = \frac{37.4}{2670}$$

$$\Rightarrow \boxed{m = 0.01}$$

Now value putting in eqn-①

$$88 = 2250m + C$$

$$\Rightarrow 88 = 2250 \times 0.01 + C$$

$$\Rightarrow 88 = 22.5 + C$$

$$\Rightarrow C = 88 - 22.5$$

$$\Rightarrow \boxed{C = 65.5}$$

Now

$$P = Wm + C$$

$$\boxed{P = 0.01W + 65.5} \quad \text{--- (3)}$$

$$W_3 = 6880$$

$$P_3 = ?$$

from eqn-③

$$P = 0.01 \times 6880 + 65.5$$

$$\Rightarrow \boxed{P = 134.3 \text{ N}}$$

(iii)

$$P_4 = 186 \text{ N}$$

$$W_4 = ?$$

$$P = Wm + C$$

$$\Rightarrow 186 = 0.01W_4 + 65.5$$

$$\Rightarrow 186 - 65.5 = 0.01 w_4$$

$$\Rightarrow 120.5 = 0.01 w_4$$

$$\Rightarrow w_4 = \frac{120.5}{0.01}$$

$$\Rightarrow \boxed{w_4 = 12050}$$

$$(iv) \quad VR = 65$$

$$\eta_{\max} = \frac{1}{m \times VR}$$

$$\Rightarrow \eta_{\max} = \frac{1}{0.01 \times 65}$$

$$\Rightarrow \boxed{\eta_{\max} = 1.53}$$

$$(v) \quad MA_{\max} = \frac{1}{m}$$

$$\Rightarrow MA_{\max} = \frac{1}{0.01}$$

$$\Rightarrow \boxed{MA_{\max} = 100}$$

- ② The law of machine is $P = 0.02W + 48$. Determine the max^m mechanical Advantage and max^m efficiency if velocity ratio is 80.

Given data

$$P = 0.02W + 48$$

$$VR = 80$$

$$C = 48$$

$$m = 0.02$$

$$\textcircled{1} \quad \begin{aligned} \text{Max}^m MA &= \frac{1}{m} \\ &= \frac{1}{0.02} \end{aligned}$$

$$\Rightarrow \boxed{\text{Max}^m MA = 50}$$

$$\eta_{max} = \frac{1}{m \times VR}$$

$$\Rightarrow \eta_{max} = \frac{1}{0.02 \times 80}$$

$$\Rightarrow \boxed{\eta_{max} = 0.625}$$

⑫ The law of machine is $P = \frac{W}{50} + 15$. The Velocity Ratio of the machine is 100.

- (i) Find max^m MA.
- (ii) Find max^m efficiency of the machine.
- (iii) Determine effort if load is 1500N.
- (iv) Determine lost if friction of effort.

Given data

$$P = \frac{W}{50} + 15$$

$$VR = 100$$

$$m = 0.02$$

$$C = 15$$

$$MA_{max} = ?$$

$$\eta_{max} = ?$$

①

MA max^m

$$MA_{max} = \frac{1}{m}$$

$$= \frac{1}{0.02}$$

$$\Rightarrow \boxed{MA_{max} = 50}$$

②

$$\eta_{max} = \frac{1}{m \times VR}$$

$$= \frac{1}{0.02 \times 100}$$

$$= \frac{1}{2} = 0.5$$

(11)

$$P = ?$$

$$W = 1500$$

$$P = Wm + C$$

$$\Rightarrow P = 1500 (0.02) + 15$$

$$\Rightarrow P = 30 + 15$$

$$\Rightarrow \boxed{P = 45 \text{ N}}$$

(12)

$$\text{Friction} = P - \frac{W}{VR}$$

$$= 45 - \frac{1500}{100}$$

$$= 45 - 15$$

$$= \boxed{30}$$

HW

①

The velocity ratio of simple machine is 20, the effort applied is 130 N. Determine efficiency, if load 1300 N?

Given data

$$VR = 20$$

$$P = 130 \text{ N}$$

$$W = 1300 \text{ N}$$

$$\eta = ?$$

Case-1 (Mechanical Advantage)

$$MA = \frac{W}{P}$$

$$= \frac{1300}{130}$$

$$\boxed{MA = 10}$$

Efficiency

$$\eta = \frac{MA}{VR}$$

$$= \frac{10}{20}$$

$$= \frac{1}{2}$$

$$\Rightarrow \boxed{\eta = 50\%}$$

②

A simple lifting machine effort of 400N raised a load 14.2 kN. If the efficiency of the machine is 55%. What is the MA and VR of the machine?

Given data

$$P = 400 \text{ N}$$

$$W = 14.2 \text{ kN} = 14,200 \text{ N}$$

$$\eta = 55\% = 0.55$$

$$MA = ?$$

$$VR = ?$$

① Mechanical Advantage

$$MA = \frac{W}{P}$$

$$= \frac{14,200}{400}$$

$$= \boxed{35.5}$$

② Efficiency

$$\eta = \frac{MA}{VR}$$

$$\Rightarrow 0.55 = \frac{35.5}{VR}$$

$$\Rightarrow 0.55 VR = 35.5$$

$$VR = \frac{35.5}{0.55}$$

$$\Rightarrow \boxed{VR = 64.54}$$

- ③ A simple machine effort of 1500N, rise a load 8.5kN. Its efficiency of the machine is 75%. What is the MA & VR of the machine?

Given data

$$P = 1500 \text{ N}$$

$$W = 8.5 \text{ kN} = 8500 \text{ N}$$

$$\eta = 75\% = 0.75$$

$$MA = ?$$

$$VR = ?$$

- ① Mechanical Advantage

$$MA = \frac{W}{P}$$

$$= \frac{8500}{1500}$$

$$\boxed{MA = 5.66}$$

- ② Efficiency

$$\eta = \frac{MA}{VR}$$

$$\Rightarrow 0.75 = \frac{5.66}{VR}$$

$$\Rightarrow 0.75 VR = 5.66$$

$$\Rightarrow VR = \frac{5.66}{0.75}$$

$$\Rightarrow \boxed{VR = 7.54}$$

- ④ The velocity ratio of simple machine is 20. The effort applied is 450N determine efficiency if load is 1400N.

Given data

$$VR = 20$$

$$P = 450N$$

$$W = 1400N$$

$$\eta = ?$$

① Mechanical Advantage

$$MA = \frac{W}{P}$$

$$= \frac{1400}{450}$$

$$MA = 3.11$$

② Efficiency

$$\eta = \frac{MA}{VR}$$

$$= \frac{3.11}{20}$$

$$\Rightarrow \eta = 0.155 \text{ or } 15\%$$

- ⑤ A simple machine effort of 200N by moving through a distance of 2m lift a load of 500N through a distance of 0.2m. Find the velocity ratio, MA and efficiency of machine.

Given data

$$P = 200N$$

$$y = 2m$$

$$x = 0.2m$$

$$W = 500N$$

$$VR = ?$$

$$MA = ?$$

$$\eta = ?$$

① Mechanical Advantage

$$MA = \frac{W}{P}$$
$$= \frac{500}{200}$$

$$\boxed{MA = 2.5}$$

② Velocity ratio

$$VR = \frac{y}{x}$$

$$\Rightarrow VR = \frac{2}{0.2}$$

$$\Rightarrow \boxed{VR = 10}$$

③ Efficiency

$$\eta = \frac{MA}{VR}$$

$$= \frac{2.5}{10}$$

$$\boxed{\eta = 25\%}$$

⑥ An effort of 400N is require to lift 500N load
is require to lift 1250N on a simple machine
Establish the law of machine & And the load
lifted by effort 450N?

Given data

$$P_1 = 400N$$

$$W_1 = 500N$$

$$P_2 = 1250N$$

$$W_2 = 1250N$$

$$P_3 = 450N$$

Case-1

$$P = mW + c$$

So,

$$P_1 = mW_1 + c$$

$$\Rightarrow 400 = 500m + c \quad \text{--- (i)}$$

Then

$$P_2 = mW_2 + c$$

$$\Rightarrow 500 = 1250m + c \quad \text{--- (ii)}$$

From eqn (i) & (ii) we get

$$500 = 1250m + c$$

$$400 = 500m + c$$

$$\hline 100 = 750m$$

$$\Rightarrow m = \frac{100}{750}$$

$$\Rightarrow m = 0.13$$

m value putting in eqn (i)

$$400 = 500m + c$$

$$\Rightarrow 400 = 500 \times 0.13 + c$$

$$\Rightarrow 400 = 65 + c$$

$$\Rightarrow c = 400 - 65$$

$$\Rightarrow c = 335$$

Now Establish law of machine

$$P = Wm + c$$

$$P_3 = W_3 m + c$$

$$\Rightarrow 450 = W_3 (0.13) + 335$$

$$\Rightarrow 450 - 335 = 0.13 W_3$$

$$\Rightarrow 115 = 0.13 W_3$$

$$\Rightarrow W_3 = \frac{115}{0.13}$$

$$\Rightarrow \boxed{W_3 = 884.61}$$

⑦ In a lifting machine effort of 88 N lifts a load of 2860 N effort of 129 N lifts a load of 4920 N.

(i) Establish law of machine.

(ii) Find effort if load 6860 N?

(iii) Find load of effort 166 N?

(iv) Find the max^m efficiency if VR of machine 65?

(v) Find the max^m MA?

Given data

$$P_1 = 88 \text{ N}$$

$$W_1 = 2860 \text{ N}$$

$$P_2 = 129 \text{ N}$$

$$W_2 = 4920 \text{ N}$$

Case-1

$$P_1 = 88 \text{ N}$$

$$W_1 = 2860 \text{ N}$$

$$P = W m + c$$

$$\Rightarrow \boxed{88 = 2860 m + c} \quad \text{--- (1)}$$

Case-II

$$P_2 = W_2 m + C$$

$$\Rightarrow \boxed{129 = 4920m + C} \quad \text{--- (II)}$$

From eqⁿ (II) & (I) we get,

$$129 = 4920m + C$$

$$88 = 2860m + C$$

$$41 = 2060m$$

$$\Rightarrow m = \frac{41}{2060}$$

$$\Rightarrow \boxed{m = 0.01}$$

m value putting in eqⁿ (I)

$$88 = 2860m + C$$

$$\Rightarrow 88 = 2860 \times 0.01 + C$$

$$\Rightarrow 88 = 28.6 + C$$

$$\Rightarrow 88 - 28.6 = C$$

$$\Rightarrow C = 59.4$$

Establish law of machine

$$\boxed{P = Wm + C}$$

$$\Rightarrow \boxed{P = 0.01W + 59.4}$$

$$P_3 = ?$$

$$W_3 = 6880 \text{ N}$$

$$\Rightarrow P_3 = mW_3 + c$$

$$\Rightarrow P_3 = (0.01) 6880 + 59.4$$

$$\Rightarrow P_3 = 128.2$$

$$P_4 = 166 \text{ N}$$

$$W_4 = ?$$

$$W_4 =$$

$$P_4 = mW_4 + c$$

$$\Rightarrow 166 = (0.01) W_4 + 59.4$$

$$\Rightarrow 166 - 59.4 = 0.01 W_4$$

$$\Rightarrow 106.6 = 0.01 W_4$$

$$\Rightarrow W_4 = \frac{106.6}{0.01}$$

$$\Rightarrow \boxed{W_4 = 10660 \text{ N}}$$

Max^m efficiency

$$\eta_{\max} = \frac{1}{m \times VR}$$

$$\Rightarrow \eta_{\max} = \frac{1}{0.01 \times 65}$$

$$\Rightarrow \boxed{\eta_{\max} = 1.53}$$

Max^m MA

$$\text{Max^m MA} = \frac{1}{m}$$

$$= \frac{1}{0.01}$$

$$\boxed{\text{Max^m MA} = 100}$$

⑧

In effort of machine effort 88 N lift a load of 2250 N and an effort 125.4 N lift a load of 4920 N.

- (i) Establish law of machine.
- (ii) Calculate the effort required to lift a load of 6880 N.
- (iii) Find the load of effort of 186 N.
- (iv) What is the max^m efficiency if VR of machine 65.
- (v) Find max^m MA?

Given data

$$P_1 = 88 \text{ N}$$

$$W_1 = 2250 \text{ N}$$

$$P_2 = 125.4 \text{ N}$$

$$W_2 = 4920 \text{ N}$$

Establish law of machine

$$P = mw + c$$

$$\Rightarrow \boxed{P = (0.01)w + 65.5}$$

(ii) $P_3 = ?$

$$W_3 = 6880 \text{ N}$$

$$P_3 = W_3 m + c$$

$$\Rightarrow P_3 = 6880 \times 0.01 + 65.5$$

$$\Rightarrow \boxed{P_3 = 134.3}$$

(iii) $P_4 = 186 \text{ N}$

$$W_4 = ?$$

$$P_4 = W_4 m + c$$

$$\Rightarrow 186 = W_4 (0.01) + 65.5$$

$$\Rightarrow 186 - 65.5 = 0.01 W_4$$

$$\Rightarrow 120.5 = 0.01 W_4$$

$$\Rightarrow W_4 = \frac{120.5}{0.01}$$

$$\Rightarrow W_4 = 12050$$

(iv) Max^m efficiency

$$\eta_{\text{max}} = \frac{1}{m \times VR} \quad VR = 65$$

$$= \frac{1}{0.01 \times 65}$$

$$\boxed{\eta_{\text{max}} = 1.53}$$

$$iv) \text{Max}^m \text{MA} = \frac{1}{m}$$

$$= \frac{1}{0.01}$$

$$\boxed{\text{Max}^m \text{MA} = 100}$$

9) The law of machine is $P = 0.02W + 49$. Determine the maximum MA, max^m efficiency, if velocity ratio 75.

Given data

$$P = 0.02W + 49$$

$$m = 0.02$$

$$C = 49$$

$$VR = 75$$

① max^m MA

$$\begin{aligned} \text{max}^m \text{MA} &= \frac{1}{m} \\ &= \frac{1}{0.02} \end{aligned}$$

$$= 50$$

② Max^m (η)

$$\begin{aligned} \eta_{\text{max}^m} &= \frac{1}{m \times VR} \\ &= \frac{1}{0.02 \times 75} \end{aligned}$$

$$\boxed{\eta_{\text{max}^m} = 0.66 \text{ or } 66\%}$$

The law of machine is $P = \frac{W}{30} + 15$ the VR of the machine is 100.

- (i) Find max^m MA
- (ii) Find max^m efficiency of machine.
- (iii) Determine effort if load is 1500N.
- (iv) Determine loss of friction.

Given data

$$\begin{aligned}P &= \frac{W}{30} + 15 \\&= \frac{1}{30} W + 15 \\&= 0.03W + 15\end{aligned}$$

$$VR = 100$$

$$m = 0.03$$

$$C = 15$$

(i)

Max^m MA

$$\begin{aligned}\text{Max}^m \text{MA} &= \frac{1}{m} \\&= \frac{1}{0.03}\end{aligned}$$

$$\boxed{\text{Max}^m \text{MA} = 33.33}$$

(ii)

max^m efficiency

$$\eta_{\text{max}^m} = \frac{1}{m \times VR}$$

$$= \frac{1}{0.03 \times 100} = 0.33$$

(iii) $P = ?$

$$W = 1500 \text{ N}$$

$$P = mw + c$$

$$= (0.03) 1500 + 0.15$$

$$\boxed{P = 60 \text{ N}}$$

(iv) Loss of friction

$$P_{\text{friction}} = P - \frac{W}{VR}$$

$$= 60 - \frac{1500}{100}$$

$$= 60 - 15$$

$$\boxed{P_{\text{friction}} = 45}$$

- ⑬ In a simple wheel and axle, radius of effort wheel is 240 mm and the axle is 40 mm. Determine the efficiency, MA & VR if load is lifted 2940 N by effort 588 N.

→ Given data:-

Wheel

$$R = 240 \text{ mm}$$

$$D = 2R$$

$$= 480 \text{ mm}$$

Axle

$$r = 40 \text{ mm}$$

$$d = 2r$$

$$= 80 \text{ mm}$$

$$P = 588 \text{ N}$$

$$W = 2940 \text{ N}$$

$$MA = ?$$

$$VR = ?$$

$$\eta = ?$$

① MA:-

$$MA = \frac{W}{P} = \frac{2940}{588} = 5$$

② VR:-

$$VR = \frac{D}{d} = \frac{480}{80} = 6$$

③ Efficiency:-

$$\eta = \frac{MA}{VR} = \frac{5}{6} = 0.83 \text{ or } 83\%$$

(Ans)

- Q4) In a simple wheel and Axle, the radius of effort wheel 180mm & Axle 50mm. Determine the efficiency. MA & VR. If load of 1429N can be lifted by 292N.

Given data:-

Wheel

$$R = 180 \text{ mm}$$

$$D = 2R$$

$$= 2 \times 180$$

$$= 360 \text{ mm}$$

Axle

$$r = 50 \text{ mm}$$

$$d = 2r$$

$$= 2 \times 50$$

$$= 100 \text{ mm}$$

$$P = 292 \text{ N}$$

$$W = 1429 \text{ N}$$

$$MA = ?$$

$$VR = ?$$

$$\eta = ?$$

① MA :-

$$MA = \frac{W}{P} = \frac{1429}{292} = \boxed{4.8}$$

② VR :-

$$VR = \frac{D}{d} = \frac{360}{100} = \boxed{3.6}$$

③ efficiency :-

$$\eta = \frac{MA}{VR} = \frac{4.8}{3.6} = \boxed{1.33} \text{ or } 133\%$$

(Ans)

- ① In a differential wheel and axle diameter of effort wheel is 400mm. The radius of axle are 150mm & 100mm. The diameter of ropes is 10mm. Find the load lifted by the effort 196N if the efficiency is 75%.

Given data:-

$$D = 400\text{mm}$$

$$r_1 = 150\text{mm}$$

$$d_1 = 2r_1 = 2 \times 150 = 300\text{mm}$$

$$r_2 = 100\text{mm}$$

$$d_2 = 2r_2 = 2 \times 100 = 200\text{mm}$$

$$\text{Ropes} = 10\text{mm}$$

then

$$D = 400 + 10 = 410\text{mm}$$

$$d_1 = 300 + 10 = 310\text{mm}$$

$$d_2 = 200 + 10 = 210\text{mm}$$

$$W = ?$$

$$P = 196\text{N}$$

$$\eta = 75\%$$

$$= 0.75$$

③ Velocity ratio

$$VR = \frac{2D}{d_1 - d_2}$$

$$= \frac{2 \times 410}{310 - 210}$$

$$VR = 8.2$$

② Mechanical Advantage:-

$$MA = \frac{W}{P} = \frac{W}{196}$$

$$MA = \frac{W}{196}$$

③ Efficiency:-

$$\eta = \frac{MA}{VR} = \frac{\left(\frac{W}{196}\right)}{8.2}$$

$$> 0.75 = \frac{W}{196 \times 8.2}$$

$$\rightarrow 0.75 = \frac{W}{1607.2} \rightarrow W = \frac{0.75 \times 1607.2}{1} = 1205.4\text{N}$$

Q. In a differential wheel and axle the diameter of wheel is 200mm and the diameter of axle are 50mm and 40mm. If 800N load lifted by the effort applied 40N. Find the efficiency of machine and ^{loss of} effort or friction.

Given data:-

$$D = 200\text{mm}$$

$$d_1 = 50\text{mm}$$

$$d_2 = 40\text{mm}$$

$$W = 800\text{N}$$

$$P = 40\text{N}$$

$$\eta = ?$$

(i) Velocity ratio

$$VR = \frac{2D}{d_1 - d_2} = \frac{2 \times 200}{50 - 40}$$
$$= \frac{2 \times 200}{10} = 40$$

$$\Rightarrow \boxed{VR = 40}$$

$$(ii) MA = \frac{W}{P} = \frac{800}{40}$$

$$\Rightarrow \boxed{MA = 20}$$

$$(iii) \eta = \frac{MA}{VR} = \frac{20}{40} \Rightarrow \boxed{\eta = 0.5 \text{ or } 50\%}$$

(iv) Loss of effort in friction

$$P_f = P \cdot \frac{W}{VR}$$

$$\Rightarrow P_f = 40 \cdot \frac{800}{40}$$

$$\Rightarrow \boxed{P_f = 20}$$

- ①7 V.V.T
 In a single purchase crab length of Handle = 120mm.
gear ratio = 6 ($\frac{T_2}{T_1}$)
 Diameter of the drum = 50mm
 It lifted load 900N by the
 applying of effort 100N. Calculate efficiency of the machine.

Given data:-

$$R = 120 \text{ mm}$$

$$\eta = ?$$

$$\frac{T_2}{T_1} = 6 \text{ (Gear ratio)}$$

$$d = 50 \text{ mm}$$

$$W = 900 \text{ N}$$

$$P = 100 \text{ N}$$

$$D = 2R = 2 \times 120 = 240 \text{ mm}$$

> Velocity Ratio :-

$$VR = \frac{DT_2}{dT_1} = \left(\frac{D}{d}\right) \left(\frac{T_2}{T_1}\right)$$

$$= \frac{240}{50} \times 6$$

$$= 4.8 \times 6 = 28.8$$

> Mechanical Advantage :-

$$MA = \frac{W}{P}$$

$$= \frac{900}{100} = 9$$

> Efficiency :-

$$\eta = \frac{MA}{VR} = \frac{9}{28.8}$$

$$= 0.31 = 31\%$$

- 18) A double purchase crab has the following dimension
 length of the handle = 120cm.
 Radius of the load drum = 25cm.
 load lifted = 600N.
 No. of teeth on pinion gear = 30 & 40.
 No. of teeth on spur gear = 60 & 80.
 efficiency of the machine = 60%. Find the effort required.

Given data:-

$$R = 120\text{cm}$$

$$D = 2R = 2 \times 120 = 240\text{cm}$$

$$r = 25\text{cm}$$

$$d = 2r = 2 \times 25 = 50\text{cm}$$

$$W = 600\text{N}$$

$$T_1 \& T_3 = 30 \& 40$$

$$\left. \begin{array}{l} T_1 = 30 \\ T_3 = 40 \end{array} \right\} \text{pinion}$$

$$\left. \begin{array}{l} T_2 = 60 \\ T_4 = 80 \end{array} \right\} \text{spur}$$

$$\eta = 60\% = 0.6$$

$$P = ?$$

Velocity Ratio:-

$$VR = \frac{DT_2 T_4}{dT_1 T_3} = \frac{240 \times 60 \times 80}{50 \times 30 \times 40}$$

$$VR = 19.2$$

Efficiency:-

$$\eta = \frac{MA}{VR}$$

$$\Rightarrow MA = \eta VR$$

$$\Rightarrow MA = 0.6 \times 19.2$$

$$\Rightarrow \left(\frac{W}{P} \right) = 11.52$$

$$\Rightarrow W = 11.52P$$

$$\Rightarrow \frac{600}{11.52} = P$$

$$\Rightarrow P = 52.08\text{N}$$

- 19) In a simple purchase crab length of the handle = 160mm
 No. of pinion gear = 20, Spur gear = 160
 diameter of drum = 35mm
 load lifted = 1500N
 effort = 75N
 $\eta = ?$

Given data:

$$R = 160 \text{ mm}$$

$$D = 2R = 2 \times 160 = 320 \text{ mm}$$

$$T_1 = 20$$

$$T_2 = 160$$

$$d = 35 \text{ mm}$$

$$W = 1500 \text{ N}$$

$$P = 75 \text{ N}$$

$$\eta = ?$$

Velocity Ratio :-

$$VR = \frac{DT_2}{dT_1}$$

$$= \left(\frac{320}{35} \right) \times \frac{160}{20}$$

$$= \frac{320 \times 160}{35 \times 20}$$

$$VR = 73.14$$

Mechanical Advantage :-

$$MA = \frac{W}{P} = \frac{1500}{75}$$

$$MA = 20$$

Efficiency :-

$$\eta = \frac{MA}{VR} = \frac{20}{73.14}$$

$$\eta = 0.27$$

$$\eta = 0.27 \text{ or } 27\%$$

Q80 In a double purchase crab as the following dimensions length of the handle = 160 mm.

Radius of the drum = 35 mm

Load lifted = 800 N

No of teeth on pinion gear = 60 & 80

No of teeth on spur gear = 120 & 140

$$\eta = 75\% = 0.75$$

Find the effort?

Given data:-

$$R = 160 \text{ mm}$$

$$D = 2R = 260 \times 2 = 320 \text{ mm}$$

$$r = 35 \text{ mm}$$

$$d = 2r = 2 \times 35 = 70 \text{ mm}$$

$$W = 800 \text{ N}$$

$$T_1 \text{ \& } T_3 = 60 \text{ \& } 80$$

$$\left. \begin{array}{l} T_1 = 60 \\ T_3 = 80 \end{array} \right\} \text{ pinion}$$

$$\left. \begin{array}{l} T_2 = 120 \\ T_4 = 140 \end{array} \right\} \text{ spur}$$

$$\eta = 75\% = 0.75$$

$$P = ?$$

Velocity ratio:-

$$VR = \frac{DT_2 T_4}{dT_1 T_3} = \frac{320 \times 120 \times 140}{70 \times 60 \times 80} = 16$$

Efficiency:-

$$\eta = \frac{MA}{VR}$$

$$\Rightarrow MA = \eta VR$$

$$\Rightarrow MA = 0.75 \times 16$$

$$\Rightarrow \left(\frac{W}{P}\right) = 12$$

$$\Rightarrow W = 12P$$

$$\Rightarrow 800 = 12P$$

$$\Rightarrow P = \frac{800}{12} \Rightarrow \boxed{P = 66.66 \text{ N}}$$

Q1) In a worm & worm wheel, the worm wheel has 40 teeth. The diameter of worm pulley is 180 mm, the diameter of load drum is 120 mm with efficiency of 55%. the worm & worm wheel lifted load is 3800 N. Find effort.

→ Given data:-

$$T = 40$$

$$D = 180 \text{ mm}$$

$$d = 120 \text{ mm}$$

$$W = 3800 \text{ N}$$

$$P = ?$$

→ Velocity ratio:-

$$VR = \frac{DT}{d}$$

$$= \frac{180 \times 40}{120} \Rightarrow \boxed{VR = 60}$$

→ Efficiency:-

$$\eta = \frac{MA}{VR}$$

$$\Rightarrow \eta VR = MA$$

$$\Rightarrow 0.55 \times 60 = \frac{W}{P}$$

$$\Rightarrow \frac{W}{P} = 33$$

$$\Rightarrow \frac{W}{33} = P$$

$$\Rightarrow \frac{3800}{33} = P$$

$$\Rightarrow \boxed{P = 115.1 \text{ N}}$$

- Q. In a double threaded worm and worm wheel, the no. of teeth 80. The diameter of effort wheel is 300mm. & the diameter of load drum is 160mm. If efficiency of the machine is 45%. Determine (1) velocity ratio (2) Effort required to lift a load 120N.

→ Given data:-

$$n = 2$$

$$W = 120N$$

$$T = 80$$

$$D = 300mm$$

$$d = 160mm$$

$$\eta = 45\% = 0.45$$

$$VR = ?$$

$$P = ?$$

Velocity ratio:-

$$VR = \frac{DT}{nd} = \frac{300 \times 80}{2 \times 160} = 75$$

$$\Rightarrow [VR = 75]$$

Efficiency:-

$$\eta = \frac{MA}{VR}$$

$$\Rightarrow \eta VR = MA$$

$$\Rightarrow 0.45 \times 75 = \frac{W}{P}$$

$$\Rightarrow 33.75 = \frac{120}{P}$$

$$\Rightarrow 33.75P = 120$$

$$\Rightarrow P = \frac{120}{33.75}$$

$$\Rightarrow [P = 3.55N]$$

- Q3) The pitch of screw Jack is 7.5 mm. The length of the lever arm at the end of effort applied is 450 mm find the effort required to raise the load 500 N. If the efficiency is 45%.

→ Given data:-

$$P(\text{small}) = 7.5 \text{ mm (pitch)}$$

$$L = 450 \text{ mm}$$

$$P = ? (\text{Effort})$$

$$W = 500 \text{ N}$$

$$\eta = 45\% = 0.45$$

→ Velocity ratio:-

$$VR = \frac{2\pi L}{p(\text{small})}$$

$$= \frac{2 \times \frac{22}{7} \times 450}{7.5} \quad \left(\because \pi = \frac{22}{7} \right)$$

$$\boxed{VR = 377.14}$$

→ Efficiency:-

$$\eta = \frac{MA}{VR}$$

$$\Rightarrow MA = \eta VR$$

$$\Rightarrow \left(\frac{W}{P} \right) = 0.45 \times 377.14$$

$$\Rightarrow \frac{W}{P} = 169.7$$

$$\Rightarrow \frac{W}{169.7} = P$$

$$\Rightarrow \frac{500}{169.7} = P$$

$$\Rightarrow \boxed{P = 2.94 \text{ N}}$$

Q4) A screw Jack thread of pitch 5mm the effort is applied at the end of lever five hundred / 500mm long. Find the effort. If load is 10kN in the efficiency is 40%.

Given data:-

$$P(\text{small}) = 5\text{mm (Pitch)}$$

$$l = 500\text{mm}$$

$$P = ? \text{ (Effort)}$$

$$W = 10\text{KN} = 10 \times 1000 = 10000$$

$$\eta = 40\% \text{ or } 0.40 / 0.4$$

$$VR = \frac{2\pi l}{P(\text{small})}$$

$$\therefore \eta = \frac{2 \times \frac{22}{7} \times 10000}{1 \times 5}$$
$$= 12571.4$$

$$\text{efficiency } (\eta) = \frac{MA}{VR}$$

$$0.4 = \frac{MA}{12571.4}$$

$$\Rightarrow MA = 12571.4 \times 0.4$$

$$\Rightarrow \frac{W}{P} = 5028.56$$

$$\Rightarrow \frac{10000}{P} = 5028.56$$

$$\Rightarrow P = \frac{10000}{5028.56}$$

$$\Rightarrow \boxed{P = 1.98\text{N}}$$